

Modelling and Simulation of Rapidly Changing Road Gradients

Robert Timothy Wragge-Morley, Guido Herrmann, and Stuart Burgess
University of Bristol

Phil Barber
Jaguar Land-Rover

ABSTRACT

In vehicle dynamics modelling, the road profile is generally treated in one of two ways; either the gradient is a property that changes over a length scale far greater than that of the vehicle's wheelbase, or as a very detailed road surface model for determining the behaviour of vehicle suspensions. Occasionally, for modelling the behaviour of off-road vehicles, step-climbing manoeuvres are modelled. We propose an extension of these step-climbing models to a general, continuously varying road gradient model for cases where the distance over which the large gradient change occurs are of similar length-scale as the vehicle wheelbase. The motivation behind this work comes from a road gradient and vehicle mass estimation problem where it was noticed that very sudden gradient changes have a significant impact on the powertrain, but in a way that is not proportional to the attitude change of the vehicle. We present an analysis applicable to these cases where the road gradient change is substantially different from the vehicle attitude change. The analysis is built up from a very basic step climbing exercise to a general case solution for two dimensional longitudinal vehicle dynamics. Methods of handling road gradient and gradient change for modelling purposes are discussed. A method is proposed that defines the gradient change as an input to a backwards dynamic model that allows us to compute torque required to achieve a pre-defined datum speed during a gradient transition. The results of simulations for simple gradient change cases are shown. We discuss the implication of these road gradient transients for vehicle parameter estimation based on powertrain behaviour.

CITATION: Wragge-Morley, R., Herrmann, G., Burgess, S., and Barber, P., "Modelling and Simulation of Rapidly Changing Road Gradients," *SAE Int. J. Passeng. Cars - Mech. Syst.* 9(1):2016, doi:10.4271/2016-01-1663.

1. INTRODUCTION

The vast majority of gradient estimation work focuses on macro-scale changes of gradient, such as those found on highways. In this case, it is certainly the case that the road gradient is, as most established literature [1] [2] [3] [4] [5] asserts, a property that affects the whole vehicle equally, and so can be treated as an individual parameter at any point in time. However, there is a family of specific cases, perhaps more relevant to off-road driving, where the road gradient changes very suddenly; for the sake of argument, we shall suppose this to mean the road gradient changing appreciably within the wheelbase length of the vehicle. Since road gradient estimation is often carried out using driveline torque as an input [6] [7] [8] it is important to consider the relationship between driveline torque and road gradient. In this paper we present an analysis of the kinematics and dynamics of a rapidly changing gradient scenario, which should be of interest to vehicle dynamics and control engineers seeking to understand the fluctuations in powertrain load and expected accelerations in such a situation and develop solutions with these dynamics in mind.

Substantial work has been carried out into the estimation of macro-scale road gradient as a vehicle parameter. This is often undertaken in conjunction with the estimation of vehicle mass, since the two are

coupled through the vehicle dynamics by Newton's Second Law. The authors of this paper have devoted a substantial research effort to estimation problems of this type [6] [7] [9] [10] using a novel adaptive filtering system as an estimation algorithm. The interest in this research is driven by the increase in levels of functionality and autonomy in vehicle embedded systems and the requirement for accurate vehicle system models.

Numerous approaches to estimation problems of this type also exist in the literature, including work by Sahlholm et al. [11] [12] [13] considering the learning of road gradient profile over multiple passes along the same route by heavy duty vehicles. Recursive Least Squares methods are common for estimation of multiple parameters, for instance, Vahidi [14] estimates road gradient and mass simultaneously. Many approaches use a combination of RLS and Kalman Filtering methods to simultaneously estimate road gradient and vehicle mass, including Raffone [8] and Vahidi [14]. Nonlinear observer structures are also used, by the current authors [9] [10] [15] [7] and by McIntyre [16] and Rajamani [17].

The other extreme of length scale for road gradient is estimation of surface roughness imperfections, which are either generally examined as a frequency domain problem for suspension and design for Noise Vibration and Harshness (NVH) [2] [18], or sometimes as a

estimation problem in their own right. The problem of developing a 'profilometer' was initially tackled in the 1960s by Spangler et al. [19] at General Motors Research in order to develop models for the analysis of suspension performance, whilst Sayers et al [20] developed a 'longitudinal profile analyser' at LCPC¹ as part of the International Road Roughness Experiment. More recently, Imine et al [21] used both the 'lpc' method and a sliding mode observer method to estimate road profile with some success; Doumiati. et al [22] have employed Kalman Filtering [23].

In the case where the gradient change in the vehicle length scale is minimal, the resistive load due to gravity experienced by the drivetrain is a function of that road gradient. However, in the case where the road gradient changes substantially within the wheelbase length of the vehicle, it is clear that the work done against gravity must still be proportional to the rate of height gained by the centre of mass. In the changing gradient case, this is now dependent on the gradient of the road at the two contact points and the instantaneous inclination of the vehicle, as well as the rate of change of inclination. Naturally, the more realistic vehicle dynamics are included in such a problem, the more complex it gets.

A similar case often presented in the literature [24] considers a vehicle which climbs a small step. This is a common problem for off-road vehicle dynamics and may be used in design of powertrain and tyre sizing; of course this problem may be examined at differing levels of complexity, and is frequently limited to fairly simple tyre models to ease analysis. The work presented in this paper is concerned with what may be considered an extension of the 'step climbing' problem. Several illustrative cases are presented in [section 2](#) that examine the behaviour of a vehicle with its front and rear contact patches on roads of different gradients. These cases are used to develop a set of geometrical relations for a general problem containing two road gradients. Implementation of the kinematics in stepwise, discrete time simulation is presented in [section 3](#) and a backwards model of the vehicle dynamics, where driving force is calculated from the kinematic relationships based on a defined speed, is presented in [section 4](#). Several different approaches to handling the problem in simulation are discussed and results are presented in [section 5](#).

2. AN ILLUSTRATION OF THE CHANGING GRADIENT PROBLEM

In order to demonstrate how the locus of the position of the centre of mass of a two-axled, rigid vehicle varies whilst undergoing a gradient transition, we shall provide a series of examples beginning with a simple, albeit unrealistic, right angle step climbing exercise. Initially we shall consider the geometric relationships and then build up a general model of the dynamics of the gradient change scenario.

Right Angle Step

This example is essentially the same as the classic 'ladder sliding down a wall' problem. Each contact point is resting on a smooth flat plane. The angle of the plane on which the front contact point rests is θ_f and the plane on which the rear rests is at an angle θ_r . The two planes meet at a sharp join. The vehicle body is represented by a rigid

bar of length L_c instantaneously inclined at an angle θ_c from the horizontal and of mass m . The centre of mass, for purposes of simplification shall be considered to be at some length along this bar, L_f from the front contact point and L_r from the rear contact point. In the analysis of the following examples, we shall consider the kinematics from the instant that the front contact point begins its new trajectory so that the distance from the rear contact point to the junction of the two planes at time $t = 0$ is equal to the wheelbase length L_c and the displacement of the front and rear contact points on their respective trajectories is s_f and s_r .

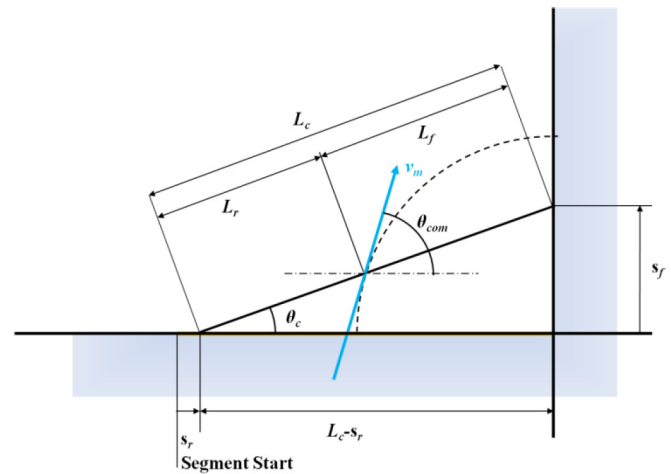


Figure 1. The simplest way of illustrating the changing gradient problem is by examining a simple step climbing example. The labelled angles in this illustration are the angle of inclination of the vehicle θ_c and the angle of the tangent to the centre of mass trajectory θ_{com} .

In this case, we may see that the relationship between front and rear displacement is, in the case of a right angle step, defined by Pythagoras' theorem.

$$L_c^2 = (L_c - s_r)^2 + s_f^2 \quad (1)$$

$$s_f = \sqrt{2L_c s_r - s_r^2} \quad (2)$$

The locus in this case is an elliptical path, or if the centre of mass is exactly halfway along the vehicle, a circular one. The gradient instantaneously experienced by the centre of mass is based on the relationship between the horizontal and vertical displacement of the rear and front axes respectively, which we have already derived. So the angle of inclination of the vehicle as a function of the displacements at a given instant is:

$$\theta_c = \tan^{-1} \left(\frac{s_f}{L_c - s_r} \right) \quad (3)$$

Since this case is a right angle, the path taken by the centre of mass may be split very neatly into its horizontal and vertical components:

$$\underline{s}_{com} = [s_r + L_r(\cos \theta_c - 1)]\mathbf{i} + \left[\frac{L_r}{\sin \theta_c} \right]\mathbf{j}$$

¹ Laboratoire central des ponts et chaussées

where $s_f \equiv L_c \sin \theta_c$.

Thus it may be shown that the angle of the instantaneous gradient of the centre of mass in this case is,

$$\theta_{com} = \tan^{-1} \frac{ds_f}{ds_r} \quad (5)$$

General Case for Planar 'Ramps'

This example concerns itself with the case where both the front and rear contact patches come to rest on ramps of non-zero gradient. We shall label the segments "f" and "r" for the front and rear contact respectively. A diagram explaining the nomenclature associated with this case may be seen in [Figure 2](#). Ignoring for the moment the actual transition from one ramp to the other, we can easily write relationships for the before and after transition cases where the two contact points move in parallel to one another, such that:

Table 1. Description of trajectories when both front and rear contact points are on the same plane.

Before	After
$\theta_{com} = \theta_{R_r}$	$\theta_{com} = \theta_{R_f}$
$\underline{s}_{com} = s_r \cos \theta_{R_r} \underline{i} + s_r \sin \theta_{R_r} \underline{j}$ $\equiv s_f \cos \theta_{R_r} \underline{i} + s_f \sin \theta_{R_r} \underline{j}$	$\underline{s}_{com} = s_r \cos \theta_{R_f} \underline{i} + s_r \sin \theta_{R_f} \underline{j}$ $\equiv s_f \cos \theta_{R_f} \underline{i} + s_f \sin \theta_{R_f} \underline{j}$

in these cases, $s_f = s_r = s_{com}$.

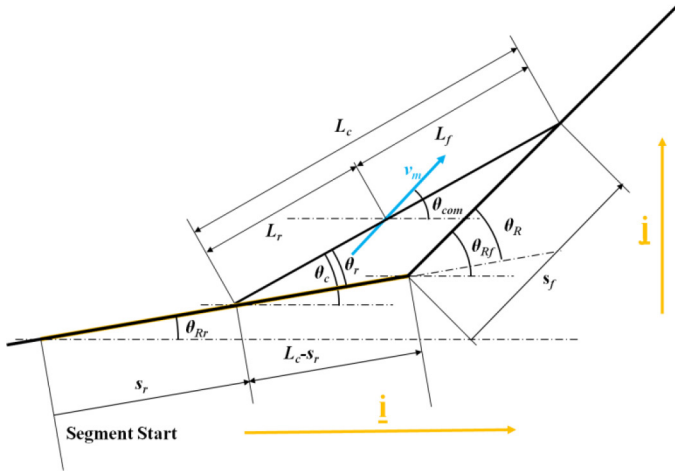


Figure 2. The 'General' case for two planar ramps meeting has non-zero road angle on each ramp, once again we consider the segment which starts at the point where the front wheel begins its new trajectory on the forward ramp. In this case, there are several additional angles to be considered and these are shown in the above diagram.

It is convenient for us to be able to write the angle of inclination of the vehicle body in terms of the road angles. Taking the triangle formed instantaneously by the wheelbase and the ramps, we may achieve this by rearranging the cosine rule as a quadratic equation ([Equation \(6\)](#)) giving a solution in the standard form ([Equation \(7\)](#)) with the terms described by Equations (8) - (10).

$$0 = b^2 - (2a \cos C)b + (a^2 - c^2) \quad (6)$$

$$b = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (7)$$

$$\alpha = 1 \quad (8)$$

$$\beta = -2a \cos C \quad (9)$$

$$\gamma = a^2 - c^2 \quad (10)$$

where:

$$c \equiv L_c \quad (11)$$

$$a \equiv L_c - s_r \quad (12)$$

$$b \equiv s_f \quad (13)$$

$$C \equiv \pi - \theta_R = \pi - (\theta_{R_f} - \theta_{R_0}) \quad (14)$$

so that:

$$\alpha = 1 \quad (15)$$

$$\beta = -2(L_c - s_r) \cos(\pi - \theta_R) \quad (16)$$

$$\gamma = s_r^2 - 2L_c s_r \quad (17)$$

and:

$$\begin{aligned} s_f &= (L_c - s_r) \cos(\pi - \theta_R) \\ &\pm \sqrt{(L_c^2 - 2L_c s_r + s_r^2) \cos^2(\pi - \theta_R) - s_r^2 + 2L_c s_r} \end{aligned} \quad (18)$$

This allows us to write an expression for the inclination of the vehicle wheelbase in terms of the ramp angles; where $\theta_c(k)$ is calculated by applying the sine rule to the triangle formed by the road planes and the vehicle wheelbase, as illustrated in [Figure 2](#). (n.b. the base of this triangle is already inclined at the angle (θ_{R_r}) :

$$\theta_c = \theta_{R_r} + \sin^{-1} \left(\frac{s_f}{L_c} \sin(\theta_{R_f} - \theta_{R_r}) \right) \quad (19)$$