

Overview of the RIFF Technique: Source Identification, Defect Detection and in-situ Material Properties Measurement by Verification of the Local Motion Equation

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ABSTRACT

Identification of vibration sources, defects and/or material properties consists generally in solving inverse problems. The called RIFF method (French acronym meaning Windowed and Filtered Inverse Solving) is one way to solve this kind of inverse problem. The basic principle of the RIFF approach consists in measuring vibration displacement on a meshgrid in a local area of interest, injecting measured data in the motion equation and calculating the searched unknown. Compared to other usual inverse techniques, the RIFF method has the curious particularity of needing the knowledge of the local motion equation only. Boundary conditions, sources or dynamic behaviors outside the area of interest can be completely ignored, whereas they are required for the direct problem solving. The searched unknown can then be identified locally with respect to the frequency and can be mapped by using a scanning process of the area of interest. However, as in all inverse methods, the RIFF approach is very sensitive to uncertainties in measured data, so that it is practically impossible to apply it without a regularization technique. Several approaches based on the use of low-pass wavenumber filtering are presented. Initially developed for vibration source identification, the first examples shown concern location and quantification of point forces and/or moments. A special focus is also proposed on aeroacoustic excitation identification. Since the method corresponds to a local verification of the motion equation, defects can be located. An experimental example on a notch detection is shown on a plate. Finally, the recent development on identification of elastic and damping material properties is presented. Examples showing the possibility to characterize composite materials in medium and high frequency ranges and to realize maps of material characteristics are presented. Actual works and perspectives in industrial applications are discussed at the end of the presentation.

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INTRODUCTION

Vibration and vibroacoustic comportments of structures are always important subjects of mechanical engineering. Usually, the main goal of these studies is to calculate, or to simulate, vibrations or acoustic radiations due to some sources. To do that, engineers need to develop mechanical models describing the physical comportments in the studied problem. These developments need to identify sources, dynamic laws and values of physical parameters used in models. Physically, when the problem is to assess the response of the structure, the problem is called direct problem, since it corresponds to the effect assessment. On the contrary, when the sources, or the parameters of a given model are the searched quantities, the problem is called inverse problem, because the calculation consists in using measured effects in order to assess causes or the link between causes and effects. Identification problems can then be viewed as inverse problems. A lot of inverse problems are studied in the literature. Usually they have a common principle, that consists in finding data, minimizing a distance between measured effects and effects calculated from a direct calculation. Physically, inverse problems have the property to be very sensitive to errors in measured effects, so that they cannot be solved without a regularization technique. Several

regularization techniques exist, they consist in adding a constraint in the cost function to minimize. This added term implies a better robustness of the inverse problem, whereas the residue obtained by the calculation of the distance between measured effects and effects calculated from a direct calculation corresponds to the accuracy.

The technique exposed in this paper was developed from two decades and is called the RIFF method. The acronym RIFF comes from initials of the French definition "Résolution Inverse Filtrée Fenêtrée", that means Windowed and Filtered Inverse Solving. In the literature, this technique was also called "Force Analysis Technique" (FAT). Indeed, the RIFF method was first developed to assess forces due to vibration sources using a well known model of the structure and vibration measurements. Its application was studied for several kinds of structures, by using the analytical motion equation for simple structures, like beams [1][2], plates [3][4], shells [5][6] or Finite Element models [7, 8, 9, 10] for structures having more complex geometries. The ability to reconstruct several kinds of excitations was also studied. In the following, the case of point mechanical excitations and the case of aeroacoustic excitations are shown. As it will be shown in this paper, the main originality of this technique is in the fact that the inverse mechanical problem can be seen as a

local problem, because the dynamic equilibrium of the structure is always based on Fundamental Principles of Dynamics of the elementary parts of the structure. The inverse problem can then be solved without all knowledge required by direct problems. In other words, the inverse problem can be solved on each elementary part without any knowledge on the rest of the structure, like boundary conditions, other excitations and other local dynamic comportments. Curiously, the vibration inverse problem seems to be simpler than the direct problem, the philosophy of the RIFF method seeks then to benefit from this particularity.

The basic principle of the RIFF method is to verify the local equilibrium of the structure. If external excitations are well known on some parts of the structure, the RIFF technique can then be used to identify other things. For example, it can be used for defect detection in some parts where no external excitation is applied. This idea was studied by several authors [7][11][12] and an example is presented in this paper. In the same vein, the RIFF technique can also be used to identify model parameters. The last example exposed in this paper comes from recent and actual works [13][14][15]. It is focused on the identification of equivalent elastic and damping properties of composite materials. In comparison with classical modal techniques, the main originality remains the fact that the proposed method is local, so that Young's modules and damping coefficients can be spatially identified at all frequencies.

BASIC PRINCIPLE OF THE RIFF TECHNIQUE

Use of the Analytical Motion Equation

The basis of the RIFF method consists in verifying the local dynamic equilibrium of an elementary domain of the structure. This equilibrium can be given by an analytic differential equation. Usually, the RIFF method is used in the frequency domain such that harmonic motion equations are taken into account and excitations are characterized at each studied frequency. Considering the example of a plate excited by a harmonic force distribution, the local motion equation can be written as:

$$D\Delta^2 w - \rho h \omega^2 w = F, \quad (1)$$

where Δ^2 represents the bi-Laplacian operator, D , ρ , h are respectively the rigidity, the mass density and the thickness of the plate, w is the out of plane displacement and F is the force distribution. The usual direct problem consists in calculating w knowing F and the parameters of the plate. In this case, all values of the force distribution F in the space domain and boundary conditions must be known, because this problem corresponds to a differential equation solving. Now, if w is known and the problem is to calculate F or one of the parameters of the plate in a specific location, the motion equation (1) seems to be sufficient, forces outside the studied region and boundary conditions can be completely ignored. This remark reveals the local aspect of inverse problems such that they can be solved without all required knowledge of direct problems. The RIFF Method is attempting to use this advantage.

In Practice, the displacement field w cannot be measured continuously and spatial derivatives in the bi-Laplacian operator cannot be assessed analytically. A discrete form of Eq.(1) must then be used. Using a regular mesh, the discrete form of the motion equation can be written as:

$$D(\delta_{i,j}^{4x} + \delta_{i,j}^{4y} + 2\delta_{i,j}^{2x2y}) - \rho h \omega^2 w_{i,j} = F_{i,j}, \quad (2)$$

where i and j indicates indexes of the point where the force distribution is calculated, $\delta_{i,j}^{4x}$, $\delta_{i,j}^{4y}$, $\delta_{i,j}^{2x2y}$ symbolize Finite difference schemes approximating the fourth derivatives of w at the point (i,j) . For example, by using a first order development of Taylor series, the scheme $\delta_{i,j}^{4x}$ corresponds to:

$$\delta_{i,j}^{4x} = \frac{1}{\Delta_x^4} (w_{i+2,j} - 4w_{i+1,j} + 6w_{i,j} - 4w_{i-1,j} + w_{i-2,j}), \quad (3)$$

where Δ_x represents the constant spacing between points in the x direction.

Regularization by Windowing and Filtering

Even if the look of the proposed technique seems not to be an inverse problem solving, it has physically the same difficulty of inverse problems that is its huge sensitivity to errors in input data. The amplification of noise is here due to the calculation of fourth spatial derivatives, which cannot be calculated precisely from measured data that contains inherent uncertainties. The amplification of noise is also more important in the high wavenumber domain such that a way to regularize the inverse problem is to use a low-pass wavenumber filtering. In practice, it is made by a convolution product with a finite point spread function that avoid a spatial integration on the whole domain of the structure:

$$F_{i,j}^{regularized} = \Delta_x \Delta_y \sum_{k,l} F_{k,l} \cdot h_{i-k,j-l}, \quad (4)$$

where $h_{i,j}$ is to the discrete point spread function of the filter. Any kind of low-pass filter can be used. Usually, the following point spread function is used. It corresponds to 2D sinc functions weighted by 2D Hanning windows:

$$h_{i,j} = \frac{1}{N_{f,i,j} \Delta_x \Delta_y} \left(1 + \cos \frac{k_c \cdot i \cdot \Delta_x}{2} \right) \left(1 + \cos \frac{k_c \cdot j \cdot \Delta_y}{2} \right) \sin(k_c \cdot i \cdot \Delta_x) \sin(k_c \cdot j \cdot \Delta_y), \quad (5)$$

for $i \cdot \Delta_x$ and $j \cdot \Delta_y \in [-2\pi / k_c : 2\pi / k_c]$ and $h_{i,j} = 0$ otherwise. In this expression, the parameter k_c corresponds to the cut-off wavenumber of the filter.

Because the measured area has finite dimensions, there is a spatial truncation in the space domain. In order to avoid the Gibbs phenomenon due to smoothing of truncations, it is recommended to

apply, before filtering, a windowing attenuating slowly the truncations near boundaries. Usually, a Tuckey window is used, where the weighted length is set equal to the cutoff wavelength of the filter.

The level of regularization is an important parameter that must be tuned. In the RIFF method, the level of regularization is given by the value of the cutoff wavenumber k_c used in the filtering procedure. In [1], the study of the optimal value of k_c is made by the comparison of exact and noisy simulations. Except at the resonances, where it appears to be impossible to regularize the technique, an optimum value of k_c can be obtained. It depends linearly only on the level of noise and the square root of the frequency. The following empirical rule was then defined:

$$k_c = a.k_N, \quad (6)$$

where a is a coefficient called "regularisation parameter" and k_N is the natural bending wavenumber of the plate depending on the frequency by the well known dispersion equation:

$$k_N = \sqrt[4]{\frac{\rho h}{D}} \sqrt{\omega}. \quad (7)$$

The adjustment of the regularisation parameter a depends on the level of noise. Its value is usually chosen from the following rule:

- $a = 4$ for very accurate measurements, typically made in laboratory with deterministic signals,
- $a = 2$ for accurate measurements, typically made in laboratory with random signals,
- $a = 1$ for measurements made on industrial machine in a noisy environment.

Regularization by a Corrected Finite Difference Scheme

The corrected finite difference scheme consists in developing a scheme approximating spatial derivatives with a control of the natural low-pass wavenumber filtering of the scheme. With such a scheme, the use of windowing and filtering can be avoided and the number of required points is that of the used scheme at all studied frequencies. The principle of the corrected finite difference scheme is completely described in [16] for beams and plates. In the case of beams, the used discrete motion equation of the beam is defined by:

$$EI\mu^4 \delta_i^{4x} - \rho S \omega^2 w_i = F_i, \quad (8)$$

where E , I , ρ , S are respectively the Young Modulus, the section inertia, the mass density and the section of the beam, δ_i^{4x} is the same development given in eq. (3) and μ^4 is a correction coefficient defined by:

$$\mu^4 = \frac{k_N \cdot \Delta}{\sqrt{2 - 2 \cos(k_N \cdot \Delta)}}. \quad (9)$$

With this correction, the used finite difference scheme corresponds to a low-pass filtering where the cutoff wavenumber k_c is equal to the bending wavenumber of the beam k_N . However, the severity of the filter depends on the number of points per wavelength, so on the frequency. Figure 1 shows the wavenumber response of this filter for several numbers of points per wavelength.

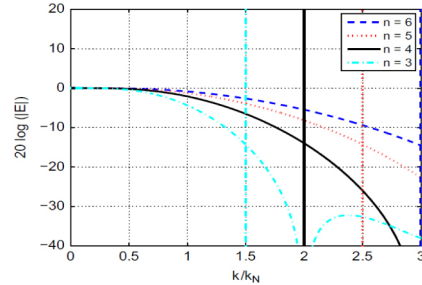


Figure 1. Transfer functions of the low-pass wavenumber filter corresponding to the corrected Finite difference scheme (from [16]) of a beam. k_N corresponds to the flexural wavenumber, n corresponds to the number of points per wavelength.

In the case of plates, obtaining a pure low-pass wavenumber filtering is not possible analytically, because a singularity at the bending wavenumber of the plate (corresponding to a circle in the k_x - k_y plan) cannot be entirely eliminated. The proposed Finite difference scheme is then based on the elimination of the singularity only for three directions: x , y and the diagonal. The corrected scheme contains then two coefficients μ^4 and ν^4 , and has this expression:

$$D(\mu^4 \delta_{i,j}^{4x} + \mu^4 \delta_{i,j}^{4y} + 2\nu^4 \delta_{i,j}^{2x2y}) - \rho h \omega^2 w_{i,j} = F_{i,j},$$

$$\text{with } \mu^4 = \frac{k_N^4 \cdot \Delta^4}{4(1 - \cos(k_N \cdot \Delta))^2} \text{ and } \nu^4 = \frac{k_N^4 \cdot \Delta^4}{8 \left(1 - \cos\left(\frac{k_N \cdot \Delta}{\sqrt{2}}\right)\right)^2} - \mu^4. \quad (10)$$

Figure 2 shows the transfer function of this filter at one frequency in the k_x - k_y plan. The singularity is not completely eliminated, but it is highly attenuated.

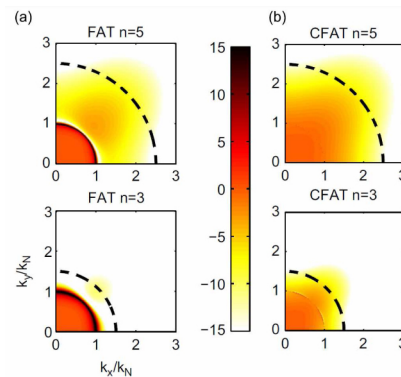


Figure 2. Transfer functions of the low-pass wavenumber filter corresponding to a) the finite difference scheme given in Eq. (2), b) the corrected finite difference scheme given in Eq. (10) (figure obtained from [16]).