

# A Robust Stability Control System for a Hybrid Electric Vehicle Equipped with Electric Rear Axle Drive

Jose Velazquez Alcantar and Farhad Assadian  
University of California

## ABSTRACT

Optimizing/maximizing regen braking in a hybrid electric vehicle (HEV) is one of the key features for increasing fuel economy. However, it is known [1] that maximizing regen braking by braking the rear axle on a low friction surface results in compromising vehicle stability even in a vehicle which is equipped with an ESP (Enhanced Stability Program). In this paper, we develop a strategy to maximize regen braking without compromising vehicle stability. A yaw rate stability control system is designed for a hybrid electric vehicle with electric rear axle drive (ERAD) and a “hang on” center coupling device which can couple the front and rear axles for AWD capabilities. Nonlinear models of the ERAD drivetrain and vehicle are presented using bond graphs while a high fidelity model of the center coupling device is used for simulation. A robust yaw rate stability controller, utilizing Youla parameterization, is proposed which uses the center coupling device to distribute regen braking torque from the rear axle to the front axle while using the maximum amount of regen braking possible to help improve fuel efficiency. It is shown through simulation studies that the proposed controller stabilizes a vehicle cornering whilst braking on a low friction surface while using the maximum amount of regen braking possible. The controller is also shown to be robust to time delays in the system. The resulting control system helps improve overall fuel efficiency during all braking maneuvers by maximizing the amount of regen braking available from the system at any given time.

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## INTRODUCTION

Hybrid Electric Vehicles (HEVs) have the capability to achieve better fuel efficiency as compared to their conventional counterparts partly due to the fact that HEVs can recuperate energy during braking which would otherwise be lost as heat dissipation in conventional friction brakes. Currently, several HEV drivetrain layouts are readily available in the market such as series, series-parallel, parallel, etc. In an effort to provide AWD capabilities and improve performance, several manufacturers have begun to use the Electric Rear Axle Drive (ERAD) drivetrain layout. HEV's equipped with ERAD place an electric machine on the rear axle of the vehicle. By doing so, the vehicle can achieve AWD capabilities and improve performance. However, during braking, ERAD has the capability of destabilizing the vehicle because the electric machine brakes the rear axle. Thus, in an effort to reduce the chance of destabilizing the vehicle, HEV's with ERAD often do not take full advantage of the regenerative braking capabilities of the electric machine. Therefore, designing a control system for HEV's with ERAD that can maintain vehicle stability and maximize regenerative braking can improve the overall fuel efficiency of the vehicle in all driving situations.

A literature review on HEVs with ERAD shows that researchers have approached the problem in several different ways. For example, it has been shown that the vehicle can be stabilized by a yaw rate controller using active front/rear steering in coordination with torque control of the rear axle[2]. Traditional torque delivery control of the rear axle

has also been explored with high fidelity ADAMS vehicle models and fuzzy logic[3]. Moreover, several works have also analyzed and developed models of actuators such as limited slip coupling devices used for mechanical AWD coupling for ERAD vehicles[4]. This work approaches the problem by controlling a mechanical AWD coupling device to stabilize the vehicle and allow the ERAD to take full advantage of the regenerative braking capabilities of the electric machine. In this way, fuel efficiency is maximized and the vehicle remains stable during all braking maneuvers.

The paper is broken up as follows: section two introduces the development of the drivetrain and vehicle models used as a testbed for the control system. Section three develops a robust yaw rate stabilizing control strategy with the objective of stabilizing the vehicle and utilizing the maximum amount of regenerative torque available from the motor. Section four presents simulation results in which the robust yaw rate controller is compared to an uncontrolled vehicle undergoing a braking-whilst-turning maneuver on a low friction surface. Finally, conclusions are presented in section five.

## SYSTEM MODELING

Models of sufficient detail are required to analyze the system dynamics. Bond graphs allow easy and intuitive modeling of multi energy domain systems; therefore, all system modeled in this paper are modeled using Bond Graphs [5]. In this section, a model of an

HEV drivetrain with an electrified rear axle is first developed. A nonlinear four wheel planar vehicle is then introduced and integrated with the drivetrain model.

### Drivetrain Model

The drivetrain layout of the HEV analyzed in this paper is shown in Figure 1. An internal combustion engine drives the front axle via a controllable clutch which connects a conventional planetary gearbox to the front differential. The rear axle of the vehicle is driven by an electric motor. The front and rear driveshafts can be coupled via a “hang on” center coupling device which can be controlled to transfer torque from one driveshaft to the other when there is a speed difference or alternatively, lock the two driveshafts together. The focus of this study is full EV mode operation, wherein the engine clutch is open. Thus, the engine is not included in the model.

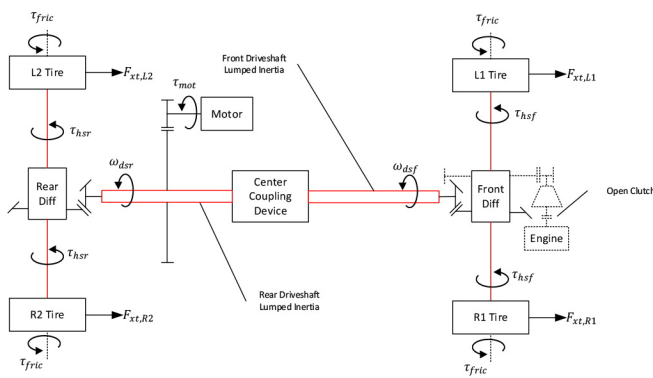


Figure 1. Schematic of a HEV drivetrain with an ERAD

The inertia of the transmission and front differential are lumped into the inertia of the front driveshaft. Similarly, the inertia of the electric motor and rear differential are lumped into the inertia of the rear driveshaft. The front and rear halfshafts are assumed to be of the same stiffness and are each lumped into an equivalent front and rear halfshaft. It has been shown that the halfshaft compliances are actually in series and not in parallel [6]. Only in the case when the two halfshafts are of the same stiffness do they act like two springs in parallel when the equivalent stiffness is computed numerically. Thus, the equivalent compliance is calculated in series as

$$C_{eq} = \frac{1}{K_{eq}} = \frac{1}{K_{left}} + \frac{1}{K_{right}} \quad (1)$$

For this study, it is assumed that the dynamics of the electric motor driving the rear axle are much faster than the drivetrain dynamics and are neglected. Thus, the dynamics of the drivetrain model arise from the items highlighted in red. It should be noted that each wheel also contributes a state to the model.

The electric motor regen torque limits are characterized as a function of forward speed and are plotted in Figure 2. The regen torque capacity curve represents the maximum amount of regen braking torque at the wheel that the motor can deliver at a given forward speed. Finally, the bond graph representation of the model is given in Figure 3.

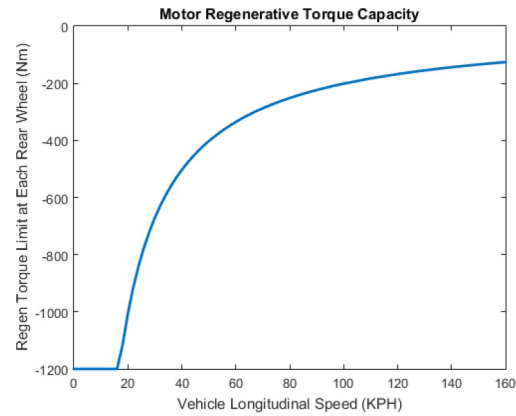


Figure 2. Electric motor regen torque capacity

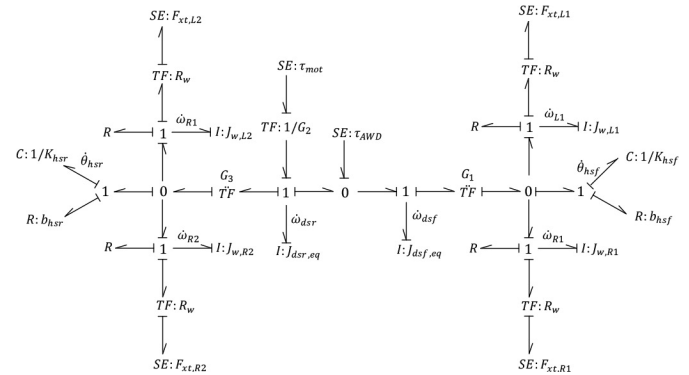


Figure 3. Bond graph model of HEV with an ERAD

In the bond graph model above, the gear ratios  $G_1$  and  $G_3$  are defined as

$$G_1 = G_3 = \frac{2}{G_{final\ drive}} \quad (2)$$

Meanwhile,  $G_2$  is the motor gear down ratio. The resistive elements on each wheel inertia represent the friction brake torques on each wheel. The resulting model outputs eight state equations given in Appendix I. The states and inputs of the model are summarized in Table 1.

Table 1. Drivetrain states and inputs

| State          | Description                         | State                                                 | Description                      |
|----------------|-------------------------------------|-------------------------------------------------------|----------------------------------|
| $\theta_{hsf}$ | Front halfshaft displacement, [rad] | $\omega_{L1}$                                         | Front left wheel speed, [rad/s]  |
| $\omega_{dsf}$ | Front driveshaft speed, [rad/s]     | $\omega_{R1}$                                         | Front right wheel speed, [rad/s] |
| $\omega_{dsr}$ | Rear driveshaft speed, [rad/s]      | $\omega_{L2}$                                         | Rear left wheel speed, [rad/s]   |
| $\theta_{hsr}$ | Rear halfshaft displacement, [rad]  | $\omega_{R2}$                                         | Rear right wheel speed, [rad/s]  |
| Input          |                                     | Description                                           |                                  |
| $\tau_{mot}$   |                                     | Motor torque, [Nm]                                    |                                  |
| $\tau_{AWD}$   |                                     | AWD coupling torque from center coupling device, [Nm] |                                  |
| $\tau_{fric}$  |                                     | Total friction brake torque, [Nm]                     |                                  |

Although the bond graph model above denotes the tire longitudinal forces,  $F_{xt,ij}$  as inputs, they are merely the interaction points between the drivetrain model and the vehicle inertial states. The authors chose to lump the wheel inertias with the drivetrain model and treat the vehicle model as a chassis.



### Center Coupling Device

The center coupling device is a high fidelity nonlinear model in the form of an S-function. The device operates on the speed difference between the front and rear driveshafts. A controllable hydraulic valve operates a clutch that can transfer torque from one driveshaft to the other. The amount of torque transfer is proportional to the speed difference of the driveshafts. Additionally, the device also features an electric motor that can “pre-charge” the system to obtain a quicker response. Finally, the device also has the capability to lock the front and rear driveshafts whenever the speed difference is small. Under normal operation, the center coupling device remains open thereby decoupling the front and rear axles. The device only engages during events where AWD capabilities are required.

### Vehicle Model

The vehicle model used in this study is the four wheeled planar vehicle model shown in Figure 4. Note that because the wheel inertias were included in the drivetrain model, the vehicle model only outputs three states, the vehicle longitudinal, lateral and yaw velocities. Hence, only the vehicle chassis is drawn.

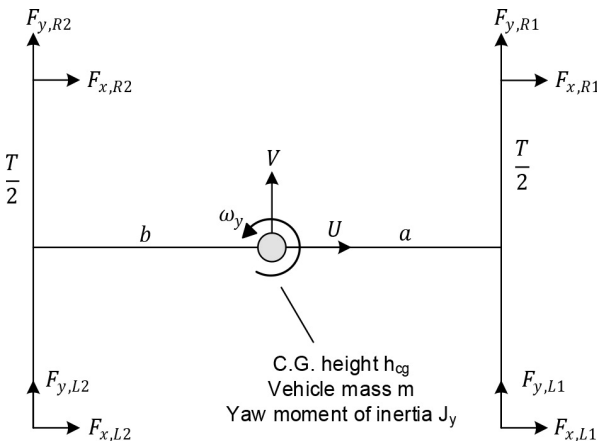


Figure 4. Schematic of four wheeled planar vehicle chassis

The vehicle chassis interacts with the drivetrain model via the longitudinal tire forces. Note that all of the tire forces are resolved in the vehicle body fixed axis. The tire forces are transformed from the tire frame to the vehicle frame as

$$\begin{bmatrix} F_{x,i1} \\ F_{y,i1} \end{bmatrix} = \begin{bmatrix} \cos(\delta) & -\sin(\delta) \\ \sin(\delta) & \cos(\delta) \end{bmatrix} \begin{bmatrix} F_{xt,i1} \\ F_{yt,i1} \end{bmatrix} \quad i = L, R$$

$$\begin{bmatrix} F_{x,i2} \\ F_{y,i2} \end{bmatrix} = \begin{bmatrix} F_{xt,i2} \\ F_{yt,i2} \end{bmatrix} \quad i = L, R$$
(3)

Here,  $\delta$  is the front tire steer angle input to the vehicle. The tire slip angles and slip ratios are defined as

$$\alpha_{ij} = \tan^{-1} \left( \frac{V_{yt,ij}}{V_{xt,ij}} \right)$$

$$\sigma_{ij} = \frac{R_w \omega_{ij} - V_{xt,ij}}{\max(V_{xt,ij}, R_w \omega_{ij})} \quad i = L, R \quad j = 1, 2$$
(4)

Here,  $R_w$  is the rolling radius of the tire,  $V_{xt}$  and  $V_{yt}$  are the longitudinal and lateral velocities of the wheel center in the tire frame. Normal tire forces are calculated statically with the addition of longitudinal weight transfer. During braking, it is assumed that longitudinal weight transfer is a much more dominant effect as compared to lateral weight transfer. Longitudinal weight transfer is calculated algebraically through the vehicle longitudinal acceleration  $a_x$ . The normal tire forces at each wheel are calculated as

$$N_f = \frac{mgb}{2(a+b)} - \frac{mh_{cg}a_x}{2(a+b)}$$

$$N_r = \frac{mga}{2(a+b)} + \frac{mh_{cg}a_x}{2(a+b)}$$
(5)

Having defined tire slip angles, slip ratios and normal tire forces, the final tire forces are calculated using the nonlinear Magic Formula (MF) tire model [7]. The planar vehicle model outputs three nonlinear state equations:

$$\dot{U} = \frac{1}{m} (mV\omega_y + F_{x,L1} + F_{x,R1} + F_{x,L2} + F_{x,R2})$$
(6)

$$\dot{V} = \frac{1}{m} (-mU\omega_y + F_{y,L1} + F_{y,R1} + F_{y,L2} + F_{y,R2})$$
(7)

$$\dot{\omega}_y = \frac{1}{J_y} \left( -\frac{T}{2} (F_{x,L1} + F_{x,L2}) + \frac{T}{2} (F_{x,R1} + F_{x,R2}) + a(F_{y,L1} + F_{y,R1}) - b(F_{y,L2} + F_{y,R2}) \right)$$
(8)

### CONTROLLER DESIGN

The objective of the controller designed in this section is to stabilize the vehicle when under hard braking on a low friction surface while using the maximum amount of available regen braking torque to improve fuel efficiency.

This section is broken up as follows: A yaw rate target generator is first developed to calculate a yaw rate target based on the driver inputs; a braking torque distribution strategy is introduced that is used to actively distribute regen and friction brake torques; the center coupling device as a smart actuator is explained in more detail and its response is analyzed; a linear vehicle model is then introduced which uses a linearized MF tire model; a robust yaw rate controller is then derived from the linear vehicle model using the Youla parameterization; finally, the complete control structure is outlined.

#### Yaw Rate Target Generator

The purpose of the yaw rate target generator is to obtain an estimate of the desired yaw rate commanded by the driver. The yaw rate target generator is a two state linear bicycle car model whose tire forces are scaled by an estimate of the road friction coefficient. It is shown that whenever the tires are saturated, the road friction coefficient can be estimated from the ratio of lateral force acting on the vehicle and gravitational acceleration[8]. Note that although many road friction estimation techniques have been explored in the literature, this work

uses a simple technique for computational efficiency since the yaw rate target generator would have to run real time if it was implemented in the vehicle.

$$\mu_{est} = \frac{a_y}{g} \quad (9)$$

The road friction estimate is used as an input along with the tire steer angle,  $\delta$ , and vehicle forward speed,  $U$ , into the yaw rate target generator. The state equations for a bicycle car model are given as

$$\dot{V} = \frac{1}{m}(-mU\omega_y + F_f + F_r) \quad (10)$$

$$\dot{\omega}_y = \frac{1}{J_v}(aF_f - bF_r) \quad (11)$$

The lateral tire forces are calculated as

$$F_f = \mu_{est} C_f \alpha_f = \mu_{est} C_f \left( \delta - \frac{V - a\omega_y}{U} \right) \quad (12)$$

$$F_r = \mu_{est} C_r \alpha_r = \mu_{est} C_r \left( \frac{b\omega_y - V}{U} \right) \quad (13)$$

Here,  $C_f$  and  $C_r$  are the lateral tire stiffness coefficients. The output of interest is the yaw rate of the bicycle car model. Note that  $\mu_{est}$  scales the lateral tire forces and ultimately also saturates the yaw rate of the bicycle car model. The yaw rate output is ultimately used as a target to the yaw rate controller which controls the center coupling device.

The yaw rate target generator was tuned to match the yaw rate response of the nonlinear vehicle model within the linear region of tire operation on a high road friction surface. Figure 5 shows the step and sinusoidal responses for a 60 degree steering wheel input (using a steering wheel to tire gear ratio of 17) at a forward speed of 50 KPH. The yaw rate response of the target generator agrees well with that of the nonlinear vehicle model. Thus, the evaluator gives a good estimate of the actual yaw rate of the vehicle.

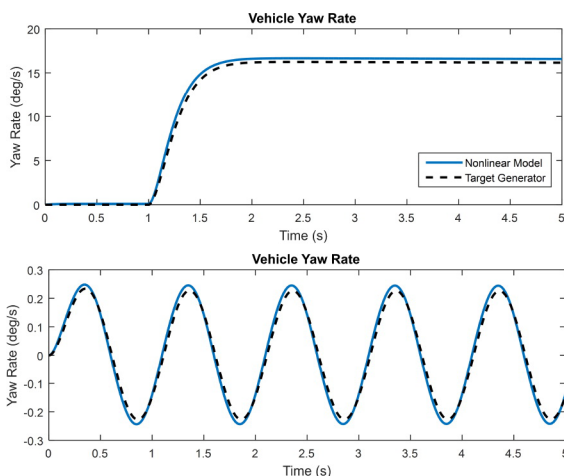


Figure 5. Yaw rate response of nonlinear vehicle and yaw rate evaluator

## Braking Torque Distribution

Braking torques on HEV's can be realized via two actuators; regen braking torque from the electric motor or via conventional friction brakes. In an effort to maximize fuel efficiency, it is ideal to use regen braking torque as much as possible and only use friction brakes when absolutely needed. In this paper, the maximum amount of regen braking torque available from the electric motor at any given speed is characterized by the curve given in Figure 2. A simple algorithm is used to distribute regen and friction braking torques:

$$\begin{aligned} &\text{if } \tau_{request} \leq \tau_{regen,max} \\ &\quad \tau_{regen} = \tau_{request}, \tau_{fric} = 0 \\ &\text{else} \\ &\quad \tau_{regen} = \tau_{regen,max}, \tau_{fric} = \tau_{request} - \tau_{regen} \\ &\text{end} \end{aligned} \quad (14)$$

The algorithm in (14) ensures that if the total brake request is below the maximum regen braking torque limit, then all of the braking will be realized via regen braking the rear axle. If the total brake request is greater than the regen torque limit, braking will be realized by applying the maximum amount of regen brake torque to the rear axle and using friction brakes to evenly distribute the rest of the request to all four wheels.

## Center Coupling Device as a Smart Actuator

The robust yaw rate stabilizing controller is designed as a high level controller that outputs an AWD coupling torque command to the center coupling device. The center coupling device features its own low level PID controller, depicted in Figure 6, to deliver the command torque coming from the high level yaw rate controller.

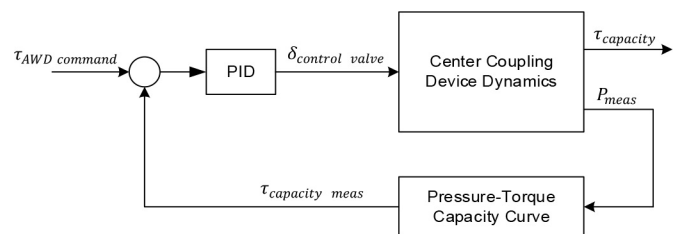


Figure 6. Control structure of the center coupling device low level controller

The center coupling device is controlled via a valve command signal. A pressure measurement is made within the device which is correlated to the torque capacity via a Pressure-Torque Capacity curve. Generally speaking, the low level PID controller is not tunable by the user since it is designed and tuned by the manufacturer.

To show the response of the device, Figure 7 shows the response of the center coupling device with the low level PID controller when a step  $\tau_{AWD}$  command of 200 Nm is used. As can be seen, the device has a quick response and follows the command signal nicely.