

# A First Principles Based Approach for Dynamic Modeling of Turbomachinery

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## ABSTRACT

As the cost and complexity of modern aircraft systems increases, emphasis has been placed on model-based design as a means for reducing development cost and optimizing performance. To facilitate this, an appropriate modeling environment is required that allows developers to rapidly explore a wider design space than can cost effectively be considered through hardware construction and testing. This wide design space can then yield solutions that are far more energy efficient than previous generation designs. In addition, non-intuitive cross-coupled subsystem behavior can also be explored to ensure integrated system stability prior to hardware fabrication and testing. In recent years, optimization of control strategies between coupled subsystems has necessitated the understanding of the integrated system dynamics. To this end, a dynamic vapor cycle modeling toolset known as the AFRL Transient Thermal Management and Optimization (ATTMO) toolset was developed to address two-phase flow systems. This toolset has been further expanded to include components typical of air cycle topologies. Current air-cycle modeling tools rely heavily on user supplied performance maps to predict turbomachinery output. This approach has a variety of limitations. First, at a conceptual design level, modifications to a specific component design requires the generation of a new map; thereby, limiting one's ability to rapidly evaluate and optimize across a wide design space. Second, interpolation routines in map-based approaches fail to yield precise solutions near critical operating points, such as compressor stall lines, due to unknown data outside of the stall point. As a result, development of control strategies around the stall margin of a machine is difficult due to the model induced instabilities or incorrect predictions from interpolation around those operating points. Lastly, modeling startup and shutdown requires torque prediction near or at zero speed, which for map-based approaches is ill-defined. To address these limitations, first principles models of traditional turbomachinery components have been developed and will be discussed in this paper. Conservation of mass, energy, and momentum are applied to capture appropriate volume dynamics relevant to plant and controls engineers. Enthalpy based calculations derived from machine geometry and fluid flow conditions allow for design optimization of machine parameters while still maintaining accurate performance predictions. These approaches have been implemented in the open-source Simulink toolset, ATTMO. Based on user defined system architecture and associated design parameters, a time-domain simulation for an integrated system analyses can be formed.

**CITATION:** McCarthy, P., Niedbalski, N., McCarthy, K., Walters, E. et al., "A First Principles Based Approach for Dynamic Modeling of Turbomachinery," *SAE Int. J. Aerosp.* 9(1):2016, doi:10.4271/2016-01-1995.

## INTRODUCTION

Turbomachinery modeling has been studied and performed for decades. Currently, there are many software toolsets available ranging in complexity, capability, and applicability. In particular, the National Aeronautics and Space Administration (NASA) has worked extensively in the development of design and sizing of

turbomachinery components. Specifically, the NASA Glenn Research Center in Cleveland, OH provides extensive software available for download related to sizing and design of compressors, turbines, compressible flows, fluid dynamics, and additional thermal, structural, acoustic, and electromagnetic modeling [1, 2, 3, 4]. In addition to NASA's software, companies like Concepts NREC

provide wide-ranging software capabilities for preliminary sizing and design of turbomachinery which, like the NASA software, is largely developed from first principles based calculations [5]. First principle implies physics and enthalpy-based calculations and correlations derived from machine geometry and fluid flow conditions, with conservation of mass, energy, and momentum applied. This method allows for design optimization of machine parameters while still maintaining accurate performance predictions over a wide range of turbomachinery designs. Another widely utilized software based on first principle derivations is numerical propulsion system simulation (NPSS), an often utilized toolset for steady-state calculations [6] via map-based turbomachinery calculations. It consists of vapor cycle and air cycle components for developing integrated system studies. Other software companies, such as Simerics Inc. [7], SoftinWay Inc. [8], and PCA Engineers Limited [9], provide detailed CFD calculations for modeling turbomachinery components. All of these software toolsets are capable of generating accurate performance maps of turbomachinery that can then be used in steady-state or transient modeling toolsets.

Despite the extensive capabilities available for estimating turbomachinery performance data, map-based transient studies suffer from several limitations. Firstly, modifications to a specific component design frequently require the generation of a new map; thereby, limiting one's ability to rapidly evaluate and optimize across a wide design space. Secondly, within a specific study, interpolation routines in map-based approaches fail to yield precise solutions near critical operating points, such as compressor stall lines due to unknown data outside of the stall point. In turn, control strategies around the stall margin of a machine are difficult due to model induced instabilities or incorrect predictions from interpolation around those operating points. Finally, modeling startup and shutdown requires torque prediction near or at zero speed, which for map-based approaches is ill-defined.

Based on the motivations described above, work has been explored regarding a first principles based transient toolset for air cycle machine (ACM) and turbomachinery based studies as an alternative to the traditional map-based performance calculations. The toolset is being developed as an extension of the AFRL Transient Thermal Modeling and Optimization (ATTMO) toolset [10-11]. Given the extensive capabilities of the ATTMO toolset in transient modeling of vapor cycle systems (VCSs) and additional heat transfer specific components, development of a first principles, transient ACM toolset for direct integration with ATTMO based components will allow for highly complex, integrated system studies that could make significant contributions in modeling and simulation of aerospace environments. Details of the initial first principles design calculations, and supporting transient calculations are provided below. As a preliminary verification study, steady-state performance maps were developed and correlated to the NASA based toolsets. As a representation of future capabilities of the ACM components developed, a final verification study was then performed via a system level transient model, the specifics of which are provided below.

## METHODOLOGY

In this effort, four main ACM components were developed: radial turbine, centrifugal compressor, axial turbine, and axial compressor. Additionally, for integrated dynamic studies, shaft and bearing loss components were developed and utilized within a Simulink environment. A first principles based design code was developed for each component based on relevant publications. In a first principles based code, ideal gas law and isentropic pressure rise equations are modified to account for losses within the ACM component. Examples of losses that result in inefficiencies in ACM performance include clearance between turbine/compressor blade and housing, incidence and deviation flow angles on rotor/stator blades, and friction losses. Detailed equations related to each ACM component are provided later in the report. Once design codes were developed for each component, Simulink blocks were established with dynamic calculations applied to the design equations for accurate transient performance.

While many references were employed in developing these components, the NASA publications are extensive and well detailed warranting a distinctive note [1, 2, 3, 4]. Additionally, books by Gorla [12] and Dixon [13] provided wide-ranging information regarding the design and performance of the turbomachinery established in this report, along with the relevant fluid mechanics required for thermodynamic analysis.

Figure 1 provides simplified two-dimensional schematics of radial turbine and centrifugal compressor components. In a radial turbine, air is input at the volute and forced through stationary nozzle blades, and then through the rotational impeller blades. The work performed by the fluid on the impeller blades, generates torque on the supporting shaft and results in a reduction of pressure and temperature of the fluid. In a nearly opposite performance, a centrifugal compressor is rotated by its supporting shaft forcing air in at the impeller and subsequently through the stationary diffuser (nozzle) blades and out through the volute. The work performed by the compressor on the fluid generates a rise in pressure and temperature of the air. Figure 2 provides example schematics for the axial turbine and axial compressor components, where stator blades are stationary and rotor blades rotate with the supporting shaft. The same general work generation of the radial turbine applies to the axial turbine, while the same work generation of the centrifugal compressor applies to the axial compressor.

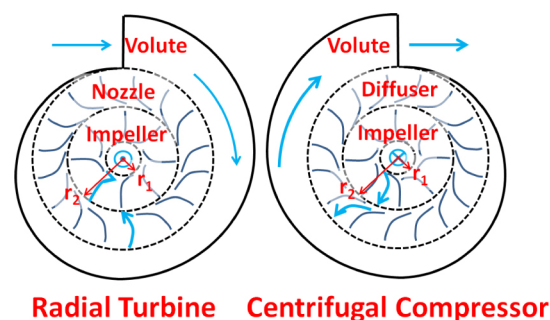


Figure 1. Radial turbine (left) and centrifugal compressor (right) schematics.

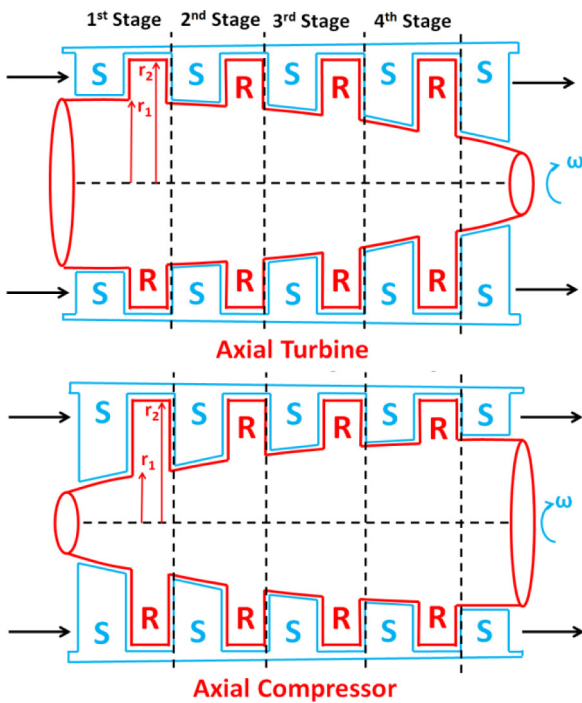


Figure 2. Axial turbine (top) and axial compressor (bottom) schematics.

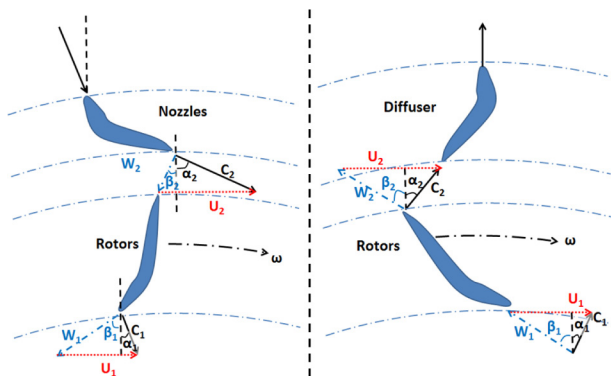


Figure 3. Radial turbine (left) and centrifugal compressor (right) velocity triangles.

Prior to providing detailed calculations for the various components, velocity diagrams (or velocity triangles) should be defined. Example velocity triangles are shown in Figure 3 for the radial turbine and centrifugal compressor, and velocity triangles for the axial components are provided in Figure 4. The absolute velocity ( $C$ ), rotational velocity ( $W$ ), and turbine/compressor velocity ( $U$ ) are plotted. The absolute flow angles ( $\alpha$ ) will approximately match the blade angles of the stationary nozzle and stator blades, while the rotational flow angles ( $\beta$ ) will approximately match the blade angles of the rotational rotor blades, as demonstrated in Figures 3 and 4. For the equations provided within this report, flow velocities are defined as being positive if they match the direction shown in the schematic below, which depends on the direction of rotation of the ACM component. As shown in Figure 3, absolute velocities are positive in the direction of rotor rotation and rotational velocities are positive in the opposite direction of the rotor rotation. As shown in Figure 4, axial turbine absolute and rotational velocities are positive in the direction of rotor rotation at the inlet of the rotor, but are positive in the opposite direction of rotor rotation at the rotor outlet. For axial

compressors, absolute velocities are always positive in the direction of rotor rotation while rotational velocities are always positive in the opposite direction of rotor rotation.

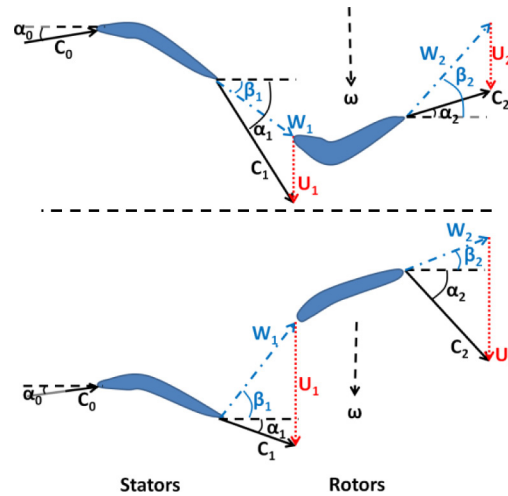


Figure 4. Axial turbine (top) and axial compressor (bottom) velocity triangles.

Turbomachinery can operate at high rotational speeds, generating large velocities within the air flow that can achieve supersonic conditions. As such it is important to differentiate between static, total, and rotational fluid properties. Static fluid properties contain no contributions due to velocity, while total properties contain contributions due to absolute velocities and rotational properties contain contributions due to rotational velocities. The static, total, and rotational temperatures are related by the following equations:

$$T_t = T_s (1 + (\gamma - 1) M_t^2 / 2) \quad (1)$$

$$T_r = T_s (1 + (\gamma - 1) M_r^2 / 2) \quad (2)$$

where  $t$  refers to total,  $s$  refers to static, and  $r$  refers to rotational,  $T$  is temperature,  $\gamma$  is the heat capacity ratio, and  $M$  is the fluid Mach number. The Mach numbers utilized above are calculated as:

$$M_t = C / c \quad (3)$$

$$M_r = W / c \quad (4)$$

$$c = \sqrt{\gamma R_{sp} T_s} \quad (5)$$

where  $R_{sp}$  is the specific gas constant of air, and  $c$  is the speed of sound. Pressure relations are then defined as:

$$P_t = P_s (1 + (\gamma - 1) M_t^2 / 2)^{\gamma / (\gamma - 1)} \quad (6)$$

$$P_r = P_s (1 + (\gamma - 1) M_r^2 / 2)^{\gamma / (\gamma - 1)} \quad (7)$$

The enthalpy of the fluid is a function of the temperature, and is utilized in the impeller/rotor work calculations. Recognizing that for high Mach numbers, total and/or rotational temperatures and pressures can be much larger than the static properties, it is critical to establish that only static properties are used for calculating fluid properties such as density, viscosity, and specific heat. Total and rotational properties will still play crucial roles in the ACM calculations, as is shown later in this section.

### Radial Turbine Design

The first ACM component developed in this effort was the radial turbine. The radial turbine consists of three main segments: the volute, the nozzle (stator blades), and the impeller (rotor blades). Within and between each segment, losses can occur. Volute losses were established based on work performed by Vilim at Argonne National Laboratory [14]. Nozzle losses were taken from work by Benson [15] and expansion and impeller losses were derived from NASA software [1]. Across the volute and nozzle, it is assumed that no total enthalpy losses occur. Total pressure losses will occur, though they are typically small. Velocities are calculated at volute inlet/outlet and nozzle inlet/outlet in order to ensure accurate calculation of fluid properties which contribute significantly in loss calculations.

In the radial turbine, the fluid performs work on the impeller blades resulting in a drop in enthalpy of:

$$dh_{imp} = U_2 C_{\theta 2} - U_1 C_{\theta 1} \quad (8)$$

where  $U_2$  is the impeller velocity at its outer diameter (inlet),  $C_{\theta 2}$  is the rotational component of fluid absolute velocity at the impeller inlet,  $U_1$  is the impeller velocity at its inner diameter (outlet), and  $C_{\theta 1}$  is the rotational component of fluid absolute velocity at the impeller outlet. With the impeller work known, the outlet to inlet temperature ratio can be calculated as:

$$T_{t1}/T_{tv} = 1 - dh_{imp}/c_p T_{tv} \quad (9)$$

where  $T_{tv}$  is the volute inlet total temperature,  $T_{t1}$  is the impeller exit total temperature, and  $c_p$  is the specific heat of the fluid. In turn the pressure ratio is calculated as:

$$P_{t1}/P_{tv} = (1 - (1 - T_{t1}/T_{tv})/\eta)^{\gamma/(\gamma-1)} \quad (10)$$

where  $P_{tv}$  is the volute inlet total pressure,  $P_{t1}$  is the impeller exit total pressure, and  $\eta$  is the total efficiency of the radial turbine. Finally, the efficiency of the turbine is defined as:

$$\eta = \frac{dh_{imp}}{(dh_{imp} + dh_{vf} + dh_{vexp} + dh_N + dh_{Nexp} + dh_{VS} + dh_{inc} + dh_{iexp})} \quad (11)$$

where  $dh_{vf}$  are volute losses due to friction,  $dh_{vexp}$  are volute expansion losses due to the change in area of the flow path along the volute,  $dh_N$  are the nozzle losses,  $dh_{Nexp}$  are nozzle exit expansion losses,  $dh_{VS}$  are friction losses within the vaneless space between the nozzle and impeller,  $dh_{inc}$  are impeller incidence and deviation flow losses, and  $dh_{iexp}$  are impeller exit expansion losses.

In order to maintain a concise report, details of the loss equations are provided in the [Appendix](#) section, and many more specific calculations are detailed in the references provided.

### Centrifugal Compressor Design

The majority of calculations utilized for the centrifugal compressor were adopted from work performed by Gravdahl [16, 17, 18, 19]. Gravdahl has worked extensively on dynamic modeling of centrifugal compressors with a particularly emphasis on control strategies around the stall region. His work, along with that executed by Jiang [20], provided the foundation for the compressor modeling performed in this report. In turn, the turbomachinery loss calculations provided by NASA [2] were studied and implemented. In this report, the NASA/Gravdahl calculations were utilized for impeller and diffuser loss calculations, while volute losses were developed from the turbine loss equations provided by Vilim [14].

The centrifugal compressor can be treated as an inversion of the radial turbine with fluid entering the inner diameter of the impeller, and exiting out of the volute. In a compressor, the impeller performs the work on the fluid generating a rise in enthalpy and pressure. Equation 8 is again utilized to describe the change in enthalpy across the compressor, with the additional assumption that  $C_{\theta 1}$  is zero as the fluid entering the impeller will have no rotational component. The temperature and pressure rise across the impeller are then described by:

$$T_{tv}/T_{t1} = 1 + dh_{imp}/c_p T_{t1} \quad (12)$$

$$P_{tv}/P_{t1} = (1 + \eta(T_{tv}/T_{t1} - 1))^{\gamma/(\gamma-1)} \quad (13)$$

The efficiency of the compressor is defined as:

$$\eta = \frac{dh_{imp}}{(dh_{imp} + dh_{vf} + dh_{vexp} + dh_{inc} + dh_{Nf} + dh_{if} + dh_{bid} + dh_{mix})} \quad (14)$$

where  $dh_{inc}$  are impeller and diffuser incidence flow losses,  $dh_{Nf}$  are the diffuser friction losses,  $dh_{if}$  are the impeller friction losses,  $dh_{bid}$  are impeller blade loading losses, and  $dh_{mix}$  are impeller fluid mixing losses.

Total enthalpy is assumed to be constant from the impeller exit to the volute exit, while total pressure is allowed to rise. Velocities are again evaluated at impeller inlet/outlet, diffuser inlet/outlet, and volute inlet/outlet in order to accurately calculate static fluid properties and loss equations. For both the radial turbine and centrifugal compressor, the velocity between the impeller and diffuser (nozzle) is typically

most susceptible to choked flow conditions (Mach number greater than 1), and therefore must be monitored closely when designing these components. Further details regarding centrifugal loss correlations are provided in the [Appendix](#).

### Axial Turbine Design

Axial components consists of alternating stator and rotor blade segments in which stator blades are placed along the stationary outer casing and rotor blades are placed along the diameter of the rotating shaft. The axial component design calculations vary significantly from the radial components. Axial turbine losses have been well-studied and are accurately depicted via correlations developed by Ainley and Mathieson [21]. Sawyer's book on turbine theory and design provides loss correlations for stator and rotor blades that have precisely predicted axial turbine losses for decades [22]. As such, they were utilized in this effort for axial turbine design considerations. Additionally, incidence loss calculations for off-design conditions were adopted from work by Horlock [23].

For the axial turbine stator blades, total enthalpy is assumed constant across blade length. The change in total pressure is calculated via isentropic assumptions/losses where the static enthalpy of air at the stator exit under isentropic expansion ( $h_{1i}$ ) is calculated as:

$$h_{1i} = h_{1s} - 0.5C_1^2 \left( \varepsilon_{st} + 0.5 \sin(\alpha_0 - \beta_{sb_0})^2 \right) \quad (15)$$

where  $h_{1s}$  is the actual stator outlet static enthalpy,  $C_1$  is the absolute fluid velocity at the stator exit,  $\varepsilon_{st}$  is the stator loss coefficient and,  $\alpha_0$  is the stator inlet absolute flow angle, and  $\beta_{sb_0}$  is the stator inlet blade angle. The isentropic outlet temperature of the fluid can be calculated as a function of the isentropic outlet enthalpy, and in turn the static pressure at the outlet ( $P_{1s}$ ) can be calculated from the isentropic relation:

$$P_{1s} = P_{0s} (T_{1i}/T_{0s})^{\gamma/(\gamma-1)} \quad (16)$$

where  $P_{0s}$  is the static inlet pressure,  $T_{1i}$  is the isentropic stator outlet temperature, and  $T_{0s}$  is the inlet static temperature. With the static pressure established, total pressure can be calculated via [Equation 6](#).

The stator outlet conditions correspond to the rotor inlet conditions. For the rotor blade section, the work performed by the fluid on the rotor blades is calculated as:

$$dh_{imp} = U_2 C_{\theta 2} + U_1 C_{\theta 1} \quad (17)$$

where 2 refers to the rotor exit, and 1 refers to the rotor inlet. There is a change in sign in [Equation 17](#) compared to [Equation 8](#). This is due to the change in sign that was established with the flow triangles for axial turbines in [Figure 4](#). The outlet total enthalpy of the rotor blade section is then simply:

$$h_{2t} = h_{1t} - dh_{imp} \quad (18)$$

where  $h_{1t}$  is the rotor inlet total enthalpy and  $h_{2t}$  is the rotor outlet total enthalpy. The rotor outlet static pressure is calculated in a manner similar to the stator blade section where:

$$h_{2i} = h_{2s} - 0.5W_2^2 \left( \varepsilon_{rt} + 0.5 \sin(\beta_1 - \beta_{rb_1})^2 \right) \quad (19)$$

where  $h_{2i}$  is the static enthalpy at the exit of the rotor under isentropic expansion,  $h_{2s}$  is the actual static enthalpy at the rotor exit,  $W_2$  is the rotational velocity at the rotor exit,  $\varepsilon_{rt}$  is the rotor loss coefficient,  $\beta_1$  is the rotational velocity angle at the rotor inlet, and  $\beta_{rb_1}$  is the rotor blade angle at the rotor inlet. Once again, rotor outlet isentropic temperature is calculated as a function of isentropic enthalpy and the static outlet pressure of the rotor is calculated as:

$$P_{2s} = P_{1s} (T_{2i}/T_{1s})^{\gamma/(\gamma-1)} \quad (20)$$

where  $T_{2i}$  is the isentropic rotor outlet temperature. If another stage of the turbine exists, the outlet conditions of the rotor are the same as the inlet conditions of the next stator segment. This process of calculating exit enthalpy/pressure along each turbine segment is continued for each stage of the turbine.

Once each segment is evaluated, total turbine pressure ratio ( $PR_T$ ) and exit total temperature under isentropic expansion ( $T_{ets}$ ) are calculated as:

$$PR_T = P_{0t}/P_{et} \quad (21)$$

$$T_{ets} = T_{0t} (1/PR)^{\gamma/(\gamma-1)} \quad (22)$$

where  $T_{0t}$  is the turbine inlet total temperature,  $P_{0t}$  is the turbine inlet total pressure, and  $P_{et}$  is the turbine exit total pressure. Turbine exit total isentropic enthalpy is calculated as a function of exit total isentropic temperature and in turn the total turbine efficiency ( $\eta_T$ ) is calculated as:

$$\eta_T = (h_{0t} - h_{et}) / (h_{0t} - h_{ets}) \quad (23)$$

where  $h_{0t}$  is the turbine inlet total enthalpy,  $h_{et}$  is the turbine outlet total enthalpy, and  $h_{ets}$  is the turbine exit total isentropic enthalpy.

Further details regarding the stator and rotor loss coefficients can be found in Chapter 4 of Sawyer's book, written by Carmichael [22].

### Axial Compressor Design

Though the axial compressor can principally be evaluated similarly to the axial turbine, a different strategy was utilized for calculating axial compressor performance. Extensive details regarding available loss

correlations from Tuncer [24] and Falck [25] were researched for this effort. Ultimately, the design calculations were adopted from work developed by Tournier [26]. Similar to the axial turbine, it is assumed that total enthalpy is constant along the stator blade. Loss calculations are handled differently in the compressor though. The change in total pressure along the stator blade is calculated as:

$$P_{1t} = P_{0t} - Y_{st}(P_{0t} - P_{0s}) \quad (24)$$

Where  $P_{0s}$  is the stator static inlet pressure,  $P_{0t}$  is the stator total inlet pressure,  $P_{1t}$  is the stator total outlet pressure, and  $Y_{st}$  is the stator loss coefficient. For calculating the work performed by the stator blades on the fluid, a 'rothalpy' term ( $I$ ) is introduced and for the stator outlet it is defined as:

$$I_1 = h_{1s} + (W_1^2 - U_1^2) \quad (25)$$

where  $h_{1s}$  is the static enthalpy at stator exit,  $W_1$  is the rotational velocity at the stator exit, and  $U_1$  is the turbine speed at the stator exit.

The rothalpy is assumed to be constant along the rotor blade length. From this assumption, the rotor outlet enthalpy is calculated as:

$$h_{2s} = I_1 - (W_2^2 - U_2^2) \quad (26)$$

where  $h_{2s}$  is the static enthalpy at rotor exit,  $W_2$  is the rotational velocity at the rotor exit, and  $U_2$  is the turbine speed at the rotor exit. The pressure rise across the rotor is calculated as a function of rotational pressure:

$$P_{2r} = P_{1r} - Y_{rt}(P_{1r} - P_{1s}) \quad (27)$$

where  $P_{2r}$  is the rotor rotational outlet pressure,  $P_{1r}$  is the rotor rotational inlet pressure,  $P_{1s}$  is the rotor static inlet pressure, and  $Y_{rt}$  is the rotor loss coefficient. All forms of static and total properties are then calculated via the relations established by [Equations 1, 2, 3, 4, 5, 6, 7](#).

Once each segment is evaluated, total compressor pressure ratio ( $PR_C$ ) and exit total temperature under isentropic compression ( $T_{ets}$ ) are calculated as:

$$PR_C = P_{et}/P_{0t} \quad (28)$$

$$T_{ets} = T_{0t}(PR)^{(y-1)/y} \quad (29)$$

The total compressor efficiency ( $\eta_C$ ) is then calculated as:

$$\eta_C = (h_{ets} - h_{0t})/(h_{et} - h_{0t}) \quad (30)$$

Further details regarding the stator and rotor loss coefficients can be found in Tournier's publication [26].

### Radial Dynamics

With radial, centrifugal, and axial ACM component thermodynamic and loss calculations established, dynamic Simulink blocks were developed. Following the established ATTMO technique, inlet total pressure, outlet total enthalpy, and outlet flow rate were calculated as transient states. Transient equations were established for the radial turbine and centrifugal compressor based on work by Gravdahl [16, 17, 18, 19]. The loss equations described prior were utilized to estimate pressure ratios, efficiencies, and compressor/turbine work and torque values that drive the state equations. The inlet pressure state was calculated as:

$$dP_{0t}/dt = T_{0t}R_{sp}(\dot{m}_{in} - \dot{m}_{out})/V \quad (31)$$

where  $V$  is the turbine/compressor volume,  $\dot{m}$  is the fluid flow rate, and  $t$  is time. With inlet flow rate known, the outlet flow rate of the compressor is calculated as:

$$d\dot{m}_{out}/dt = V^2(PR_C P_{0t} - P_{2t})/(2D) \quad (32)$$

where  $D$  is the nozzle/diffuser outer diameter, inlet and outlet total pressure ( $P_{0t}$  and  $P_{2t}$  respectively) are known state variables at a given time step, and  $PR_C$  is the pressure ratio resulting from the design equations.  $PR_C * P_{0t}$  equates to  $P_{2t}$  calculated from the design code and as such, [Equation 32](#) drives the outlet flow rate based on the difference between the outlet total pressure value at the previous time step, and the outlet total pressure value calculated from the design code. Similarly, the radial turbine outlet flow rate is calculated as:

$$d\dot{m}_{out}/dt = V^2(P_{0t}/PR_T - P_{2t})/(2D) \quad (33)$$

The outlet enthalpy state equation for the centrifugal compressor is:

$$dh_{2t}/dt = (\dot{m}_{in}h_{0t} - \dot{m}_{out}h_{2t} + \dot{m}_{out}dh_{imp})/\rho_0V \quad (34)$$

where  $\rho_0$  is the inlet density of the fluid. Similarly for the radial turbine, the outlet enthalpy is calculated as:

$$dh_{2t}/dt = (\dot{m}_{in}h_{0t} - \dot{m}_{out}h_{2t} - \dot{m}_{out}dh_{imp})/\rho_0V \quad (35)$$

For the Simulink component models, density values are calculated at the inlet and exit of the turbine/compressor with an additional value calculated between the nozzle and impeller. Each density value is utilized appropriately for calculating velocity and loss calculations.

### Axial Dynamics

Different transient state calculation techniques were required for the axial components. State variables were only required at the inlet and outlet of the radial turbine and centrifugal compressor blocks where