

# Efficiency and Durability Predictions of High Performance Racing Transmissions

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## ABSTRACT

Efficiency and durability are key areas of research and development in modern racing drivetrains. Stringent regulations necessitate the need for components capable of operating under highly loaded conditions whilst being efficient and reliable. Downsizing, increasing the power-to-weight ratio and modification of gear teeth geometry to reduce friction are some of the actions undertaken to achieve these objectives. These approaches can however result in reduced structural integrity and component durability. Achieving a balance between system reliability and optimal efficiency requires detailed integrated multidisciplinary analyses, with the consideration of system dynamics, contact mechanics/tribology and stress analysis/structural integrity.

This paper presents an analytical model to predict quasi-static contact power losses in lubricated spur gear sets operating under the Elastohydrodynamic regime of lubrication. Tooth Contact Analysis (TCA) is used to predict variations in contact loads, local surface curvature and rolling and sliding velocities. This is combined with an extrapolated oil film thickness formula available in literature, to predict instantaneous lubricant film thickness and sub-surface stresses. Subsequently, viscous and boundary friction are estimated, enabling calculation of power losses.

The presented methodology has been used to investigate the effects of parabolic tip relief on power loss and induced sub-surface stresses. The results of this investigation are also presented.

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## INTRODUCTION

Transmissions of high performance racing vehicles can routinely transmit contact loads in excess of 20 kN. Thus, small variations in gear meshing efficiency can have a significant effect on the magnitude of power loss. These transmissions are also particularly compact, yielding highly concentrated tooth meshing contacts. Coupled with relatively high rolling and sliding velocities, small variations in the contact footprints can result in noticeable change in transmission efficiency. A methodical approach capable of predicting the parameters which affect transmission efficiency in such a system is the key to making informed engineering decisions ranging from appropriate geometrical modifications to lubrication strategies.

During operation, loaded gear teeth routinely experience contact pressures exceeding 1-2 GPa. This results in significant piezo-viscous action, with the lubricant behaving as an amorphous solid, whilst the solid bodies in contact undergo elastic deformation, a condition referred to as Elastohydrodynamic Lubrication (EHL).

Reliable estimation of lubricant film thickness is required for accurate evaluation of contact friction, comprising that generated by the shear of a thin lubricant film, as well as boundary friction due to potential asperity interaction on the contiguous rough surfaces. Ertel [1] was

the first to incorporate both the localised elastic deformation of mating surfaces and the piezo-viscous action of the lubricant. He solved the piezo-viscous hydrodynamic problem while assuming that the surfaces were elastically deformed in accord with the small strain Hertzian contact theory.

Dowson and Higginson [2] produced a series of computed solutions over a wide range of speeds, loads and material properties and derived regression equations expressed in terms of three non-dimensional groups to predict minimum film thickness for line contact configuration, such as a rigid roller against a semi-infinite elastic solid. In the 1960s, Archard and Cowking [3] and Ranger et al [4] produced full numerical solutions of the 2D EHL problem for point contact. Hamrock and Dowson [5] extended these to the elliptical contact footprint geometry. They provided extrapolated regression formulae for central and minimum lubricant film thickness. Chittenden et al. [6] later extended the range of geometrical configurations to include the effect of angled entrainment flow into the contact, which occurs in some gearing applications such as those of hypoid gear pairs. In such contacts significant side leakage flow occurs away from the main flow in the direction of entraining motion along the semi-minor half-width of the elliptical point contact as demonstrated experimentally by Töp and Gohar [7]. Jalafi-Vahid

et al. [8] provided a detailed numerical analysis using isothermal Newtonian EHL, which agreed with the interferometric studies of Thorp and Gohar [7], thus underpinned the significance of side leakage flow from such elliptical point contact geometries, which affect the film thickness, thus the estimated friction.

Early solutions assumed fully flooded inlets, with isothermal Newtonian conditions, and usually at low to medium contact loads, owing to the limitations in computational power at the time. The issue of inlet starvation required use of inlet reverse flow boundaries such as those of Tipei [9] which were confirmed by the experimental works by Johns-Rahnejat and Gohar [10] and recently through combined experimental and numerical investigation in [11], which show reduced film thickness, thus increasing friction as would be expected. These conditions in transmission systems at high load and increased shear rate lead to thin thermo-elastohydrodynamic conditions investigated in some detail by Evans and Johnson [12].

Evans and Johnson [12] constructed traction maps for lubricants of known rheological properties in which different regimes of shear behaviour are described, depending on the operating conditions; load, speed and temperature. The lubricant behaviour was deemed to be either Newtonian, Eyring, viscoelastic, or elastic-plastic. A series of expressions for traction coefficients was derived for each of these regimes with which viscous friction of the lubricant can be evaluated.

Prediction of the film thickness using existing EHL formulations requires prior knowledge of instantaneous equivalent radii of curvature of meshing teeth pairs, as well as the sliding and rolling velocities of the contacting surfaces.

Tooth Contact Analysis (TCA) is employed for this purpose, which comprises a combination of finite element technique and Hertzian theory in order to obtain accurate representation of gear teeth contacting geometry as well as the calculation of the necessary kinematic and loading parameters for a quasi-statically loaded teeth pair. Although the classical methods of gear contact analysis are faster and computationally efficient [13], they do not take into account the effect of complex tooth modifications.

Numerous studies have been carried out in an effort to predict transmission efficiency. Mohammadpour et al. [14] developed a tribodynamic model to study the transient nature of transmission efficiency and NVH (noise, vibration and harshness) performance under various driving conditions. They combined multi-body dynamics with an analytical approach for non-Newtonian lubricant behavior in order to determine transmission efficiency. Xu et al. [15] carried out a parametric studies using a numerical thermal EHL model to assess the effect of gear geometry, tooth modifications, operating conditions, lubricant properties, manufacturing and assembly errors, and surface finish on mechanical efficiency of gears. Results of this study were validated by comparing the predictions with experimental measurements made on a spur gear rig.

The current study incorporates TCA and analytical non-Newtonian thermo-elastohydrodynamics (TEHL) to predict the instantaneous lubricant film thickness and friction in gear teeth contacts through typical meshing cycles. The simulated conditions are typical of high

performance racing transmissions during severe conditions of a race. The study also assesses the effect of gear tip relief and crowning on power losses over a single meshing cycle. Structural integrity of teeth under maximum contact pressures are also ascertained using the maximum reversing cyclic orthogonal sub-surface stresses.

## METHODOLOGY

The method integrates TCA, analytical TEHL and sub-surface stress analysis. The instantaneous radii of curvature, rolling and sliding velocities and normal load are obtained for a complete meshing cycle. These parameters form the input to the formulations for the instantaneous estimations of EHL film thickness, viscous friction, boundary friction and stress distributions.

### Film Thickness

The lubricant film thickness under the instantaneous operating conditions is obtained using Chittenden et al. [6] lubricant film thickness expression:

$$h_{c0}^* = 4.31 U_e^{0.68} G_e^{0.49} W_e^{-0.073} \left\{ 1 - \exp \left[ -1.23 \left( \frac{R_y}{R_x} \right)^{2/3} \right] \right\} \quad (1)$$

where, the prevailing contact conditions are represented by the following non-dimensional parameters:

$$W_e = \frac{\pi W}{2 E_r R_x^2}, \quad U_e = \frac{\pi \eta_0 U_r}{4 E_r R_x}, \quad G_e = \frac{2}{\pi} (E_r \alpha) \quad \text{and} \quad h_{c0}^* = \frac{h_{c0}}{R_x}$$

where,  $W$  is the normal load,  $E_r$  is the reduced elastic modulus of contact,  $R_x$  is the radii of curvature along the lubricant entrainment direction,  $\eta_0$  is the lubricant viscosity,  $U_r$  is the speed of entraining motion,  $\alpha$  is the pressure viscosity coefficient, and  $h_{c0}$  is the central film thickness.

### Viscous Friction

Evans and Johnson [12] implemented a method to analyse viscous friction in EHL contacts. With the high loads experienced by gears in racing transmissions, the lubricant in the mesh teeth-pair contacts operates in the Eyring traction regime. Consequently, the coefficient of friction is calculated using [12]:

$$\mu = 0.87 \alpha \tau_0 + 1.74 \frac{\tau_0}{\bar{p}} \ln \left[ \frac{1.2}{\tau_0 h_{c0}} \left( \frac{2 K \eta_0}{1 + 9.6 \xi} \right)^{1/2} \right] \quad (2)$$

where,  $\tau_0$  is the lubricant Eyring stress,  $\bar{p}$  is the average pressure at the contact point,  $K$  is the lubricant thermal conductivity, and  $\xi$  is given by:

$$\xi = \frac{4}{\pi} \frac{K}{h_{c0}/R} \left( \frac{\bar{p}}{E' R K' \rho' c' U_r} \right)^{1/2}$$

where,  $R$  is the contact radii of curvature,  $E'$  is the reduced elastic modulus,  $K'$ ,  $\rho'$ , and  $c'$  are the thermal conductivity, density, and specific heat capacity of the solid.



The generated friction due to viscous shear of the lubricant film is expressed as:

$$f_v = \mu W \quad (3)$$

### Boundary Friction

The thin lubricant films in the meshing contacts of loaded gear teeth pairs in racing transmissions are comparable to the roughness of surface asperities. Consequently, asperity interaction and therefore boundary friction is to be expected.

Greenwood and Tripp [16] developed a method to evaluate the generated boundary friction as the result of direct interaction of asperities on the counter face contacting surfaces. The method assumes a Gaussian height distribution of surface asperities. When mixed or boundary regimes of lubrication occur, Stribeck's oil film parameter:  $1 < \lambda = \frac{h_{c0}}{\sigma} < 2.5$ , which is updated at each time step based on the estimated EHL film thickness, specifies the fraction of the load carried by the asperities in the apparent contact area,  $A$  [16]:

$$W_a = \frac{16\sqrt{2}}{15} \pi (\xi\beta\sigma)^2 \sqrt{\frac{\sigma}{\beta}} E' A F_{5/2}(\lambda) \quad (4)$$

where,  $\beta$  is the average asperity tip radius,  $\sigma$  is the composite RMS surface roughness, and the statistical function  $F_{5/2}(\lambda)$  for a Gaussian distribution of asperities is obtained as [17]:

$$F_{5/2} = \begin{cases} -0.004\lambda^5 - 0.057\lambda^4 - 0.29\lambda^3 - 0.784\lambda^2 - 0.784\lambda - 0.617 & \text{for } \lambda < 2.5 \\ 0; & \text{for } \lambda \geq 2.5 \end{cases} \quad (5)$$

The roughness parameter  $(\xi\beta\sigma)$  for steel surfaces is generally in the range 0.03-0.07.  $\sigma/\beta$  is a representation of average asperity slope and is in the range of  $10^{-4}$  to  $10^{-2}$  [18]. In the current study, it is assumed that  $\xi\beta\sigma = 0.055$  and  $\sigma/\beta = 10^{-3}$ .

Asperity friction should be considered in mixed and boundary regimes of lubrication. A thin adsorbed film exists at the summit of the asperities or is entrapped in their inter-spatial valleys. This thin adsorbed film is subjected to non-Newtonian shear, thus boundary friction  $f_b$  is given as [18]:

$$f_b = \tau_L A_a \quad (6)$$

where,  $A_a$  is the asperity area and  $\tau_L$  is the lubricant's limiting shear stress [19]:

$$\tau_L = \tau_0 + \varepsilon P_m \quad (7)$$

where,  $\varepsilon$  is the slope of the lubricant limiting shear stress-pressure, and the mean (Pascal) pressure  $P_m$  is:

$$P_m = \frac{W_a}{A_a} \quad (8)$$

The asperity contact area is expressed as [16]:

$$A_a = \pi^2 (\xi\beta\sigma)^2 A F_2(\lambda) \quad (9)$$

The statistical function  $F_2(\lambda)$  is calculated as [18]:

$$F_2(\lambda) = \begin{cases} -0.002\lambda^5 - 0.028\lambda^4 - 0.173\lambda^3 + 0.526\lambda^2 - 0.804\lambda - 0.500 & \text{for } \lambda < 2.5 \\ 0; & \text{for } \lambda \geq 2.5 \end{cases} \quad (10)$$

### Power Loss

The instantaneous power loss is determined by taking into account the calculated viscous and boundary friction contributions as:

$$P_{loss} = (f_v + f_b) U_s \quad (11)$$

where,  $U_s$  is the sliding velocity of the contact.

### Sub-surface Stresses

The main concern with respect to structural integrity and reliability of gearing transmission is fatigue spalling resulting from sub-surface stress field. Figure 1 shows an elastic half-space loaded over the strip  $(-b < x < a)$  by normal pressure  $p(x)$  and tangential traction  $q(x)$  distributed in any arbitrary manner.

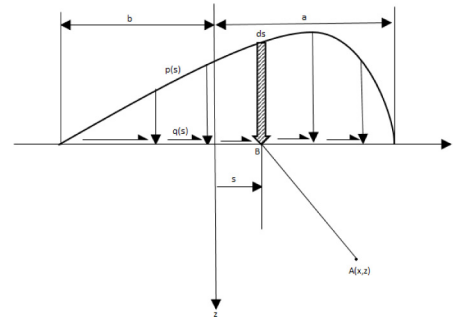


Figure 1. Normal pressure and tangential traction distribution over the solid surface (after Johnson [20])

The sub-surface stress field induced by  $p(x)$  and  $q(x)$  at any point A within the body of contacting solids is found according to Johnson [20] as:

$$\sigma_x = -\frac{2z}{\pi} \int_{-b}^a \frac{p(s)(x-s)^2 ds}{[(x-s)^2 + z^2]^2} - \frac{2}{\pi} \int_{-b}^a \frac{q(s)(x-s)^3 ds}{[(x-s)^2 + z^2]^2} \quad (12)$$

$$\sigma_z = -\frac{2z^3}{\pi} \int_{-b}^a \frac{p(s) ds}{[(x-s)^2 + z^2]^2} - \frac{2z^2}{\pi} \int_{-b}^a \frac{q(s)(x-s) ds}{[(x-s)^2 + z^2]^2} \quad (13)$$