

Control Allocation for Multi-Axle Hub Motor Driven Land Vehicles

Qian Wang and Beshah Ayalew
Clemson University

Amandeep Singh
US Army, TARDEC

ABSTRACT

This paper outlines a real-time hierarchical control allocation algorithm for multi-axle land vehicles with independent hub motor wheel drives. At the top level, the driver's input such as pedal position or steering wheel position are interpreted into desired global state responses based on a reference model. Then, a locally linearized rigid body model is used to design a linear quadratic regulator that generates the desired global control efforts, i.e., the total tire forces and moments required track the desired state responses. At the lower level, an optimal control allocation algorithm coordinates the motor torques in such a manner that the forces generated at tire-road contacts produce the desired global control efforts under some physical constraints of the actuation and the tire/wheel dynamics. The performance of the proposed control system design is verified via simulation analysis of a 3-axle heavy vehicle with independent hub-motor drives.

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INTRODUCTION

Multi-axle land vehicles with drive actuation on more than one axle offer improved stability and traction on various road surfaces. This is possible via the optimization of load distribution and maximization of the utilization of individual tire-road contact by exploiting the redundancy of the drive system. The redundancy could also be exploited to boost fault tolerance and reliability. Going further, replacing traditional axle drive systems with wheel hub motors brings efficiency to the independent control of the wheel torque with fast and accurate actuation [1]. Nevertheless, the increased redundancy of independent hub motor control commonly results in an over-actuated vehicle system, which requires elaborate control allocation (CA) schemes. Ref [2] gave an overview of CA methods applied in vehicle motion control. The methods are often designed as part of a hierarchical vehicle control structure: at the top, a motion control algorithm is used to generate the global control efforts needed to satisfy the overall motion objectives; at the bottom, a CA algorithm coordinates the actuators to generate the desired global control efforts. Such a hierarchical design of CA system has been suggested in plenty of previous works concerning 4-wheel electric vehicles with access to torque control of hub motors [3,4,6]. This paper treats the case of multi-axle land vehicles with hub motor drives, which involve even more actuation redundancies.

Researchers in [3] introduced a linear quadratic regulator (LQR) based dynamic yaw-moment control (DYC) method to generate global yaw moment needed to track the desired lateral response of a reference model under the input of driver's steering and vehicle

speed. Weighted least square method was used in [4,5] to resolve the distribution of longitudinal forces to each wheel from the global yaw moment under the hard constraint of the moment balance equation. The objective was set to minimize the magnitude of distributed force at each wheel, or slip ratio and its rates of change. Tire slip ratios, slip angles, and the states like vehicle slip angle and yaw rate are obtained by designing estimators [6,7]. In the above works, it was possible to obtain explicit solutions of the CA problem as linear tire model was used and the coupling between longitudinal and lateral tire force generation was not considered as a constraint. However, these simplifications are not practical in limit conditions outside of the regime of linearity.

To ensuring the feasibility of the CA solution, global force tracking error can be added as a soft term in the objective function, as presented in [8], in addition to other terms might related to efficiency of tire usage [9] or energy dissipation of tire slip [10]. Optimization of this objective function was done subject to a linear approximation of friction ellipse constraint [11]. These resulted in a quadratic programming problem suitable for real-time application on a 4-wheel vehicle. The distributed forces were then used to generate the control variables based on a reversed tire model [9]. However, the linear approximation of the friction ellipse is conservative which leads to wasting part of the tires' potential. Consider the nonlinear tire model as constraint in the CA can further improve the full usage of the tires, as pointed out in [12]. In this case, local controllers, like traction controller, could become of limited need as the wheel dynamics and its effects on the tire force generation are already integrated in the CA. The downside is the application of nonlinear model leads to

nonlinear programming problems. However, such problems can be efficiently solved via sequential quadratic programming algorithms [13].

As already pointed out, almost all of the existing works in this area have applied to two-axle vehicles. In this paper, we adopt the hierarchical control framework and pose and solve the required CA problem for multi-axle (more than 2 axle) land vehicles with hub motor drives. The contribution of the paper includes the modeling framework for control of such vehicles as well as the mathematical formulation of the CA problem in a way that leads to improvement in the performance of the vehicles in standard and aggressive handling maneuvers.

The rest of the paper is organized as follows. First, the mathematical model of the multi-axle vehicle and the tire used for developing the CA method are introduced. The next section describes the control system framework, including the hierarchical structure with motion controller at the top and CA, at the bottom, for the multi-axle vehicle with hub motors. Finally, the effectiveness of the proposed CA method is illustrated via the simulations of a 3-axle heavy vehicle with hub-motor drives.

MODELING

Vehicle Model

Ignoring the rolling and pitching effect and assuming a flat road, the general model of the planar motions, including longitudinal, lateral and yaw motions, for n -axle driven vehicle are shown in Figure 1. Therein, only the first axle is assumed to be steerable with the road wheel steer angle δ . d_i and l_i represent the wheelbase of the i -th axle and relative distance between the i -th axle and the CG, based on the vehicle coordinate with the origin located at the CG. The plane motions are described by the following rigid-body equations:

$$m\dot{V}_x = mV_y\dot{\psi} - 0.5C_a\rho aV_x^2 + F_x \quad (1)$$

$$m\dot{V}_y = -mV_x\dot{\psi} + F_y \quad (2)$$

$$I\ddot{\psi} = M_z \quad (3)$$

where V_x , V_y and $\dot{\psi}$ are the longitudinal velocity, lateral velocity and yaw rate of the vehicle. m and I are the total mass and yaw inertia of the vehicle. C_a is the drag coefficient, ρ is the density of air, a is the frontal area of the vehicle. Here, only the longitudinal air drag is considered. F_x , F_y and M_z represent the total longitudinal force, total lateral force and total yaw moment accumulated from the tire forces generated at the contact patches, which are given by:

$$F_x = \sum_{i=1}^n (F_{xl,i} + F_{xr,i}) \quad (4)$$

$$F_y = \sum_{i=1}^n (F_{yl,i} + F_{yr,i}) \quad (5)$$

$$M_z = \sum_{i=1}^n (-F_{xl,i}d_i/2 + F_{xr,i}d_i/2 + F_{yl,i}l_i + F_{yr,i}l_i) \quad (6)$$

where $F_{x\Box,i}$, $F_{y\Box,i}$ are the lateral and longitudinal tire forces of the i th axle in vehicle coordinate. In the following discussions, we shall use the subscript symbol $\Box$ to refer to the left side (l) or right side (r) of the vehicle for all relevant parameters and variables.

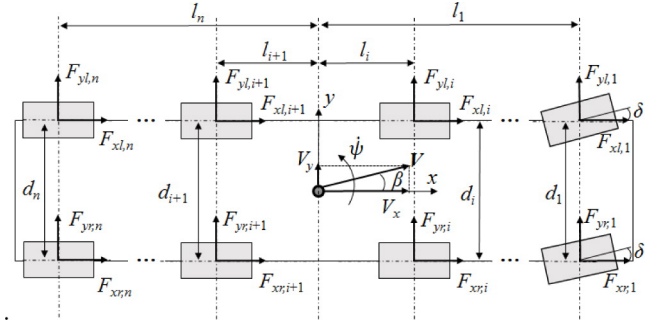


Figure 1. Schematic of a vehicle with n -axes

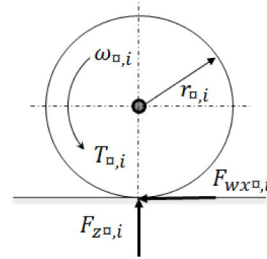


Figure 2. Schematic for wheel dynamics

Wheel and Tire Model

In this paper, we assume the same wheel and tire are installed at all the axles. The dynamics for either left or right side, represented by $\Box$, of wheel in axle i is shown in Figure 2 and given by:

$$J_{w\Box,i}\dot{\omega}_{\Box,i} = T_{\Box,i} - F_{wx\Box,i}r_{\Box,i} \quad (7)$$

Where $\omega_{\Box,i}$ is the wheel speed, $J_{w\Box,i}$ and $r_{\Box,i}$ are the spinning inertia and the effective radius of the wheel. $T_{\Box,i}$ is the torque applied by the motor or braking pad on the wheel. $F_{wx\Box,i}$ and $F_{wy\Box,i}$ are the longitudinal and lateral forces generated from the contact patch in tire coordinate, which can be transferred, as shown in Figure 3, to the vehicle coordinate by:

$$\begin{bmatrix} F_{x\Box,i} \\ F_{y\Box,i} \end{bmatrix} = \begin{bmatrix} \cos \delta & -\sin \delta \\ \sin \delta & \cos \delta \end{bmatrix} \begin{bmatrix} F_{wx\Box,i} \\ F_{wy\Box,i} \end{bmatrix} \quad (8)$$

If no road wheel steering angle is applied, as those on the non-front axles, the tire forces in tire-coordinate will be the same as those in the vehicle coordinate.

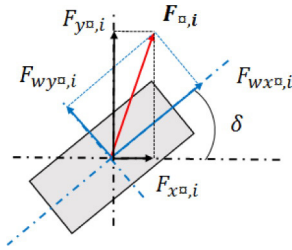


Figure 3. Relation between tire forces in tire coordinate and vehicle coordinate for steerable axle

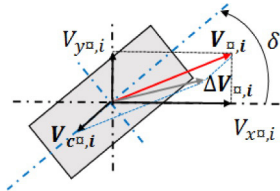


Figure 4. Wheel velocity relationships in different coordinates

We adopt the simplified Pajeka tire model proposed in [14] and defined in vehicle coordinates to calculate the tire forces. The tire force vector $F_{v,i} = [F_{xv,i} \ F_{yv,i}]^T$ and the peak value of the tire force $\bar{F}_{v,i}$ are given by:

$$F_{v,i} = \bar{F}_{v,i} \sin[C_{v,i} \operatorname{atan}(B_{v,i} \|s_{v,i}\|/\mu_{v,i})] s_{v,i} / \|s_{v,i}\| \quad (9)$$

$$\bar{F}_{v,i} = \mu_{v,i} F_{z0v,i} [1 + k_{Fz0v,i} (F_{z0v,i} - F_{zv,i}) / F_{z0v,i}] \quad (10)$$

with the friction coefficient of the contact patch $\mu_{v,i}$, the tire model parameters, including the shape factor $C_{v,i}$, stiffness factor $B_{v,i}$, digression factor $k_{Fz0v,i}$ and nominal load $F_{z0v,i}$, and wheel vertical load $F_{zv,i}$.

Due to the redundancy of the multi-axle configuration, the individual vertical load of each wheel is statically indeterminate. To overcome this difficulty, we adopt the hierarchical static balance method, including a compression based algorithm, proposed in [15], to approximate the vertical load at each axle, and include a roll-angle based algorithm to approximate the vertical load at the left and right wheel of each axle. Readers are referred to the [Appendix](#) of this paper for further details of how this is done.

In (9), the forces are computed based on the slip vector $s_{v,i} = [s_{xv,i} \ s_{yv,i}]^T$ defined by:

$$s_{v,i} = -\Delta V_{v,i} / \|V_{v,i}\| \quad (11)$$

where $\Delta V_{v,i}$ is the relative velocity vector at the contact patch, obtained by

$$\Delta V_{v,i} = V_{cv,i} + V_{v,i} \quad (12)$$

with the wheel circumference velocity vector $V_{cv,i} = -\omega_{v,i} r_{v,i} [\cos \delta \ \sin \delta]^T$ and wheel center velocity vector $V_{v,i} = [V_{xv,i} \ V_{yv,i}]^T$, as shown in [Figure 4](#).

From [Figure 1](#), $V_{v,i}$ on the left or right side of the vehicle can be determined based on the states and the kinematic parameters of the global motion of the vehicle by:

$$V_{l,i} = \begin{bmatrix} V_x - d_i \dot{\psi} / 2 \\ V_y + l_i \dot{\psi} \end{bmatrix} \quad (13)$$

$$V_{r,i} = \begin{bmatrix} V_x + d_i \dot{\psi} / 2 \\ V_y + l_i \dot{\psi} \end{bmatrix} \quad (14)$$

CONTROL SYSTEM FRAMEWORK

The whole aim of the control framework at controlling the vehicle is to track the response of a reference model, which might have specifically designated performance, by coordinating the torque of the hub-motor (and/or wheel brake) with appropriate control allocation (CA). The framework follows the hierarchical structure, as shown in [Figure 5](#). It mainly consists of 3 modules including the reference model, global motion controller and CA. Each of these modules is described in further detail below.

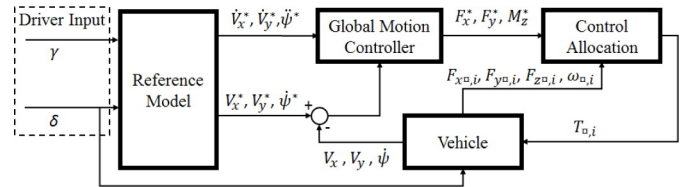


Figure 5. Control framework

Reference Model

The reference model is used to generate the reference (desired) response X^* composed of longitudinal velocity V_x^* , lateral velocity V_y^* and yaw rate $\dot{\psi}^*$ for a given driver input of road wheel steering angle δ and interpreted gas/brake pedal position γ . The reference model can be represented by the following standard transfer functions [4]:

$$V_x^* = \frac{K_V}{1 + T_V s} \gamma \quad (15)$$

$$V_y^* = \frac{A_{Vy} (1 + T_{Vy} s)}{1 + \frac{2\zeta_{Vy}}{\omega_{n,Vy}} s + \frac{1}{\omega_{n,Vy}^2} s^2} \delta \quad (16)$$

$$\dot{\psi}^* = \frac{A_{\dot{\psi}} (1 + T_{\dot{\psi}} s)}{1 + \frac{2\zeta_{\dot{\psi}}}{\omega_{n,\dot{\psi}}} s + \frac{1}{\omega_{n,\dot{\psi}}^2} s^2} \delta \quad (17)$$