

Study on Path Following Control Method for Automatic Parking System Based on LQR

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ABSTRACT

The Automatic Parking System (APS) is consisted of environmental perception, path planning and path following. As one of the key technologies in APS, path following module controls the lateral movement of the vehicle during the parking process. A mature path following module should meet all the performance indexes of high precision, fast convergence, convenient tuning and good passenger comfort. However, the current path following control methods can only meet parts of the performance indexes, instead of all. In order to satisfy all the performance indexes above, a path following control method based on Linear Quadratic Regulator (LQR) is proposed in this paper. Firstly, the linearization of the non-linear vehicle kinematic model was done to establish a linear system of the path following error. Secondly, LQR optimal control was used to achieve the closed-loop control of this linear system to guarantee its stability and fast convergence property. In this process, optimal quadratic performance index of LQR can be achieved by balancing the weights of the state variables and inputs through the matrix Q and R. For this reason, the control parameters can be conveniently tuned and calibrated. Thirdly, the feasibility of this method was verified through co-simulation of Simulink and Carsim and field tests of Roewe E50 electric vehicle, and the path following error can be limited within 10 centimeters in both parallel parking and vertical parking.

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INTRODUCTION

The typical solution for Automatic Parking System (APS) controller is realized by the hierarchical mode of Environmental Perception, Path Planning and Path Following [1, 2]. The advantage of this structure is that each module has a clear division of responsibilities, making it easier to achieve complex intelligent behavior. As one of the key technologies in APS, path following module uses the steering angle instructions to control the vehicle to travel along the path and guarantee the good passenger comfort.

In [3,4], a method of pure pursuit was used to solve the path following problem. In the papers, look-ahead distance was found to play a decisive role in improving precision. So in [4], S.Campbell introduced the relationship between look-ahead distance and velocity to correct the look-ahead distance. However, this method had some problems under large curvature conditions, such as slow response and low precision. There were also some papers which proposed improved method based on pure pursuit. Muhammad et al.[5](2012) proposed a stable trajectory tracking method for autonomous vehicle.

This control algorithm used the kinematic model to predict future coordinate of the vehicle and the future estimated lateral error by comparing it with the target position. Based on the lateral error, the desired steering angle was got. Compared with traditional pure pursuit, the algorithm proposed by this paper improved the control precision.

A.Benalia et al.[6](2003) linearized the vehicle kinematic model by using the feedback linearization. A second order sliding mode controller were used for vehicle trajectory tracking. The simulation result showed that the second order sliding mode control can not only make the vehicle track the references trajectory but also showed robustness to initial error conditions. However, the chattering with high frequency in sliding mode control system is the barrier to the application to the field test, because it would cause the steering wheel chattering and reduce the passenger comfort.

Kunpeng Cheng et al.[7,8](2013) proposed a path following algorithm. In the paper, a new vehicle model of covered distance was obtained by time-scaling algorithm, and then, the new model was linearized based on feedback linearization. Finally, pole-placement method was used to control the linearized model. This path following algorithm had the advantages of small amount of computations and high following precision. However, the algorithm needed to assign three poles, which complicated the parameter tuning work.

Some indexes, such as high precision, fast convergence, convenient tuning and good passenger comfort, are the essential elements of the path following module. However, the path following control methods put forward by most researches, such as the pure pursuit, pole assignment control and sliding mode control, can only meet parts of the performance indexes, instead of all. In order to satisfy all the

performance indexes above, on the basis of feedback linearization in Kunpeng Cheng[7,8], a path following control method based on Linear Quadratic Regulator(LQR) is proposed in this paper.

This paper is organized as follows. The vehicle kinematic model of time was transferred to a new vehicle model with respect to covered distance by time-scaling algorithm. Then the linearization of the nonlinear vehicle kinematic model based on differential flatness theory was done to establish a linear system of the path following error. After that, LQR optimal control was used to achieve the closed-loop control of this linear system to guarantee its stability and fast convergence property. To ensure the passenger comfort in longitudinal direction, the velocity was designed with continuous acceleration during the parking process. In the end, field tests were implemented to validate the proposed control method based on the Roewe E50 electric vehicle.

FEEDBACK LINEARIZATION

Kinematic Vehicle Model

Since the vehicle's velocity is relatively low during the parking process, the impact of lateral dynamic of the tire can be neglected and the kinematic vehicle model is used in path following as shown in Figure 1.

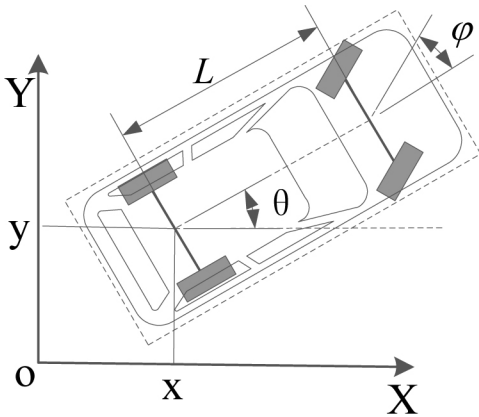


Figure 1. Kinematic vehicle model

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos \theta(t) \\ \sin \theta(t) \\ \frac{\tan \varphi(t)}{L} \end{bmatrix} v \quad (1)$$

where L denotes wheel base; (x, y, θ) denotes the posture of the vehicle in global coordinate; φ denotes steering angle; and v denotes linear velocity at the center of the rear axle.

For the sake of simplicity, the steering actuator of a vehicle can be regarded as a first order dynamics, as shown in Equation 2.

$$\dot{\varphi}(t) = -T^{-1}\varphi(t) + T^{-1}u_1(t) \quad (2)$$

where T denotes the system time constant and u_1 denotes the desired front wheel steering angle.

In order to transfer the vehicle model to the time-invariant model, a new time scale s_d which means the desired driving distance is introduced and then the vehicle model can be rewritten in this time scale[9,10]. The time-scaling function is defined as Equation 3.

$$\frac{ds_d(t)}{dt} = \frac{v(t)}{u_2(t)} \quad (3)$$

where u_2 is the time-scaling input.

By applying the chain rule to Equation 1 and Equation 2, the vehicle model with respect to the new time scale s_d is shown as Equation 4, and the derivation process of the Equation 4 was described in[7] in detail.

$$\begin{bmatrix} x'(s_d) \\ y'(s_d) \\ \theta'(s_d) \\ \varphi'(s_d) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T^{-1}v^{-1} \end{bmatrix} \bar{u}_1(s_d) + \begin{bmatrix} \cos \theta(s_d) \\ \sin \theta(s_d) \\ L^{-1} \tan \varphi(s_d) \\ -T^{-1}v^{-1}\varphi(s_d) \end{bmatrix} \bar{u}_2(s_d) \quad (4)$$

where $\bar{u}_1(s_d) = u_1(s_d)u_2(s_d)$, $\bar{u}_2(s_d) = u_2(s_d)$, $(-)'$ is the abbreviation of $d(-)/ds_d$.

Feedback Linearization of the Vehicle Model

According to the conclusion in [7], the system (4) is a typical differential flatness system, which can be transformed to a linear system through dynamic feedback, static feedback and coordinate transformation. The flow chart of the controller[7] is shown in Figure 2.

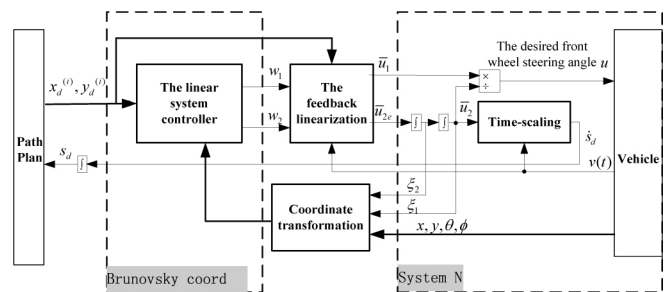


Figure 2. The flow chart of the controller

The linearized system through feedback linearization is denoted as System L:

$$\begin{aligned} z'_1 &= z_2, & z'_2 &= z_3, & z'_3 &= w_1 \\ z'_4 &= z_5, & z'_5 &= z_6, & z'_6 &= w_2 \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{z} &= (z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6)^T \\ \text{where} \quad \mathbf{z} &= (x \ x' \ x'' \ y \ y' \ y'')^T \end{aligned}$$



Two new inputs w_1 and w_2 are introduced here, so the key point of the path following algorithm is to design the linear controller to guarantee its stability and fast convergence property.

THE LINEAR CONTROLLER DESIGN BASED ON LQR

In this section, a linear system of path following error is obtained based on the feedback linearization of vehicle model. In order to ensure the global convergence and stability of the system, a linear controller based on LQR optimal control is proposed.

The Path Following Error Linear System

Suppose that e represents the path following error, the following error vector in x direction can be expressed as Equation 6.

$$\begin{aligned} \mathbf{e}_x &= (e_x \quad e'_x \quad e''_x) \\ &= (x - x_d \quad x' - x'_d \quad x'' - x''_d) \end{aligned} \quad (6)$$

where x denotes the actual posture of the vehicle in x direction, x_d denotes the desired posture in x direction which can be obtained from the path plan module.

From Equation 5 and Equation 6, the following error linear system in x direction and y direction can be achieved easily:

$$\begin{aligned} \mathbf{e}'_x &= \mathbf{A}\mathbf{e}_x + \mathbf{B}\mathbf{w}_1 \\ \mathbf{e}'_y &= \mathbf{A}\mathbf{e}_y + \mathbf{B}\mathbf{w}_2 \end{aligned} \quad (7)$$

$$\text{where } \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The linearized system obtained from Equation 7 can be stabilized by a state feedback of the form.

$$\begin{aligned} \mathbf{w}_1 &= -\mathbf{K}\mathbf{e}_x \\ \mathbf{w}_2 &= -\mathbf{K}\mathbf{e}_y \end{aligned} \quad (8)$$

Such that gain matrix $\mathbf{K}$ makes the eigenvalues of the closed loop system in the left half of the complex plane.

The equation of state after state feedback control can be expressed as Equation 9.

$$\begin{aligned} \mathbf{e}'_x &= (\mathbf{A} - \mathbf{BK})\mathbf{e}_x \\ \mathbf{e}'_y &= (\mathbf{A} - \mathbf{BK})\mathbf{e}_y \end{aligned} \quad (9)$$

LQR Optimal Control Application in Path Following

The linear system of following error has been achieved in the previous section. In the case, the traditional method for feedback gain acquisition, such as pole assignment or LQR optimal control, can be adopted to stabilize the linear system. Since the weight matrix of LQR optimal control corresponds to the specific physical meaning, the control parameters can be conveniently tuned and calibrated. For this reason, LQR optimal control is used to get the gain matrix $\mathbf{K}$.

Consider the linear time-invariant system:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (10)$$

with the quadratic performance index

$$J = \frac{1}{2} \int_0^\infty [\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)] dt \quad (11)$$

where $\mathbf{Q}$ is a 3×3 symmetric positive definite matrix, which represents the state variable weight matrix, $\mathbf{R}$ is constant, which represents the input variable weight matrix.

Here our goal is to find the control signal $\mathbf{u}^*(t)$ which provides not only closed loop stability but also the optimal LQR performance for the path following. The necessary and sufficient condition for optimal control signal $\mathbf{u}^*(t)$ is:

$$\mathbf{u}^*(t) = -\mathbf{K}\mathbf{x}(t), \quad \mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \quad (12)$$

with $\mathbf{P}$ a positive definite solution of a Riccati equation.

$$-\mathbf{P}\mathbf{A} - \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} - \mathbf{Q} = 0 \quad (13)$$

For x direction, the equation of state of the path following error has been deduced as follows:

$$\begin{aligned} \mathbf{e}'_x &= (\mathbf{A} - \mathbf{BK})\mathbf{e}_x \\ &= (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})\mathbf{e}_x \end{aligned} \quad (14)$$

With the quadratic performance index:

$$\begin{aligned} J &= \frac{1}{2} \int_0^\infty [\mathbf{e}_x^T(s_d)\mathbf{Q}\mathbf{e}_x(s_d) + \mathbf{w}_1^T(s_d)\mathbf{R}\mathbf{w}_1(s_d)] ds_d \\ &= \frac{1}{2} \int_0^\infty \left[\begin{pmatrix} e_x & e'_x & e''_x \end{pmatrix} \begin{pmatrix} Q_{11} & & \\ & Q_{22} & \\ & & Q_{22} \end{pmatrix} \begin{pmatrix} e_x \\ e'_x \\ e''_x \end{pmatrix} + \mathbf{w}_1^T(s_d)\mathbf{R}\mathbf{w}_1(s_d) \right] ds_d \\ &= \frac{1}{2} \int_0^\infty [Q_{11}e_x^2 + Q_{22}e'^2_x + Q_{33}e''^2_x + \mathbf{R}\mathbf{w}_1^2(s_d)] ds_d \end{aligned} \quad (15)$$