

Computational Efficiency Improvements in Topography Optimization using Reanalysis

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ABSTRACT

To improve fuel economy, there is a trend in automotive industry to use light weight, high strength materials. Automotive body structures are composed of several panels which must be downsized to reduce weight. Because this affects NVH (Noise, Vibration and Harshness) performance, engineers are challenged to recover the lost panel stiffness from down-gaging in order to improve the structure borne noise transmitted through the lightweight panels in the frequency range of 100-300 Hz where most of the booming and low medium frequency noise occurs. The loss in performance can be recovered by optimized panel geometry using beading or damping treatment. Topography optimization is a special class of shape optimization for changing sheet metal shapes by introducing beads. A large number of design variables can be handled and the process is easy to setup in commercial codes. However, optimization methods are computationally intensive because of repeated full-order analyses. In this paper, we propose using a Modified Combined Approximations (MCA) reanalysis method integrated with topography optimization to improve the computational efficiency. Using a tire tub example, we will demonstrate that the topography optimization can become more efficient if integrated with reanalysis.

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INTRODUCTION

Designs with minimum weight are sought in all industries primarily for cost reduction reasons. For automobile manufacturers, because of stringent Corporate Average Fuel Economy (CAFÉ) requirements, meeting fuel economy targets demands a minimum weight solution. Lighter vehicles benefit from a smaller engine, better fuel economy and lower emissions. However most of the time, NVH performance conflicts with a lightweight design as lighter body panels are vulnerable to increased airborne and structural noise levels in the mid and high frequency range.

To meet NVH functional requirements and at the same time have a lightweight design, we use optimization. For panels with minimum weight and increased stiffness for good NVH performance, a typical optimization process starts with gage sizing and ends up with bead optimization. The latter introduces beading to increase lost stiffness from gage reduction. Finite element analysis has been a well-established numerical tool which is integrated with optimizers to aid in designing optimal products. To achieve an optimized design, several finite element analysis runs are performed which can be very costly. Each optimization iteration involves a sensitivity analysis to determine the most efficient way to move towards the optimized design by properly updating the design variables. This results in a modified design for which a costly eigenvalue analysis must be performed to assess its NVH performance (free or forced vibration response). In this paper, we propose using a reanalysis technique to reduce the optimization time for NVH problems involving finite element analysis.

Reanalysis methods estimate the eigenvalues and eigenvectors of a modified design without performing a full-order eigenvalue analysis. Instead, a reduced-order model is formed using an appropriate basis and then solved for its eigenvalues and eigenvectors. The eigenvectors of the modified design are finally obtained by multiplying the basis with the eigenvectors of the reduced system. The need for efficient and accurate reanalysis techniques has increased as analysis models have become large and complex.

In general, reanalysis methods are based on local and global approximations. Local approximations are efficient but are effective only for small structural perturbations. On the other hand, global approximations are computationally expensive and preferable for larger structural perturbations. Reanalysis methods were first applied on linear static problems in the early 1970s [1]. The combined approximations (CA) method developed by Kirsch [2] combines the strength of both local and global approximations and is quite accurate even for larger modifications. The CA method uses a combination of binomial series or a Neumann expansion (local approximation) and a reduced basis (global approximation). CA was originally applied to linear static problems and was later extended to linear dynamics problems [3]. A modified combined approximation (MCA) method was introduced in [4] for reanalysis of dynamic problems with many dominant mode shapes.

Recently, Bogomolni [5] has shown that reanalysis using combined approximations (CA) can be successfully applied to topology optimization of free vibration problems. However to the best of our knowledge, reanalysis has not been used yet to improve the computational effort of topography optimization.

In this paper, we present a methodology to reduce the computational effort associated with repeated eigenvalue analyses in topography optimization of vibratory problems. We propose integrating the MCA reanalysis method with commercial optimization software such as MSC/NASTRAN. We will demonstrate the efficiency of the proposed approach using a tire tub example.

INTRODUCTION TO REANALYSIS AND REVIEW OF EXISTING REANALYSIS METHODS

Reanalysis methods use the eigenvectors of the original (baseline) model to form a basis which is then used to estimate the eigenvectors of a modified model without performing a costly full-order eigenvalue analysis.

The eigenvalue problem for an undamped system with stiffness $\mathbf{K}_{n \times n}$ and mass $\mathbf{M}_{n \times n}$ is given by

$$\mathbf{K}_{n \times n} \Phi_{n \times m} = \mathbf{M}_{n \times n} \Phi_{n \times m} \Lambda_{m \times m} \quad (1)$$

where n represents the number of degrees of freedom (DOF) and m represents the number of kept modes. If a perturbation is applied on the existing structure, the modified system matrices are

$$\mathbf{K}_p = \mathbf{K}_0 + \Delta \mathbf{K}, \quad \mathbf{M}_p = \mathbf{M}_0 + \Delta \mathbf{M} \quad (2)$$

where $\mathbf{K}_0$, $\mathbf{M}_0$ and $\mathbf{K}_p$, $\mathbf{M}_p$ are the stiffness and mass matrices of the original (baseline) and modified systems, respectively.

In reanalysis, a modal model is formed using the following reduced matrices

$$\mathbf{K}_r = \Phi_0^T \mathbf{K}_p \Phi_0, \quad \mathbf{M}_r = \Phi_0^T \mathbf{M}_p \Phi_0 \quad (3)$$

where Φ_0 are the eigenvectors of the original (baseline) design, and the following eigenvalue problem is solved

$$\mathbf{K}_r \Theta = \Lambda \mathbf{M}_r \Theta \quad (4)$$

The reduced eigenvector Θ is then projected back to the physical domain to obtain the eigenvectors Φ_p of the modified design as

$$\Phi_p = \Phi_0 \Theta \quad (5)$$

Next, we briefly review the Combined Approximations (CA) and the Modified Combined Approximations (MCA) reanalysis methods.

Combined Approximations (CA)

For the CA method [6, 7, 8, 9, 10, 11, 12], the eigenvectors at a new design are approximated using a linear combination of basis vectors. A subspace basis is formed as

$$\mathbf{R}_{n \times s} = [\mathbf{R}_{n \times m}^1 \quad \mathbf{R}_{n \times m}^2 \quad \dots \quad \mathbf{R}_{n \times m}^s] \quad (6)$$

where s is the number of basis vectors normally in the range of 3-6 depending on accuracy and efficiency.

For a modified design, the stiffness and mass matrices are given by

$$\mathbf{K}_p = \mathbf{K}_0 + \Delta \mathbf{K}, \quad \mathbf{M}_p = \mathbf{M}_0 + \Delta \mathbf{M} \quad (7)$$

where $\mathbf{K}_0$ and $\mathbf{M}_0$ are the stiffness and mass matrices of the baseline (original) design and $\Delta \mathbf{K}$ and $\Delta \mathbf{M}$ represent large perturbations. The basis vectors in Equation (6) are generated using Equations (8) and (9)

$$\mathbf{R}_{n \times m}^1 = \mathbf{K}_0^{-1} \mathbf{M}_p \Phi_0 \quad (8)$$

$$\mathbf{R}_{n \times m}^j = -\mathbf{K}_0^{-1} \Delta \mathbf{K} \mathbf{R}_{n \times m}^{j-1}, j = 2, 3, \dots, s \quad (9)$$

where Φ_0 are the eigenvectors of the baseline design. For the perturbed (new) design, a reduced model is obtained with the following stiffness and mass matrices

$$\mathbf{K}_{n \times n}^R = \mathbf{R}_{n \times m}^T \mathbf{K}_p \mathbf{R}_{n \times m}, \quad \mathbf{M}_{n \times n}^R = \mathbf{R}_{n \times m}^T \mathbf{M}_p \mathbf{R}_{n \times m} \quad (10)$$

and an eigenvalue problem is solved using the $\mathbf{K}_{n \times n}^R$ and $\mathbf{M}_{n \times n}^R$ expressions of Equation (10) to calculate the eigenvectors Θ of the reduced model (Equation 4). These eigenvectors are then projected back to the physical domain to obtain the following approximate eigenvectors Φ_p of the new design as

$$\Phi_p = \mathbf{R}_{n \times s} \Theta \quad (11)$$

Because the basis vectors are linearly dependent, Gram Schmidt orthogonalization must be performed to avoid numerical errors. The CA method is efficient for problems where the number of desired modes is low. In such a case, the computational effort of stiffness matrix decomposition (Equations 8 and 9) dominates.

The CA method has the following three main advantages:

1. It only requires a single matrix decomposition of stiffness matrix $\mathbf{K}_0$ to calculate the subspace basis $\mathbf{R}$,
2. It is accurate because the basis is updated for every new design, and
3. The eigenvectors of a new design are efficiently approximated using Equation (11) where the eigenvectors Θ correspond to a much smaller eigen-problem.

Modified Combined Approximations (MCA)

The MCA method is an extension of the CA method. Zhang [4] modified the CA method such that the matrix $\mathbf{K}_0$ is inverted in the former and matrix $\mathbf{K}_p$ is inverted in the latter. The columns of the subspace basis are obtained from the following recursive equations

$$\mathbf{T}_{n \times m}^1 = \mathbf{K}_p^{-1} \mathbf{M}_p \Phi_0 \quad (12)$$

$$\mathbf{T}_{n \times m}^j = \mathbf{K}_p^{-1} \mathbf{M}_p \mathbf{T}_{n \times m}^{j-1}, j = 2, 3, \dots, s \quad (13)$$

where $\mathbf{K}_p$ and $\mathbf{M}_p$ are the stiffness and mass matrices of the modified design. A subspace basis is then defined as

$$\mathbf{T} = [\Phi_0 \quad \mathbf{T}_1 \quad \dots \quad \mathbf{T}_s] \quad (14)$$

where s is equal to 1 or 2. Gram Schmidt orthogonalization is again performed to avoid numerical errors from linear dependency. A reduced model is defined using the following reduced stiffness and mass matrices

$$\mathbf{K}_{m \times m}^R = \mathbf{T}_{n \times m}^T \mathbf{K}_{n \times n}^p \mathbf{T}_{n \times m}, \mathbf{M}_{m \times m}^R = \mathbf{T}_{n \times m}^T \mathbf{M}_{n \times n}^p \mathbf{T}_{n \times m} \quad (15)$$

and the eigenvalue problem $\mathbf{K}^R \Theta = \lambda_0 \mathbf{M}^R \Theta$ is solved to obtain the eigenvectors Θ which are then used to approximate the eigenvectors of the modified design as

$$\Phi_p = \mathbf{R}_{n \times s} \Theta. \quad (16)$$

Although the MCA method has a slightly higher matrix decomposition cost as the matrix $\mathbf{K}_p$ must be decomposed for each new design, the overall computational effort of MCA is lower than that of CA because the forward-backwards cost in solving Equations (12) and (13) is much lower compared to CA. For this reason, MCA is computationally more efficient than CA for large size models with many modes.

STRUCTURAL OPTIMIZATION

Structural optimization can be classified as size, shape or topology optimization (Figure 1). In size optimization the mesh does not change (top panel of Figure 1). The design variables are usually thickness, beam cross-sectional areas, material properties, etc. Topometry optimization is a special class of size optimization.

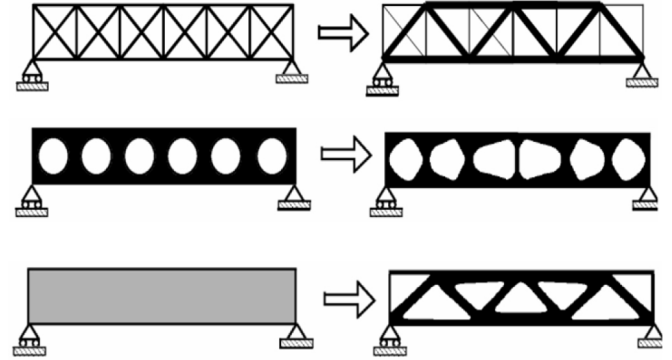


Figure 1. Structural optimization disciplines: size (top panel), shape (middle panel), and topology (low panel)

In shape optimization, we allow changes of only characteristic features such as geometry of holes (middle panel of Figure 1) without changing their number, for example. For that, geometry parameterization is very useful. Topography is a special class of shape optimization. Topology optimization [13, 14] determines the optimal material distribution or layout within a defined design domain. This generates distinct geometrical features within the domain as is evident from the last panel of Figure 1.

Topography Optimization

Topography optimization is special class of shape optimization which may result in a weight neutral design meeting all functional requirements (constraints) using an optimal pattern layout (beads). The latter results in a higher stiffness compared to a flat panel, for example. For a simply supported curved panel, the bending stiffness is given as

$$\mathbf{B} = \frac{Et^3}{12(1-\nu^2)} \times \frac{\mathbf{K}_c}{\mathbf{K}_f} \quad (17)$$

where $\mathbf{K}_c$ and $\mathbf{K}_f$ represent the stiffness of the curved and flat panels with the same material properties (e.g., Young modulus E and Poisson ratio ν) and thickness t . The bending stiffness of a curved panel is always higher than that of a flat panel. Therefore, addition of bead patterns stiffens the panels and some of the lost performance due to down gaging can be recovered.

In topography optimization, the grids of the design domain are perturbed in a direction normal to the part as shown in Figure 2. The perturbation vectors are automatically generated and are defined as [15]

$$X_i = X_{oi} + \sum_j DV_j X_{ji} \quad (18a)$$