

The Roles of Front and Rear Cornering Stiffness in Vehicle Directional Dynamics

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ABSTRACT

The transfer functions between the variables in linear bicycle models of four wheel automobiles, the input to which is steering angle, were examined. The modal response transfer functions between all of the motion outputs, with the exception of front tire slip angle, were found to be independent of front tire cornering stiffness. It is a role unique to rear cornering stiffness to define the above mentioned transfer functions. This concept was also deductively demonstrated. The concept is true in the frequency domain, therefore, it guarantees that the amplitude ratio between two outputs and the phase difference between two outputs depends only on rear cornering stiffness. However, it is necessary to be cautious, when one applies the concept to phenomena in time domain. For example, the difference in 50 % rise time of two output variables is dependent on front tire cornering stiffness.

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INTRODUCTION

Many of the vehicle directional dynamics characteristics indices of four wheeled automobiles in the linear range are dependent on both the front axle cornering stiffness and the rear axle cornering stiffness. However, there have been papers in the scientific literatures, which proposed that a sufficiently high rear cornering stiffness is more vital, compared to the choice of the front cornering stiffness, in obtaining desirable performance.[1], [2] Furthermore, there exist some vehicle directional dynamics characteristics indices, which are dependent only on rear cornering stiffness, and are independent of the front cornering stiffness. This has reminded the author that some special role must have been allocated to the rear cornering stiffness, and the front cornering stiffness cannot affect it. Therefore, an attempt was made to clarify what the role unique to the rear cornering stiffness is. This paper will explain the results of the study, covering the following contents.

1. Investigation into dependency of directional dynamics characteristics on front and rear cornering stiffness
2. Commonality among directional dynamics characteristics, which are independent of front cornering stiffness. - Transfer function between two outputs -
3. Physical meaning of determination of the transfer functions between two outputs.
4. Verification with kinetic equations.
5. Verification with deductive logic in dynamics.
6. Effects in the time domain.
7. Vehicles with a steering system only on rear axle.
8. Conclusions.

INVESTIGATION INTO DEPENDENCY OF DIRECTIONAL DYNAMICS CHARACTERISTICS ON FRONT AND REAR CORNERING STIFFNESSES

To begin with, the equations, which express directional dynamics characteristics indices, will be enumerated. Each of them will be examined, whether it depends only on one of front/rear stiffnesses, or both of them. The linear bicycle model that was studied is shown below with a schematic drawing, the kinetic equations and the nomenclature.

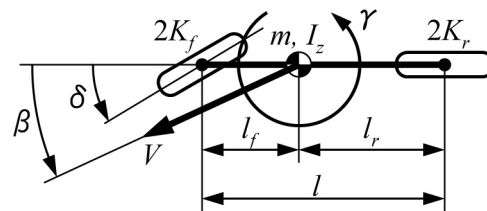


Figure 1. Schematic drawing of the bicycle model, used for the study.

$$F_f + F_r = mV(\dot{\beta} + \dot{\gamma}) \quad (1)$$

$$F_f l_f - F_r l_r = \dot{\gamma} I_z \quad (2)$$

$$F_f = 2K_f \beta_f = 2K_f (-\beta + \delta - l_f \dot{\gamma} / V) \quad (3)$$

$$F_r = 2K_r \beta_r = 2K_r (-\beta + l_r \gamma / V) \quad (4)$$

V ; Vehicle velocity [m/s]

$2K_f, 2K_r$; Cornering stiffnesses of front/rear axles [N/rad]

C_{cf}, C_{cr} ; Cornering compliances of front/rear axles [rad s²/m]

$$C_{cf} = \frac{ml_r}{2K_f l} \quad C_{cr} = \frac{ml_f}{2K_r l} \quad (5)$$

l, l_f, l_r ; Wheelbase, front/rear wheelbases [m]

m ; Vehicle mass [kg]

I_z ; Yaw moment of inertia [kg m²]

i_D ; Yaw dynamic index [-]

δ ; Front axle steer angle [rad]

γ ; Vehicle yaw velocity [rad/sec]

β ; Vehicle sideslip angle at vehicle center of gravity [rad]

β_f, β_r ; Vehicle sideslip angles at front/rear axle center [rad]

β_{fp}, β_{rp} ; Tire slip angles of front/rear tires [rad]

α ; Vehicle lateral acceleration at vehicle center of gravity [m/s²]

α_f, α_r ; Vehicle lateral acceleration at front/rear axle center [m/s²]

F_f, F_r ; Side forces of front/rear axles [N]

F_{nf}, F_{nr} ; Normalized side forces of front/rear axles [G]

$$F_{nf} = (F_f l) / (l_r mg) \quad F_{nr} = (F_r l) / (l_f mg) \quad (6)$$

g ; Gravitational acceleration [m/s²]

D ; Denominator of vehicle model transfer function [N²ms/rad]

A ; Stability factor [rad s²/m²]

ω_n ; Eigen frequency of the vehicle model [rad/sec]

ζ ; Damping factor of the vehicle model [-]

t_R ; Response time of the vehicle model [s]

Directional Dynamics Characteristics that Depend on both Front and Rear Cornering Stiffnesses

The stability factor A defines the dependency of steady state yaw rate gain on the vehicle velocity. It is expressed using the cornering compliances C_{cf} and C_{cr} as shown below.

It can be seen that the rear cornering compliance must be smaller than that of the front by a certain amount in order to obtain a desirable value of A , i.e. K_r larger than $K_f l_f / l_r$ is essential. It is apparent that both of the front and rear cornering stiffnesses K_f and K_r affect A .

$$A = \frac{C_{cf} - C_{cr}}{l} \quad (7)$$

The Eigen frequency ω_n of the bicycle model, which is derived from the characteristic equation of the vehicle model, is the index, which represents the rapidness of the rise-up of the response. ω_n is shown in Equation (8). The equation suggests that reductions of both the front and rear cornering compliances, especially that of the rear cornering compliance, is essential in obtaining a high ω_n value. It is apparent that both the front and rear cornering stiffness affect ω_n .

$$\omega_n = \sqrt{\frac{l - C_{cr} V^2 + C_{cf} V^2}{i_D C_{cf} C_{cr} V^2 l}} \quad (8)$$

Similarly, equation (9) shows the response time t_R , when the damping factor ζ is smaller than 1.0. t_R again is derived from the characteristic equation, and is a function of both the front and rear cornering stiffnesses.

$$t_R = \frac{1}{\zeta \omega_n} = \frac{I_z V}{K_f l_f (l_f + l_r i_D) + K_r l_r (l_r + l_f i_D)} \quad (9)$$

Irie et al. proposed an index, which equals to $\zeta \omega_n V$, and called it "handling capacity" in 1985 [1]. It is an index, which evaluates the vehicle motion response to the steering input at high speeds. They pointed out the importance of a sufficiently high rear cornering stiffness in allowing compatibility between a high stability factor and a high handling capacity.

Kushiro et al. studied the damping of steering wheel angle under free control, which arises from the lag of steering torque relative to steering angle [2]. They pointed out that the rear cornering stiffness strongly affects the phenomenon. Equations (10) and (11) show the transfer function of the steering wheel angle to the steering torque in a three degree of freedom vehicle model, which takes steering wheel inertia into consideration.

$$\frac{M_H(s)}{\delta_H(s)} = I_{sw} s^2 + \frac{\xi_c}{\lambda_s^2} \frac{2K_f}{D} \quad (10)$$

$$(m I_z V s^2 + 2K_r I_z s + 2K_r l_r^2 m + 2K_r l_r m V)$$

$$D = mV I_z s^2 + 2(K_f l_f^2 + K_r l_r^2)ms + 2(K_f + K_r)I_z s + 4K_f K_r l^2 / V + 2(K_r l_r - K_f l_f)mV \quad (11)$$

I_{SW} ; Moment of inertia of the steering wheel [kg m²]

λ_s ; Steering gear ratio [-]

ξ_c ; Castor trail [m]

δ_H ; Steering wheel angle [rad]

M_H ; Steering-wheel torque [Nm]

The transfer function again depends on both the front and rear cornering stiffness.

All the above examples of vehicle directional dynamics indices suggested the importance of high rear cornering stiffness, by showing that higher rear cornering stiffness and/or a smaller lag in the rear cornering force relative to the rear slip angle is necessary, in order to keep the indices' values to good levels. However, it was shown that all of them are functions of both the front and rear cornering stiffness, and none of the articles explicitly discussed that there are any indices, which depend solely on the rear cornering stiffness, and that the front cornering stiffness cannot affect them. However, there exist some vehicle directional dynamics characteristics, which depend only on rear cornering stiffness, and which are independent of front cornering stiffness. This was of interest, and an attempt was made to find the condition in which an index is independent of the front cornering stiffness.

Directional Dynamics Characteristics that Depend only on Rear Cornering Stiffness

Sakai proposed the indices "slalom diagram" and "skid function" in 1997.[3] When a sinusoidal steering input is given to an ordinary understeer vehicle, traveling at a constant velocity, the front or rear tire skids as the steering input amplitude is increased. The axle, which skids, changes depending on the vehicle velocity and the steering input frequency, and it can be determined using the slalom diagram or skid function, i.e. they determine, whether vehicle-spin or plow-out, in the limit. The skid function is the gain ratio of the rear normalized side force to the front normalized side force. Equation (12) is the transfer function between the front and rear normalized side forces.

$$\frac{F_{nr}(s)}{F_{nf}(s)} = \frac{(ml_f l_r s - I_z s + ml_f V) 2K_r l_r}{(mI_z V s^2 + 2K_r I_z s + 2K_r l_r^2 ms + 2K_r l_r mV) l_f} \quad (12)$$

It can be seen that there is no K_f term in Eq. (12), and the skid function depends solely on K_r .

There is another example of directional dynamics characteristics, which depend only on K_r . Bergman found out that the "Ease of Vehicle Control" can be represented well by the time lag of vehicle

slip angle at the center of gravity of the vehicle (vehicle C.G. hereinafter) relative to the yaw velocity [4]. There were no suitable methods to measure the vehicle slip angle easily and accurately back in the 1970's. Therefore, lateral acceleration data at the vehicle C.G. were used to substitute for the vehicle slip angle data. The lags are thought to depend on the transfer function between the yaw rate and the vehicle slip angle, which is shown in Eq. (13), and the transfer function between the yaw rate and the lateral acceleration in Eq. (14), respectively.

$$\frac{\beta(s)}{\gamma(s)} = \frac{I_z s - mV l_f + 2K_r l_r l / V}{mV l_f s + 2K_r l} \quad (13)$$

$$\frac{\alpha(s)}{\gamma(s)} = \frac{I_z V s^2 + 2K_r l_r l s + 2K_r l V}{mV l_f s + 2K_r l} \quad (14)$$

The two transfer functions again do not have K_f term in them, and depends solely on K_r .

Commonality among Directional Dynamics Characteristics, which are Independent of Front Cornering Stiffness. - Transfer Function between Two Outputs

The commonality among the abovementioned transfer functions are that they are not the transfer functions of outputs relative to the steering angle input, but those relative to the other outputs. The transfer functions of the related output variables in response to the steering angle input are written as the following equations.

Front normalized side force / steering angle input

$$\frac{F_{nf}(s)}{\delta(s)} = \frac{2K_f}{D} \left(\frac{mI_z l V s^2}{ml_r g} + \frac{2K_r I_z l s}{ml_r g} + \frac{2K_r l_r^2 l m s}{ml_r g} + \frac{2K_r l V}{g} \right) \quad (15)$$

Rear normalized side force / steering angle input

$$\frac{F_{nr}(s)}{\delta(s)} = \frac{2K_f}{D} \left(\frac{2K_r l_r l s}{g} - \frac{2K_r I_z l s}{ml_f g} + \frac{2K_r l V}{g} \right) \quad (16)$$

Yaw velocity / steering angle input

$$\frac{\gamma(s)}{\delta(s)} = \frac{2K_f}{D} (mV l_f s + 2K_r l) \quad (17)$$

Vehicle slip angle / steering angle input

$$\frac{\beta(s)}{\delta(s)} = \frac{2K_f}{D} (I_z s - mV l_f + 2K_r l_r l / V)$$

