

Source Separations and Identification by Structural Holography

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ABSTRACT

The source field reconstruction aims at identifying the excitation field measuring the response of the system. In Near-field Acoustic Holography, the response of the system (the radiated acoustic pressure) is measured on a hologram using a microphones array and the source field (the acoustic velocity field) is reconstructed with a back-propagation technique performed in the wave number domain. The objective of the present works is to use such a technique to reconstruct displacement field on the whole surface of a plate by measuring vibrations on a one-dimensional holograms. This task is much more difficult in the vibratory domain because of the complexity of the equation of motion of the structure.

The method presented here and called "Structural Holography" is particularly interesting when a direct measurement of the velocity field is not possible. Moreover, Structural Holography decreases the number of measurements required to reconstruct the displacement field of the entire plate. This method permits to separate the sources in the case of multi-sources excitations by considering them as direct or back waves. It's possible to compute the structural intensity of one particular source without the contributions of others sources.

The purpose of this paper is to introduce the Structural Holography method. The first part presents the theoretical background of the method. A numerical simulation of displacement fields generate by few sources for an infinite plate is presented in a second part. The structural intensity for each source is computed by removing the contribution of others source. Finally, some results are presented for a simply supported plate.

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INTRODUCTION

Source identification is a very important diagnostic tool in noise and vibration control. Many methods have become an increasingly powerful research tool and allow today to understand and predict the structure behavior. The method presented in this paper is called Structural Holography and allows to identify and separate sources present on a plate by remote measurements of the displacement. The goal is to reconstruct an entire two-dimensional wave field from few one dimensional sensor lines (the holograms). Therefore, this method offers a drastic reduction of measurements and allows to characterize the vibratory phenomenon when a direct measurement is impossible.

Structural Holography is based on the method applied in the acoustic domain, introduced by Williams and Maynard called Near-field Acoustical Holography (NAH). NAH is a powerful method to reconstruct the velocity distribution of a vibrating plate from the acoustic pressure hologram measured from a microphone array in the near-field [1]. Since few years, NAH, which was initially devoted to stationary sources, has evolved and hereafter can be used through RT-NAH (Real-Time Near-field Acoustic Holography) for continuously visualizing nonstationary acoustic fields [2] or through TDH (Time Domain Holography) for reconstructing sound data blocks in the time domain [3]. NAH can be also coupled with other techniques to identify vibration sources from radiated noise measurements [4].

In the vibratory domain, many methods were developed to localize and quantify the sources present on plates. Pavic [5] and Noiseux [6] have developed the structural intensimetry, defined with acoustic analogy, to analyze the flexural wave propagation in simple structures. Gavric showed the potentiality of numerical calculation of structural intensity [7, 8] and demonstrated that this approach can be used for modal models obtained from the experimental modal analysis [9]. Another method for the localization of source is the force analysis technique (FAT) also known as the RIFF technique introduced by Pézerat [10,11], which uses a finite difference scheme to discretize the equation of motion and localize the force distribution acting on a structure like beams [12], plates [11] and shells [13,14].

The present work aims at presenting another method to identify and separate sources present on plates. The separation source is a technique already applied in the wave number domain by Ming-Te Cheng [15] to separate the incident and scattered sound fields. This method is based on the principle that any waveform can be decomposed using a two-dimensional spatial Fourier transform into wave components that propagate in a known manner [15]. Although structural holography method presented in this paper is also performed in the wave-number domain, the approach is different. Indeed, structural holography provides source separation in the case of multi-source excitations by considering them as direct or back waves with only two measurement lines of the displacement.

The purpose of this paper is to introduce the structural Holography method. The first part presents the theoretical background of the method. The far-field and near-field formulations are introduced. The separation source principle is presented and applied on a noisy displacement field numerically simulated, generated by 2 sources. The study object is a simply supported plate. To increase the accuracy of the identification source, the structural intensity approach is then applied to the reconstructed displacement field.

FORMULATION OF THE PROBLEM

Structural Holography is based on the equation governing the forced flexural vibration of a thin plate:

$$\frac{Eh^3(1+j\eta)}{12(1-\nu^2)} \left(\frac{\partial^4 w(x,y,t)}{\partial x^4} + \frac{\partial^4 w(x,y,t)}{\partial y^4} + 2 \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} \right) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = F(t)\delta(x,y), \quad (1)$$

where E is the Young modulus, ν the Poisson's ratio, h the thickness of the plate, η the damping, $F(t)$ the punctual force and $w(x,y,t)$ the transverse displacement. For $y > 0$, where no external force is applied, the equation becomes in the harmonic regime $e^{j\omega t}$:

$$\nabla^4 w(x,y) - k_f^4 w(x,y) = 0, \quad (2)$$

where $k_f^4 = \frac{\omega^2 \rho h}{B}$ and $B = \frac{Eh^3(1+j\eta)}{12(1-\nu^2)}$. By applying the spatial Fourier transform (SFT) to [equation \(2\)](#) over the x direction, the wavenumber spectrum $W(k_x, y)$ is obtained. The SFT of this displacement field is then:

$$W(k_x, y) = SFT[w(x, y)] = \int w(x, y) e^{jk_x x} dx. \quad (3)$$

Process to obtain the Structural Holography formulation is the same as Near-Field Acoustical Holography (NAH). However, the application of the SFT is only performed on 1 dimension in vibratory domain (2 dimensions for NAH). This operation yields a 4th order differential equation:

$$\frac{\partial^4}{\partial y^4} W(k_x, y) - 2k_x^2 \frac{\partial^2}{\partial y^2} W(k_x, y) + (k_x^4 - k_f^4) W(k_x, y) = 0, \quad (4)$$

which admits the general solution :

$$W(k_x, y) = C_{Reva}(k_x) e^{\sqrt{k_x^2 + k_f^2} y} + C_{eva}(k_x) e^{-\sqrt{k_x^2 + k_f^2} y} + C_{Rmix}(k_x) e^{\sqrt{k_x^2 - k_f^2} y} + C_{mix}(k_x) e^{-\sqrt{k_x^2 - k_f^2} y}. \quad (5)$$

This solution has 4 coefficients which is a major difference with NAH. Unlike in acoustic domain, structural Holography requires 4 holograms to identify the 4 unknowns of [equation \(5\)](#). Coefficients $C_{Reva}(k_x)$ and $C_{eva}(k_x)$ characterize purely evanescent waves for all values of k_x . Coefficients $C_{Rmix}(k_x)$ and $C_{mix}(k_x)$ define propagating waves for $k_x \leq k_f$ and evanescent waves for $k_x > k_f$. The letter R characterizes the back-waves.

To reconstruct the displacement with structural holography method, one must follow few step describe below:

- Measure displacements with on the holograms
- Apply a STF on the holograms to identify the coefficient
- Compute the wave number spectrum $W(k_x, y)$ on the entire plate with [eq.\(5\)](#)
- Apply a ISTF on the wave number spectrum to reconstruct the displacement $w(x,y)$

However, structural holography is the general solution of a STF application on the [equation \(2\)](#). As has been shown above, this formula is valid only in zones where no external forces is applied. Therefore, this method cannot back-propagate the displacement beyond the force F . This limit is visible because it generates a very high increase of the amplitude of the reconstructed displacement. Therefore, all results present in this paper showing the reconstructed displacement before this phenomenon. Beyond, the fields are not illustrated.

Near-Field and Far-Field Formulation

In the near-field and with the presence of back waves, the formulation with 4 coefficients is used and the matrix form to identify the coefficients is:

$$\begin{pmatrix} e^{(k_x^+)y_{h1}} & e^{-(k_x^+)y_{h1}} & e^{(k_x^-)y_{h1}} & e^{-(k_x^-)y_{h1}} \\ e^{(k_x^+)y_{h2}} & e^{-(k_x^+)y_{h2}} & e^{(k_x^-)y_{h2}} & e^{-(k_x^-)y_{h2}} \\ e^{(k_x^+)y_{h3}} & e^{-(k_x^+)y_{h3}} & e^{(k_x^-)y_{h3}} & e^{-(k_x^-)y_{h3}} \\ e^{(k_x^+)y_{h4}} & e^{-(k_x^+)y_{h4}} & e^{(k_x^-)y_{h4}} & e^{-(k_x^-)y_{h4}} \end{pmatrix} \begin{pmatrix} C_{Reva}(k_x) \\ C_{eva}(k_x) \\ C_{Rmix}(k_x) \\ C_{mix}(k_x) \end{pmatrix} = \begin{pmatrix} W(k_x, y_{h1}) \\ W(k_x, y_{h2}) \\ W(k_x, y_{h3}) \\ W(k_x, y_{h4}) \end{pmatrix} \quad (6)$$

when $k_x^+ = \sqrt{k_x^2 + k_f^2}$, $k_x^- = \sqrt{k_x^2 - k_f^2}$, $W(k_x, y_{hi})$ with $i = 1, \dots, 4$ is respectively the wave number spectrum on the four holograms and y_{hi} the position of hologram i on y axis. As the evanescent waves exist only in the near-field, these coefficients can disturb the results if the holograms are positioned in the far field. In that case, taking into account the purely evanescent waves in the determination of the coefficients would lead to large errors. To avoid this behavior, the C_{Reva} and C_{eva} coefficients are not taking into account when the holograms are positioned in the far field. Therefore, [equation \(5\)](#) becomes:

$$W(k_x, y) = C_{Rmix}(k_x)e^{(k_x^-)y} + C_{mix}(k_x)e^{-(k_x^-)y}. \quad (7)$$

Thus, [equation \(7\)](#) is simplified and provides analytical formulation of the two coefficients:

$$C_{Rmix}(k_x) = \frac{-W(k_x, y_{h2}) + W(k_x, y_{h1})e^{-(k_x^-)d}}{e^{(k_x^-)y_{h1}}(e^{-(k_x^-)d} - e^{(k_x^-)d})}, \quad (8)$$

$$C_{mix}(k_x) = \frac{W(k_x, y_{h2}) - W(k_x, y_{h1})e^{(k_x^-)d}}{e^{-(k_x^-)y_{h1}}(e^{-(k_x^-)d} - e^{(k_x^-)d})}. \quad (9)$$

Source Separation Principle

The structural holography can be used to separate the sources. They are considered as forward waves or back waves and the hologram must be placed between the sources to separate. The source separation principle with holograms positioned in far-field is illustrated in [figure 1](#). In a first step, the holograms are positioned on the reference displacement $w_{sum} = w_{Fi} + w_{Fj}$ and the coefficients are identified with [equations \(8\) and \(9\)](#) (step (B)). We assume that the displacement w_{Fi} is the forward waves and w_{Fj} the back-waves. In this case, the wave number spectrum $W_{Fi}(k_x, y)$ is computed on the entire plate with the coefficient C_{mix} only ([equation \(10\)](#)) and by the ISFT, the singular behavior of the displacement w_{Fi} is obtained (step (C)). It is the same protocol with C_{Rmix} to compute $W_{Fj}(k_x, y)$ ([equation \(11\)](#)) and to reconstruct the displacement field w_{Fj} (step (D)).

$$W_i(k_x, y) = C_{mix}(k_x)e^{-(k_x^-)y}, \quad (10)$$

$$W_j(k_x, y) = C_{Rmix}(k_x)e^{(k_x^-)y}. \quad (11)$$

Filtering

The presence of noise in the measured signals causes measurement uncertainties. These small data errors are essentially located in the high wavenumber domain, amplified by back-propagation. Moreover, structural holography requires to apply the spatial Fourier transform to the holograms to identify the coefficients present in the general solution ([eq. 5](#)). As the Fourier transforms are performed with discrete data and over a finite hologram, numerical difficulties are occurred in the calculation of coefficients.

For these reasons, in order to not introduce noise during the back propagation operation, it is necessary to filter part of the evanescent waves before processing [\[16\]](#). Therefore, before computing the wavenumber spectra on the entire plate with structural holography, it is necessary to apply signal processing techniques such as filtering in the wave-number domain on the coefficients. This treatment overcomes numerical difficulties caused by the noise and the use of the Spatial Fourier Transform [\[17\]](#).

The purpose of filtering in the wave-number domain is to remove the noise in the wave number spectrum computed with structural holography. To achieve this goal, the choice of a proper filter size (k_x bandwidth) is crucial [\[18\]](#). If the filter size is too large, the noise may not be removed and the accessible distance for back-propagation can decrease. If the filter size is too small, useful information may be removed along with noise, and the results may not lead to the source positions [\[17\]](#).

In this paper, the wave-number domain filter used in the numerical experiment is a filter based on the Tukey window illustrated in [figure 2](#) [\[19\]](#):

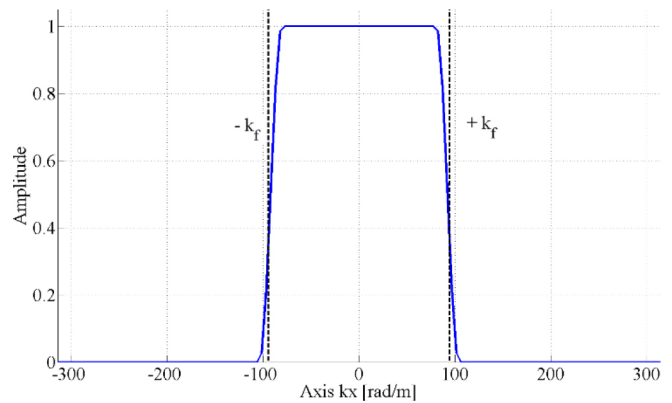


Figure 2. Filter in the wave number domain based on the Tukey window applied to the coefficients.