

Structure-Borne Noise Source Characterization from a Bayesian Point of View

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ABSTRACT

In this paper, a local method of structure-borne noise source characterization is presented. It is based on measurements of transverse displacement and local structural operator knowledge and allows to localize and quantify sources without any need of boundary condition information. To fix the instability caused by measurement noise, the regularization step inherent to inverse problem is realized with a probabilistic approach, within the Bayesian framework. When *a priori* distributions about noise and sources are considered as Gaussian, the Bayesian regularization is equivalent to the well-known Tikhonov regularization. The optimization of the regularization is then performed by the Gibbs Sampling (GS) algorithm, which is part of Markov Chain Monte Carlo (MCMC) techniques. The whole probability of the regularized solution is inferred, providing access to confidence intervals. Both simulation and measurements of a beam excited by an harmonic point source are realized to validate this approach.

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INTRODUCTION

Efforts have been made to improve methods describing vibration propagation allowing a better estimation of model outputs, as a displacement field, but the input sources remain broadly unknown. This lack of knowledge of sources leads to a bias on the outputs. Hence, the identification of vibration sources is an important topic of vibroacoustics. Originally, vibration sources identification has been investigated in [1] and [2] using the concept of structural intensity. This technique generates a field of intensity vectors, allowing to locate vibration sources without quantifying them. Then, different methods based on modal models or frequency response functions, including Transfer Path Analysis (TPA) described in [3] or Statistical Energy Analysis described (SEA) described in [4], have been developed. A review of some of these methods can be found in [5]. As these methods often need informations about the structure, boundary conditions, position of sources or number of sources, they can be interpreted as global methods in opposite to local methods which only need local information about the structure. Simultaneously, [6] and [7] developed a local approach based on the numerical discretization of the equation of motion of the structure to identify the vibration force. The difference between these two methods is the way that they deal with the measurement noise. This step represents the regularization inherent to inverse problem, as it will be introduced below. [6] passes through the wavenumber domain while [7] uses a convolution with a truncated convolution kernel. From that point of view, the second method, called RIFF (Résolution Inverse Filtrée Fenêtrée in French), is more local and will serve as a starting point for this present work. The RIFF method has been

originally developed for beams and plates, recent works adapted it to cylindrical shells [8] and to more complex structures by using a Finite Element Method [9].

As mentioned above, the measurement noise causes problems to the RIFF method and more generally to inverse problems because it is interpreted as the result of multiple unphysical sources instead of measurement perturbations. Mathematically, this instability results from the matrix linking inputs to outputs which is ill-conditioned. The ratio between its maximal and minimal eigenvalues is too high. To ensure that the estimated sources have a physical behavior, one should apply a regularization step to fix the ill-conditionness of the matrix. The Truncated Singular Value Decomposition (TSVD) method described in [10] improves the stability of a matrix by taking into account only eigenvalues higher than a specific value to ensure that the ratio between maximal and minimal eigenvalues is reasonable. As another possibility, the Tikhonov regularization, proposed in [11] is most widely adopted. The concept is to increase low eigenvalues to get an appropriate ratio. The low pass filter method presented in [7] gives similar results with a more physical approach. The optimization of the regularization (i.e. the number of eigenvalues to take into account or the threshold of the low eigenvalues) can be realized by Generalized Cross Validation (GCV) or by the L-curve proposed respectively in [12] and [13]. To find the value of an optimization parameter, the first one compares perturbation errors and regularization errors while the second one compares residuals and regularized solution. Recently, the Bayesian framework has been investigated for the regularization and optimization of inverse problems (see [14] and [15]). In this work,

the regularization step is realized within the Bayesian framework because it makes possible to take into account various *a priori* informations and to formalize them mathematically, making this approach more practical than Tikhonov method. Applications have been developed mainly in econometrics, biology and social sciences. In vibration and acoustics, [16] and [17] have used the Bayesian framework for acoustic holography, [18] has used it for sparse source identification. To go further, Markov Chain Monte Carlo (MCMC) algorithms [19] allow to infer to whole *a posteriori* probability on unknown parameters from numerical sampling, yielding to a better exploration of probabilities and confidence intervals. The Gibbs sampling [20], a special case of these methods, is particularly suited to problems where the probabilities are well-known, such as Gaussian or gamma distributions. After this introduction, the identification method based on measurements of displacement field is described. The Bayesian regularization is then introduced with Gaussian priors for the Gibbs sampling, followed by a simulation of a beam excited by a point source in order to illustrate these methods. Experimental validation also on a beam is then presented, before a conclusion.

SOURCE CHARACTERIZATION

Equation of Motion

The RIFF method is based on the equation of motion of a known structure. As an example, the method is presented on a beam within Euler-Bernoulli beam theory. The transverse displacement $w(x, t)$ of the beam satisfies [equation 1](#),

$$E(1 + j\eta) \frac{bh^3}{12} \frac{\partial^4 w(x, t)}{\partial x^4} + \rho b h \frac{\partial^2 w(x, t)}{\partial t^2} = f(x, t), \quad (1)$$

where E is the Young's modulus, η is the loss factor, b is the width, h is the thickness, $w(x, t)$ is the transverse displacement at location x and at time t , ρ is the density and $f(x, t)$ is the vibration source at location x and at time t . The principle of the method is to evaluate the transverse displacement field by measurement and, knowing the rest of the terms of left hand side of [equation 1](#), to calculate directly the vibration source applied on the structure.

Estimation of Derivatives

The temporal derivative can be simplified in harmonic motion, with temporal convention $e^{+j\omega t}$, leading to

$$E(1 + j\eta) \frac{bh^3}{12} \frac{\partial^4 w(x, \omega)}{\partial x^4} - \rho b h \omega^2 w(x, \omega) = f(x, \omega), \quad (2)$$

where ω is the angular frequency and where $w(x, \omega)$ and $f(x, \omega)$ now depend on ω . The spatial derivative is estimated by numerical differentiation, using centered finite difference method at second order, from

$$\frac{\partial^4 w(x_i, \omega)}{\partial x^4} \approx \frac{1}{\Delta_x^4} \cdot \left(w(x_{i+2}, \omega) - 4w(x_{i+1}, \omega) + 6w(x_i, \omega) - 4w(x_{i-1}, \omega) + w(x_{i-2}, \omega) \right), \quad (3)$$

where Δ_x is the constant spatial sampling rate. Substituting [equation 3](#) into [equation 2](#) allows to identify the vibration source applied at location x_i without having to evaluate physical boundary conditions and displacement all over the domain, as proposed in [equation 4](#)

$$E(1 + j\eta) \frac{bh^3}{12} \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta_x^4} - \rho b h \omega^2 w_i(\omega) = f_i(\omega). \quad (4)$$

Due to the spatial derivative, two displacement points are used at each boundary of the studied area, without allowing the evaluation of sources at these boundaries. At these points, the central stencil [1; -4; 6; -4; 1] can then be substituted by the corresponding forward stencil [3; -14; 26; -24; 11; -2] or the backward one [-2; 11; -24; 26; -14; 3] in order to increase the domain of source identification with the same number of measures. It is important to underline that [Equation 2](#) (and its discrete form in [Equation 4](#)) describes locally the dynamic comportment of an elementary part of the structure. Consequently, if displacements and derivatives of displacement are known in a subpart of the structure (by measurement), the calculation of the force distribution in this subpart is possible without other informations outside the subpart. Physical boundary conditions of the structure and displacement field outside the studied area can then be ignored. In practice, this "local" property of the inverse method presents a real advantage over global methods, which depend on boundary conditions (ie. modal analysis) and/or need a description of displacement field on the whole domain of the structure.

Matrix Formulation

In the following, the equation of motion given by [equation 4](#) will be used in matrix form

$$D \cdot w = f, \quad (5)$$

where D is a $N \times N$ symmetric matrix resulting of the discretization of the structural operator by forward, central and backward stencils, w is the $N \times 1$ vector of displacement and f is the $N \times 1$ vector of vibration sources. As each measurement includes noise, the equation of observation is given by

$$y = w + n, \quad (6)$$

where y is the $N \times 1$ vector of observed displacement and n is the $N \times 1$ vector of noise. Applying the method of source identification to the accessible quantity y yields

$$D \cdot y = f + D \cdot n$$

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n is often much lower than w but D is a differential operator. As the displacement field remains mainly smooth while the noise is completely random, the term $D \cdot n$, called aberrant forces, can quickly becomes much higher than f , making difficult the source identification. Thus, a regularization step is needed.

REGULARIZATION WITHIN THE BAYESIAN FRAMEWORK

In lots of methods, an especially in inverse methods, some quantities are badly known and sometimes completely unknown. Working with a probabilistic point of view is then a suitable approach to take into account these uncertainties. The Bayesian framework used in this work allows to include the *a priori* information in the identification process.

Bayes' Theorem

Starting from the conditional probability formula

$$[A, B] = [A | B] \cdot [B] = [B | A] \cdot [A], \quad (8)$$

where $[A]$ stands for the probability density of the random variable A , $[A, B]$ is the joint probability density for A and B and $[B | A]$ is the conditional density of B knowing A , Bayes proposed the following theorem to infer the probability of an event from both experiences and *a priori* knowledge

$$[A | B] = \frac{[B | A] \cdot [A]}{[B]} \propto [B | A] \cdot [A]. \quad (9)$$

$[A | B]$ is called the *a posteriori* probability density, $[B | A]$ is the likelihood which expresses the information extracted from the experience, $[A]$ is the *a priori* probability density from *a priori* knowledge and $[B]$ is the marginalized likelihood or the evidence. The evidence acts as a multiplicative constant to ensure that the product of the *a priori* density and the likelihood is still a probability density (the integral must be equal to 1). The shape of the *a posteriori* density is used to evaluate the most probable value corresponding to the regularized solution. As it is not impacted by the evidence, the proportional relationship is preferred in [equation 9](#) rather than the absolute equality.

Priors

Considering an additive white noise, the prior distribution on n can be expressed as

$$[n] \propto \mathcal{N}_c(n; 0, \tau_n^{-1} \cdot \mathbf{I}) \quad (10)$$

with τ_n the precision to be inferred, $\mathbf{I}$ the $N \times N$ identity matrix, and the M -dimensional multivariate complex normal density on x with mean vector μ and covariance matrix Σ is defined as

$$\mathcal{N}_c(x; \mu, \Sigma) = \frac{1}{\pi^M |\Sigma|} \exp \left((x - \mu)^H \Sigma^{-1} (x - \mu) \right). \quad (11)$$

Although the sources are unknown, their energy is limited, so the prior distribution on f can also be expressed as

$$[f] \propto \mathcal{N}_c(f; 0, \tau_f^{-1} \cdot \mathbf{I}) \quad (12)$$

with τ_f the precision to be inferred too. The two precisions being always positive, an appropriate prior density is then the gamma density

$$[\tau_n] \propto \mathcal{G}(\tau_n; \alpha_n, \beta_n) \quad (13)$$

$$[\tau_f] \propto \mathcal{G}(\tau_f; \alpha_f, \beta_f) \quad (14)$$

defined on x with parameters α and β as

$$\mathcal{G}(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot \exp(-\beta \cdot x) \quad (15)$$

Likelihood

From [equations 7](#) and [10](#), the probability on the observations y knowing the sources, the structural operator and the measurement noise (ie. the probability corresponding to the direct problem) can be expressed as

$$[y | D, f, \tau_n] \propto \mathcal{N}_c(y; D^{-1} \cdot f, \tau_n^{-1} \cdot \mathbf{I}) \quad (16)$$

Posteriors

Applying the Bayes theorem of [equation 9](#) to the unknown quantities of interest f , τ_n and τ_f yields

$$[f | D, y, \tau_f, \tau_n] \propto \mathcal{N}_c(f; 0, \tau_f^{-1} \cdot \mathbf{I}) \cdot \mathcal{N}_c(y; D^{-1} \cdot f, \tau_n^{-1} \cdot \mathbf{I}) \quad (17)$$

$$\propto \mathcal{N}_c(f; \mu_f, \Sigma_f) \quad (18)$$

with :

$$\Sigma_f = \left(D^{-H} \cdot D^{-1} \cdot \tau_n + \tau_f \cdot \mathbf{I} \right)^{-1} \quad (19)$$

$$\mu_f = \Sigma_f \cdot D^{-H} \cdot \tau_n \cdot y \quad (20)$$

where the superscript H stands for the Hermitian transformation.