

Modeling on GPS with Software-Centered Observation Errors

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ABSTRACT

Intelligent vehicles have gained increasing popularity in recent years as traffic safety and efficiency have become the major challenges faced by automotive industry. Vehicle positioning system, such as GPS, plays more and more important role on intelligent or autonomous driving. Intelligent vehicle technologies have been developed and tested mainly based on intensive field experiment under various driving scenarios. However, the large variation, uncertainty and complexity of the driving environment, including buildings, traffic and weather conditions have posed great challenges on test repeatability and system robustness. This paper proposes a GPS model considering software-centered observation errors. The focus of the research is on its error to reflect the real signals from GPS measurement. A three-dimensional position of the twenty-four GPS satellites is first calculated according to the ephemerides, and the visible satellites are selected based on the position with respect to the satellites. Then the distance between them is calculated as the ideal signal, which is further contaminated with Gaussian white noise to simulate the pseudo range that stems from various noise sources from hardware and environmental factors. Simulation has been conducted and the results are correlated well with real signals measured by an on-board GPS receiver.

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INTRODUCTION

Since GPS plays an important role on intelligent or autonomous driving, modeling GPS becomes indispensable part of simulation on intelligence driving. Although there exists various GPS models in commercial tools [1][2][3], the physics of GPS, in particular with measurement errors, has not been adequately modeled in various known virtual simulation tools. Some of them are more focused on traditional vehicle dynamics and control, or on other environmental sensors such as radar, lidar or camera. Others may focus more on simulation on vehicle driving environment with environment sensors for specific applications of vehicles and/or vehicle driving environment. However their GPS models are relatively simple and may not adequately reflect the physical phenomena of real measurement.

A signal model for GPS has been given by Michael S. Braasch [10], the error source algorithms that computes the observed pseudorange was established, the SA (Selective Availability) data was analyzed in detail and the ARMA model was finally given. The pseudorange error which stem from a variety of hardware and environmental factors can be modeled as Gauss-Markov noise [11] or Gaussian White noise according to their statistical characteristics [9]. What's more, the MATLAB simulation created by NAVSYS Corp. allows one to construct virtually any type of signaling environment for GPS [12].

However, the software-centered GPS error model we need is not the more sophisticated the better, but to meet the demands of simulation, and get close to the objective laws of the real world as far as possible. SA was turned off since 2000 so that the statistical characteristics of GPS observation error are changed to a certain extent. And no measurements were used to compare with the simulation results on the above papers.

This paper develops an on-board GPS observation error model according to the real satellite positioning process which analyzed the movements of satellite constellation, simulated the calculation of satellite receivers, and realized pseudorange positioning in pure software mode. Finally the simulation results are compared with the measured values.

The paper is organized as follows: Section II introduces GPS satellites orbit model, then calculates the visible ones from which the user can receive signals. Section III discusses the pseudorange error model according to the statistical characteristics of causes of error that have been reached by pioneers. The user position is calculated by solving the pseudorange observation equations with Newton iteration method in Section IV. Section V analyzes the simulation results and section VI concludes the paper with a brief summary.

SATELLITE ORBIT MODEL

In order to calculate the position of each satellite, the satellite orbit model needs to be established. The forces acting on the satellite are usually classified into two categories according to their effects: one is the central gravitational force; the other is the perturbative force, also known as the non-central force. The central force is the gravity of the homogeneous earth ellipsoid and determines the basic rules and characteristics of the satellite motion, which can be regarded as the ideal orbit. The non-central forces include gravitational interactions caused by the non-spherical symmetry body of the earth, unsolar attraction, atmospheric drag, light radiation pressure, the earth's tidal force and so on. The effects of the perturbative force makes the satellite's motion deviate from the ideal orbit. The ideal state of the satellite motion is called the non-perturbation motion, which is also known as the Kepler movement, and can be described by Kepler's laws.

Actually the non-perturbation motion can be described by a set of appropriate parameters. The selection of the parameters is not unique, and one set of the most widely used parameters is known as the Kepler orbit parameters or Keplerian elements [4], which is shown in Figure 1.

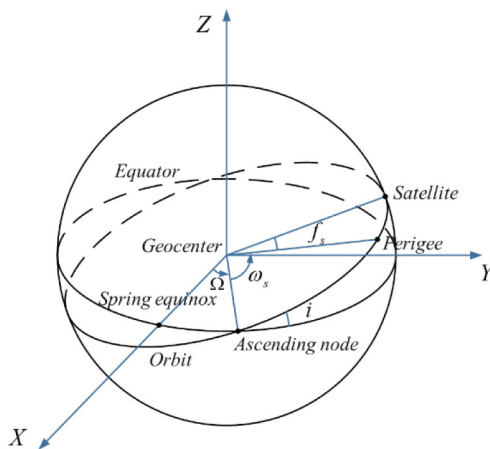


Figure 1. Keplerian elements

The motion of a Satellite moving in three-dimensional space is characterized by a position vector and a velocity vector. Each vector has three components, so the total number of values needed to define a trajectory through space is six. The traditional orbital elements are the six Keplerian elements, after Johannes Kepler and his laws of planetary motion, whose symbols, meanings and roles is shown in Table 1.

Table 1. The meaning and role of Keplerian elements

Symbol	Meaning	Role
a_s	Semimajor axis	Define the size and shape of the trajectory
e_s	Eccentricity	
Ω	Longitude of the ascending node	Define the orientation of the orbital plane.
i	Inclination	
ω_s	Argument of periapsis	Define the angle between the ascending node and the periapsis.
f_s	True anomaly	Is the function of time, and define the position of the orbiting body along the trajectory.

In general, it is reasonable and necessary to use the above six Keplerian elements to describe the orbit of the satellite motion. When the value is fixed, the space position and velocity of the satellite can be determined.

For practical convenience, the satellite position is often described by a Cartesian co-ordinate system with its origin coincide with geocenter O . The ζ_s axis is perpendicular to the orbital plane, the η_s axis is on the plane and points to the perigee and constitute the right hand system with the ξ_s axis. The ζ_s , η_s axes are shown in Figure 2.

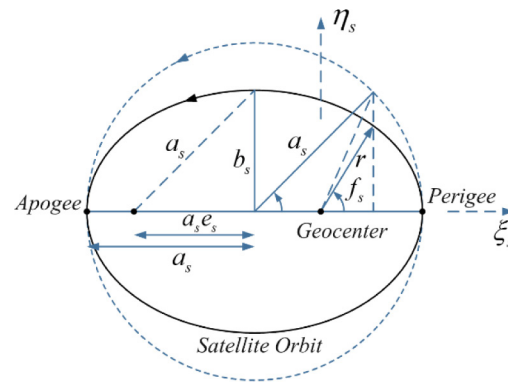


Figure 2. Satellite orbital plane and its Cartesian co-ordinate system

The coordinates of the satellite (ξ_s, η_s, ζ_s) can be described as:

$$\begin{bmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{bmatrix} = r \begin{bmatrix} \cos f_s \\ \sin f_s \\ 0 \end{bmatrix} = a_s \begin{bmatrix} \cos E - e_s \\ \sqrt{1 - e_s^2} \sin E \\ 0 \end{bmatrix} \quad (1)$$

The origin of celestial co-ordinate system and orbital co-ordinate system are all the center of mass (= geocenter), but the axes which their axes point are different. In order to make the two coordinate systems point to the same, the orbital co-ordinate system should be rotate as follows:

1. Rotate ω_s degree clockwise around the ζ_s axis to make the ξ_s axis points from Perigee to Ascending node. The rotation matrix is:

$$R_1 = \begin{bmatrix} \cos \omega_s & -\sin \omega_s & 0 \\ \sin \omega_s & \cos \omega_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

2. Rotate i degree clockwise around the ξ_s axis to make the ζ_s axis coincide with the Z axis. The rotation matrix is:

$$R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \quad (3)$$

3. Rotate Ω degree clockwise around the ζ_s axis to make the ξ_s axis points to spring equinox. The rotation matrix is:

$$R_3 = \begin{bmatrix} \cos \Omega & -\sin \Omega & 0 \\ \sin \Omega & \cos \Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

So the satellite position in celestial co-ordinate system can be expressed as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_3 R_2 R_1 \begin{bmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{bmatrix} = R_3 R_2 R_1 a_s \begin{bmatrix} \cos E - e_s \\ \sqrt{1 - e_s^2} \sin E \\ 0 \end{bmatrix} \quad (5)$$

As we just consider the ideal trajectory without perturbation, the three-dimensional position in celestial co-ordinate system also can be used as WGS-84 (World Geodetic System 1984 [5]) co-ordinates. Actually, as the satellite orbit is so complicated that it is impossible to use a simple model to calculate the satellite position, the true meaning of modeling is to meet using demand within a certain range of accuracy. The satellite orbit model not only reflects the physical characteristics of satellite trajectory, but also satisfies the functional requirements of simulation software. And the following model are built under this guiding ideology.

The GPS Constellation

Imaging that each satellite orbit shown in Figure 1 is “opened” and extended deuce on a plane, then we can get the plane projection of GPS constellation. The satellites’ orbits which are shown in Figure 3 are based on the epoch time at 00:00 on February 1st, 1970 [6]. The slope of each orbit indicates the inclination angle relative to the equatorial plane, and its rated value is 55 degrees. The horizontal axis represents the longitude of the ascending node and the values are given.

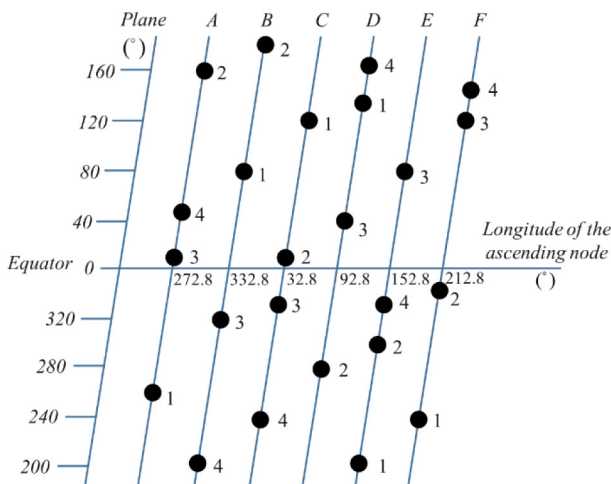


Figure 3. The plane projection of GPS constellation

The GPS satellites constellation consists of 24 satellites which located in 6 geocentric orbit planes, for each orbit consists of 4 satellites. The satellite's orbital period is half a sidereal day, or 11 hours and 58 minutes. The orbits are distributed uniformly along the equator and the major axis semidiameter of orbital ellipse is about 26560 kilometers.

Table 2. Keplerian elements

Keplerian elements	Value
a_s	26560
e_s	0.02
Ω	[32.8 92.8 152.8 212.6 272.8 332.8]
i	55°
ω_s	$\begin{bmatrix} 10 & 50 & 160 & 260 \\ 80 & 180 & 220 & 320 \\ 10 & 130 & 250 & 340 \\ 50 & 150 & 170 & 300 \\ 100 & 210 & 310 & 340 \\ 120 & 140 & 240 & 350 \end{bmatrix}$
f_s	0 (Ignore the satellites motion)

The formulas and parameter values of satellite orbit model are given, and the figure of satellites’ position is plotted in MATLAB which is shown in Figure 4.

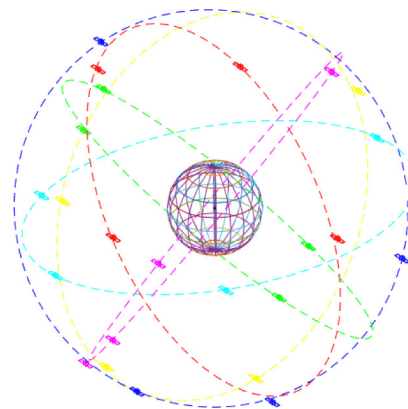


Figure 4. Satellite orbit model which is plotted in MATLAB

Calculate Visible Satellites

Using the included angle between satellite, geocenter and the user to estimate whether the satellite is blocked by earth’s shadow (Figure 5).

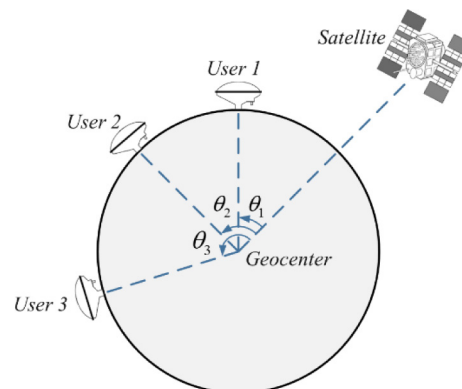


Figure 5. Schematic diagram of satellite visibility