

Sideslip Angle Estimation of a Formula SAE Racing Vehicle

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ABSTRACT

A method for estimating the sideslip angle of a Formula SAE vehicle with torque vectoring is presented. Torque vectoring introduces large tire longitudinal forces which lead to a reduction of the tire lateral forces. A novel tire model is utilized to represent this reduction of the lateral forces. The estimation is realized using an extended Kalman filter which takes in standard sensor measurements. The developed algorithm is tested by simulating slalom and figure eight maneuvers on a validated VI-CarRealTime vehicle model. Results indicate that the algorithm is able to estimate the sideslip angle of the vehicle reliably on a high friction surface track.

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INTRODUCTION

There is an ever increasing trend in the use of mechatronic systems for active chassis control systems. Such control systems impose a desired behavior on the vehicle with the help of actuation systems such as active differentials, active front and/or rear steering, electro hydraulic brakes, electric motors driving the front, the rear or all the wheels independently. As the control algorithms are based on methods such as state feedback or sliding surface controllers, the knowledge of vehicle states such as sideslip angle and yaw angle rate is becoming of paramount importance [1, 2, 3, 4, 5]. While current series production automobiles are only equipped with inexpensive sensors which can measure accelerations and yaw rate, it is a challenging task to have an accurate measurement of the sideslip angle. Although, it is possible to directly measure the slip angle of the vehicle very accurately using optical sensors, these devices are very expensive and they are impractical for series production implementation [6]. Researchers in the past have made use of Global Positioning System (GPS) based techniques to measure the sideslip angle of the vehicle [7, 8, 9, 10]. Such techniques generally involve the fusion of inertial sensor signals with the GPS signals. One major drawback of these systems is the need to have satellite signals at all times for accurate estimation. In urban environments and in tunnels, such systems are highly prone to errors due to loss of communication. Moreover, accurate GPS systems also tend to be expensive [10].

Motivated by the issues listed above, researchers have tried to investigate indirect methods such as state observers to measure sideslip angle. The dynamic model based approach uses state observers to get an estimate of the sideslip angle. The state observer relies on sensor measurement from cheap sensors such as gyroscopes and accelerometers to estimate the sideslip angle. The accuracy of

this method relies heavily on a suitable modeling of the vehicle. The most crucial part of modeling the system is to successfully represent the tire cornering stiffness as incorrect modeling tends to generate steady state errors. The challenging task in the modeling of the tire cornering stiffness is that its value is influenced by time varying parameters. Recent research works have made use of adaptive Kalman filters to estimate the cornering stiffness. The cornering stiffness is considered as a state variable in the Extended Kalman Filter (EKF) and its derivative is modeled as a random Gaussian noise in [11]. Variable variance-covariance matrices depending upon tire operating conditions are used in [12]. Above methods depend a lot on correct parametrization of the variance covariance matrices. Ray [13] used a Kalman-Bucy filter based on an eight degree of freedom model to estimate the vehicle sideslip angle and the tire lateral forces. However, due to the estimation of eight states, the computation burden is too much. The authors in [14] estimated the sideslip angle of a vehicle implementing a four degree of freedom model and the magic tire formula. Hsu et al. in [15] used the steering self-aligning torque measurement and a calibrated tire model to estimate the sideslip angle with a non-linear observer. The effect of the longitudinal forces on the reduction of the lateral forces due to the combined slip tire characteristics and on the yaw rate dynamics is largely neglected in the above methods (apart from [13]). However, this effect is no longer negligible in the Formula SAE (FSAE) electric vehicle under study as it is equipped with a torque vectoring system for Direct Yaw Control (DYC). This is due to the fact that DYC involves the generation of a yaw moment through the generation of longitudinal forces, even at constant velocity operating conditions.

The objective of this paper is to present a dynamic model-based sideslip estimation method for a racing vehicle equipped with a left right torque vectoring system. The estimation methods should be simplified enough to reduce the computation burden due to the limitation in the on-board processing power. At the same time it should also consider the influence of the longitudinal forces on the vehicle dynamics. In the presented work, the vehicle sideslip angle is estimated with an adaptive EKF which is based upon a four degree of freedom model of the vehicle. The novelty of the algorithm lies in the introduction of a simplified tire model derived from literature which is then subsequently adapted to handle the reduction of lateral tire force during the presence of longitudinal tire force.

This paper is organized in the following sections. The first section provides a description of the vehicle and the tire model incorporated in the observer. It is followed by a brief remark about the EKF used in the observer design. Next, a description of the torque vectoring algorithm and the estimation of longitudinal forces due to torque vectoring is presented. The second half of the paper describes the experimental validation of the vehicle model used for simulation and presents the results and discussion for a variety of maneuvers with different input conditions. This is followed by the conclusions and future scope of work.

SYSTEM MODELING

The vehicle under consideration is the Squadra Corse XV (SCXV) racing vehicle. It is an electric vehicle with four independent inwheel motors. Figure 1 shows the vehicle under consideration. Table 1 lists the different vehicle parameters.



Figure 1. The SCXV prototype FSAE racing vehicle.

Table 1. Vehicle parameters.

Mass(with driver)	290kg
CG longitudinal front wheel distance	948mm
Wheelbase	1580mm
Front track width	1200mm
Rear track width	1200mm
Maximum Torque per wheel	336Nm
Wheel nominal radius	265mm
Center of Gravity height	220mm
Battery maximum voltage	400 V

The mathematical models used for representing the vehicle dynamics and the tire road force interactions in the observer design are described in the following sections. Figure 2 presents the vehicle architecture and the sign conventions for the body frame of reference.

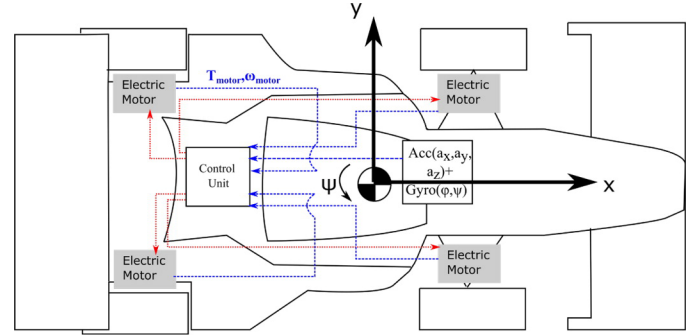


Figure 2. Vehicle architecture.

Vehicle Dynamics Model

The vehicle is represented with a four wheel model as shown in the Figure 3 [16] including the lateral velocity v , longitudinal velocity u , yaw rate r , and roll angle ϕ as the four degrees of freedom to represent the vehicle dynamics. The overall dynamics for the vehicle can be formulated by taking into account all the forces and the moments acting about the origin of the vehicle body reference frame. As a result, the system can be represented by the following equations:

$$\begin{aligned}
 m\dot{v} - m\dot{h}p &= -m\dot{u}r + F_{yf} \cos \delta_f + F_{yr} \cos \delta_r \\
 &\quad + 0.5F_{xtot} (\sin \delta_f + \sin \delta_r), \\
 m\dot{u} &= m\dot{v}r - m\dot{h}rp - F_{yf} \sin \delta_f - F_{yr} \sin \delta_r \\
 &\quad + 0.5F_{xtot} (\cos \delta_f + \cos \delta_r), \\
 I_{xx}\dot{p} - m_s h\dot{v} - I_{xz}\dot{r} &= m\dot{h}ur - B_{roll}p + (m_s gh - K_{roll})\phi, \\
 I_{zz}\dot{r} - I_{xz}\dot{p} &= aF_{yf} \cos \delta_f - bF_{yr} \cos \delta_r + 0.5aF_{xtot} \sin \delta_f \\
 &\quad - 0.5bF_{xtot} \sin \delta_r + (F_{yfl} - F_{yfr}) \sin \delta_f \frac{t_f}{2} \\
 &\quad + (F_{yrl} - F_{yrr}) \sin \delta_r \frac{t_r}{2} + M_z, \\
 \dot{\phi} &= p,
 \end{aligned} \tag{1}$$

where m is the total mass of the vehicle, h is the height of the CG above the roll axis, F_{yfl} , F_{yfr} , F_{yrl} , F_{yrr} denote the tire-ground lateral forces on the front left, front right, rear left and rear right tires respectively, F_{yf} , F_{yr} denote the sum of the left and the right tire-ground lateral force on the front and rear axles respectively, δ_f represents the average front steering angles, δ_r the average rear roll steer angle, I_{xx} , I_{xz} , I_{zz} are the moments of inertia in the planes denoted by their subscripts, t_f and t_r the front and the rear trackwidth, K_{roll} the roll stiffness, B_{roll} the roll damping, M_z the torque vectoring moment given by the controller, p the roll rate. For simplification, it is assumed that the tractive forces are equally distributed among the front and the rear axles.

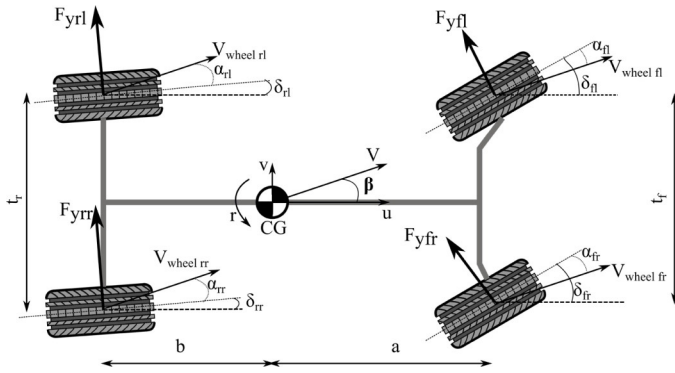


Figure 3. Four wheel vehicle model used for representing the vehicle dynamics.

Tire Force Model

A simple polynomial tire force model is chosen for the calculation of the tire forces [17]. The tire lateral force, F_{y0} is computed as follows,

$$F_{y0} = \begin{cases} -C_\alpha \alpha \left(1 - \frac{\alpha^2}{3(\alpha^o)^2} \right) & \|\alpha\| \leq \alpha^o = \frac{3\mu F_z}{2C_\alpha} \\ -\mu F_z \operatorname{sgn}(\alpha) & \text{otherwise} \end{cases} \quad (2)$$

where C_α is the tire cornering stiffness in the linear region, α is the tire sideslip angle, μ is the maximum lateral friction coefficient, F_z is the normal load on the tire. The reason for this choice is the ease of calibrating the model as it contains two tunable parameters C_α and μ only. The original model introduces the tire cornering stiffness as a constant parameter. However, the model used in the present work is augmented by modelling the cornering stiffness as a function of the normal load.

$$C_{\alpha, \text{final}} = C_\alpha \left(\frac{F_z}{F_{z \text{nom}}} \right)^n, \quad (3)$$

where $F_{z \text{nom}}$ and n are tuning parameters. Thus, in equation (2), the value of the constant C_α is replaced by $C_{\alpha, \text{final}}$ from equation (3). Furthermore, a horizontal α -shift is introduced by replacing the tire sideslip α with α_{eq} in equation (2) such that,

$$\alpha_{eq} = \alpha - \alpha_{HY} \quad (4)$$

where α_{HY} is the sideslip angle that generates the same lateral force offset due to the effects of normal load and camber thrust on the tire. Figure 4 presents a comparison of the tire lateral force versus the tire lateral slip angle properties for an experimentally validated Pacejka model [18] and the presented tire lateral force model developed in the sections above. The lateral forces are plotted for a range of normal

loads on the tire starting from 500 N up to 900 N. It can be seen that the forces have a decent amount of match in terms of magnitude up to the maximum of the tire lateral force. The model loses its accuracy for higher slip values. This is considered acceptable as the torque vectoring controller tries to keep the tire slip values within these limits. In addition, the level of match falls down a bit when the normal load is around 500 N.

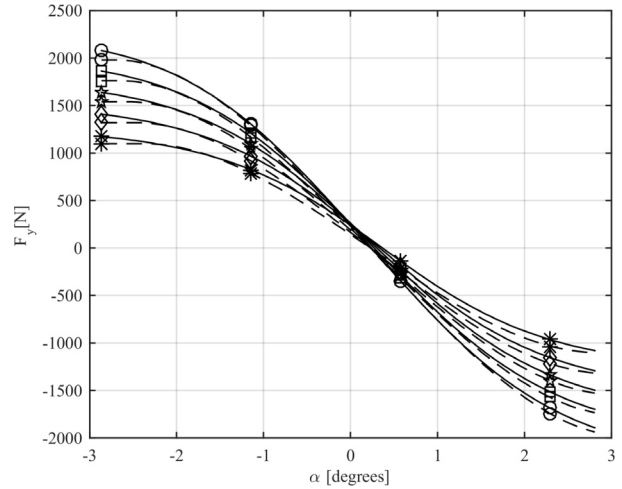


Figure 4. Comparison of tire lateral forces between the experimentally validated Pacejka's formula (solid) and the developed simplified tire model (dashed) for different values of tire sideslip angles and different normal loads (asterix 500N, diamond 600N, star 700N, square 800N, circle 900N).

However, as the normal load increases, the match improves. Since the average normal load on each tire is in the range of 800 N, it is safe to assume that the developed tire model is robust enough to model the tire forces in the observer. Additionally, one drawback of using such a model is that it is unable to take into account the influence of longitudinal forces on the lateral forces. Figure 5 shows the relationship between lateral and longitudinal tire forces [18]. It can be noticed that the envelop curve of the tire forces has a quasi-elliptical shape. This shape is also known as the friction circle of the tire [19]. It is evident that the magnitude of the lateral force decreases when the longitudinal force increases. As a result, the model is further modified by adding a correction of lateral force as a function of the longitudinal force, F_x . The final expression for the lateral force, F_y is thus given by the following equation,

$$F_y = F_{y0} + \frac{\alpha_{eq} \|F_x\|}{K}, \quad (5)$$

where K is a tuning factor which is used to model the relationship between F_y and F_x . The estimation of the tire longitudinal forces is mentioned in the section elaborating the torque vectoring controller which is presented in the following sections.