

Low-Order Contact Load Distribution Model for Ball Nut Assemblies

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ABSTRACT

Ball nut assemblies (BNAs) are used in a variety of applications, e.g., automotive, aerospace and manufacturing, for converting rotary motion to linear motion (or vice versa). In these application areas, accurate characterization of the dynamics of BNAs using low-order models is very useful for performance simulation and analyses. Existing low-order contact load models of BNAs are inadequate, partly because they only consider the axial deformations of the screw and nut. This paper presents a low-order load distribution model for BNAs which considers the axial, torsional and lateral deformations of the screw and nut. The screw and nut are modeled as finite element beams, while Hertzian Contact Theory is used to model the contact condition between the balls and raceways of the screw and nut. The interactions between the forces and displacements of the screw and nut and those at the ball-raceway contact points are established using transformation matrices. The resulting set of linear and nonlinear equations are solved iteratively using the Newton-Raphson method to obtain the contact load distribution of balls under static equilibrium. A simulation-based case study is presented to validate and highlight the benefits of the proposed model. The proposed low-order model is particularly relevant for BNAs used in automotive applications, where off-axis loads are common.

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INTRODUCTION

A ball nut assembly (BNA) consists of a ball screw and nut. It is used to convert rotary motion to linear motion (or vice versa) and it has found applications in the automotive industry (e.g., [1,2,3,4,5]) since GM's recirculating ball pitman arm gear of the early 1940s [5]. BNAs are also used in numerous manufacturing and aerospace applications. The dynamics of BNAs is very complex yet critical to the many applications that make use of them. Therefore a lot of research effort has been poured into developing models that shed some light into various dynamic phenomena related to BNAs. For instance, Lin et al. [6] and Wei et al. [7], by studying the kinematics of the balls established that the balls always undergo micro-scale slip during motion transmission. Cuttino et al. [8, 9] used a finite element (FE) model to derive a relationship between the torque generated as the ball rotates in the raceway and the angle of rotation, and they successfully predicted the micro-scale hysteretic behavior observed in the torque and displacement transmission of the preloaded nut interface.

Low-order models, i.e., models with few degrees of freedom, are particularly desirable for characterizing mechanical systems like BNAs. This is because they have low computational cost and hence can easily be used for dynamic simulations. This paper particularly addresses the development of low-order models to study the contact load distribution of the balls in BNAs. In this regard, Mei et al. [10] have developed an FE model for predicting the load distribution (i.e., the normal forces) of balls in BNAs. Their model considers the axial

deformations of the screw/nut as well as the geometric errors of the balls and raceways, by assuming that the elongation of the screw/nut under axial loads should compensate for the Hertzian contact deformations and the geometric errors of the balls and raceways. Recent extensions of their work have incorporated the effects of variable contact angles [11] and provided limited experimental validation using a planar cross section of a ball screw [12]. A critical problem with their model [10] and its extensions [11, 12] is that they assume that axial loads give rise to only axial deformations. However, Slocum [13] and Okwudire et al. [14,15,16] have proven analytically and experimentally that axial loads do in fact produce lateral deformations in BNAs. Moreover, for automotive applications (e.g. [1,2,3,4,5]), where BNAs routinely experience off-axis loads, the consideration of lateral and torsional dynamics is of particular importance in predicting contact load distribution.

Therefore, building on parts of the work by Okwudire et al. [14,15,16], this paper proposes a low-order load distribution model for BNAs which considers the axial, torsional and lateral deformations of the screw and nut. A simple case study is presented to illustrate the significant differences between the proposed model and Mei et al.'s model [10], both benchmarked against an elaborate high-order Solidworks® FE model. Conclusions and future work, which aims to incorporate geometric errors into the proposed low-order model, are discussed.

PROPOSED CONTACT LOAD DISTRIBUTION MODEL

Contact Load for a Single Ball and its Transformation to a Global Coordinate System

Consider the i^{th} ball of a BNA, sandwiched between the screw and nut as shown in **Figure 1**. Let $\delta_{S(i)}$ and $\delta_{N(i)}$ represent the deformation of the ball at the ball-to-screw interface and ball-to-nut contact points, respectively, caused by the contact force P_i . The deformations and contact force can be related by Hertzian Contact Theory as

$$\begin{aligned}\delta_{S(i)} &= C_S P_i^{2/3} \\ \delta_{N(i)} &= C_N P_i^{2/3}\end{aligned}\quad (1)$$

where C_S and C_N are the Hertzian constants of the screw and nut, respectively, determined by the material properties and geometries the ball and screw/nut. The reaction force P_i , together with all the other reaction forces $P_1, P_2, \dots, P_{N_{\text{ball}}}$ from the N_{ball} loaded balls in the BNA, cause the screw and nut to deform elastically.

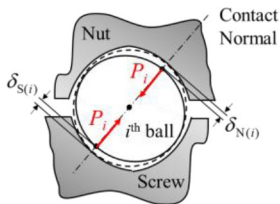


Figure 1. Ball-screw/nut interface showing the elastic deformations of a ball along its contact normal caused by contact load.

To determine the elastic deformations of the screw and nut, the contact force P_i must be transformed from a local coordinate system defined along the contact normal of each ball i to a global coordinate system defined for the BNA [14,15,16]. The procedure for determining this local-to-global coordinate system transformation is exactly the same for the screw and nut. Therefore, for the sake of brevity, only the transformations for the screw are highlighted in this paper. Details on the transformations for the nut can be found in [14,15,16].

Figure 2(a) shows the x, y, z global coordinate system for the BNA, as well as the local coordinate system (CS1 $\triangleq x_1, y_1, z_1$) whose z -axis lies along the contact normal for ball i . Angle α shown in the figure is the lead angle of the screw, given as

$$\alpha = \tan^{-1} \left(\frac{l}{\pi d} \right) \quad (2)$$

where l is the lead and d is the pitch diameter of the screw; angle β is the contact angle at the ball contact point. Using these two angles, the ball-screw contact surface can be represented as a double-inclined plane as depicted in **Figure 2(b)**. An intermediary coordinate system (CS2 $\triangleq x_2, y_2, z_2$) is defined such that its z -axis points along the axis

of the screw and its y -axis lies along the radius of the screw. The transformation between z_1 and CS2 (i.e., T_{2-z1}) is obtained using current frame rotation operators about the y and x axes, and then multiplying them by a unit vector in the z_1 direction, as

$$\begin{aligned}T_{2-z1} &= T_{2-1} \cdot [0 \ 0 \ 1]^T \\ T_{2-1} &= \text{rot}_y(-\alpha) \cdot \text{rot}_x(\beta)\end{aligned}\quad (3)$$

where rot represents a current-frame rotational operator about the axis specified by its subscript [14].

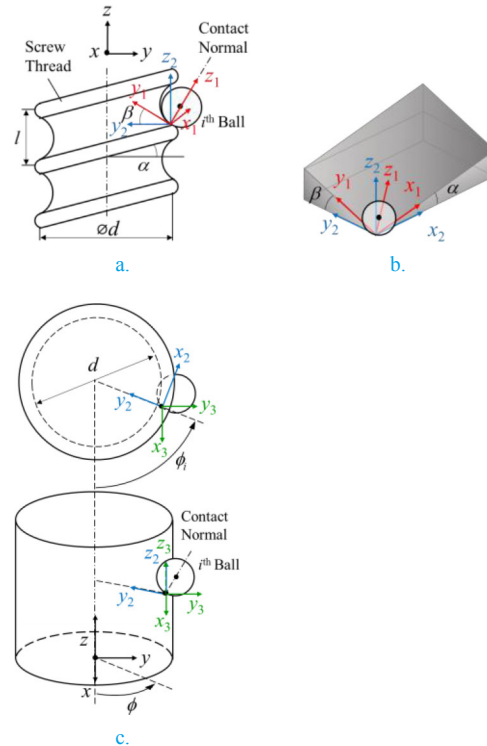


Figure 2. (a) Orientation of contact normal determined by angles α and β , and (b) double-inclined plane representation of contact surface showing coordinate systems CS1 and CS2; (c) global view of ball in BNA showing global coordinate system and coordinate systems CS2 and CS3.

Figure 2(c) shows the global view of a ball in a BNA. Another intermediary coordinate system, CS3 $\triangleq x_3, y_3, z_3$, is defined with its origin at the contact point and its x, y and z axes are parallel with those of the global coordinate system. The azimuth angle ϕ determines the position of the i^{th} ball on the circumference of the screw. Accordingly, the transformation $T_{3-2(i)}$ from CS3 to CS2 simply involves a rotation by the amount $\phi_i + \pi/2$ about the z axis; i.e.,

$$T_{3-2(i)} = \text{rot}_z \left(\phi_i + \frac{\pi}{2} \right) \quad (4)$$

The transformation between the contact normal direction z_1 and CS3 is therefore obtained as follows

$$T_{3-z1(i)} = T_{3-2(i)} \cdot T_{2-z1}$$