

Validation of a DC-DC Boost Circuit Control Algorithm

Jon Zumberge
AFRL

Michael Boyd
PC Krause & Associates

Raul Ordonez
University of Dayton

ABSTRACT

Cost and performance requirements are driving military and commercial systems to become highly integrated, optimized systems which require more sophisticated, highly complex controls. To realize benefits of those complex controls and make confident decisions, the validation of both plant and control models becomes critical. To quickly develop controls for these systems, it is beneficial to develop plant models and determine the uncertainty of those models to predict performance and stability of the control algorithms. A process of model and control algorithm validation for a dc-dc boost converter circuit based on acceptance sampling is presented here. The validation process described in this paper is based on MIL-STD 3022 with emphasis on requirements settings and the testing process.

The key contribution of this paper is the process for model and control algorithm validation, specifically a method for decomposing the problem into model and control algorithm validation stages. The other contribution includes projection of model and control uncertainty limits to areas where experimental data has not been taken. These methods will be used to compare delaying of a control action until the beginning of the next control loop versus using the control action immediately but at a non-deterministic time interval (i.e., jitter will be introduced).

CITATION: Zumberge, J., Boyd, M., and Ordonez, R., "Validation of a DC-DC Boost Circuit Control Algorithm," *SAE Int. J. Aerosp.* 9(1):2016, doi:10.4271/2016-01-2030.

INTRODUCTION

With the increase in complexity of systems fielded by both military and commercial entities, there is an ever growing need to "prove" these systems will perform to specifications in a stable manner. To address these issues, there is on-going work to include model based system engineering earlier in the development cycle. One such program is the Air Force Research Laboratories (AFRL) Integrated Vehicle Energy Technology (INVENT) program [1], [2]. To realize the benefits of model based system engineering, there is a need for validation of models, control algorithms, and software used in these complex systems at a reasonable cost [3], [4]. The goal of this work is to describe a novel methodology for nonlinear control algorithm and model validation. Here, the definition of control algorithm validation is to prove that with a pre-specified amount of risk, a control algorithm will behave in a stable manner and meet performance specifications through use of a validated model and/or through experimental testing. The key contributions discussed in this work are first the model and control algorithm validation process (see Section Model Verification and Validation Documentation), a means to calculate the differential equation uncertainty as typically used in

robust controls and decompose it into a problem of finding the model uncertainty and controller uncertainty (see Section Model and Control Uncertainty Analysis), a measurement of strong validation data versus weak validation data (see Section Model and Control Uncertainty Analysis), and projections of model uncertainty to areas not tested (see Section Model and Control Uncertainty Analysis).

Here, a dc-dc boost converter circuit will be used to demonstrate the model and control algorithm validation process. Power converters are used throughout aircraft to help regulate voltage levels and impact power quality. With advances in electrical, mechanical, and structural design on-board more electric aircraft, the electrical systems have undergone considerably higher level of stresses, thus reducing power quality [5], [6]. Better designs of robust controllers for power converters that can handle the disturbances presented on the aircraft can improve power quality and lead to components with longer life expectancy. By taking advantage of the model validation approaches presented here, the testing of new control laws can be reduced, avoiding the typical trial and error methods.

Previously in [7], model validation was performed for the boost circuit under investigation here. Several improvements to the model and testing capability have been performed after that initial work. Those improvements include addition of a higher order diode model and building of a printed circuit board (PCB) for the boost circuit rather than use of a bread board. As such the model uncertainty has changed, but will only be presented here. The same basic process for model validation is also followed, with the additional step of prediction. This paper will focus on the control algorithm validation and how to use the model validation uncertainty estimate in the control algorithm design. The work in [8] looked at model and control uncertainty with the improvements discussed above and used a controller based on National Instruments (NI) hardware. The NI hardware and operating system introduced a time delay in the output of the control law to the boost circuit of one sample time to ensure a low amount of jitter. In this paper, different control hardware and operating system (OS) based on a dSpace's MicroLabBox is used. With this new hardware and OS, the delay time between the output of the control to the boost circuit has been reduced to about half, but considerably more jitter was noticed (i.e., the process of analog to digital conversion, calculation of the control command, and digital to analog conversion requires a significant time ($1/2$ loop time $\sim 35 \mu s$) and is not deterministic). A comparison based on model and control uncertainty will be done to examine the trade-offs that occur between delay time and jitter.

BACKGROUND

Classical robust controls [9, 10, 11, 12] guarantee stability by determining uncertainty bounds then designing an appropriate controller to obtain the needed stability. In [9, 13, 14, 15] various methods of identifying uncertainty or uncertainty bounds are described, typically by comparing a known model to experimental output. These works focus on linear systems and attribute the non-linearities of the systems in question to the uncertainties. These works then use a linear design approach to develop a controller for the system that will be stable for the uncertainties identified. In this paper, the control model uncertainty identification is done by examining the residual dynamics and applying nonlinear control to guarantee stability, in this case backstepping control.

In [16] and [17] backstepping control is performed on an ideal boost circuit similar to the one used by [18]. In [16] an adaptive and non-adaptive backstepping approach are designed and tested in simulation with an unknown / varying load resistance. In [17] the authors consider parameter uncertainty in the inductor, capacitor, resistor, and input voltage but do not consider model form uncertainty which is one focus of this paper. In [17] the authors make use of nonlinear damping to handle uncertainty and find a boundary condition, which is dependent on the variation, whereas in this paper the bound will be fully dependent on the parameters chosen for the control law while the control law is dependent on the uncertainty bounds.

George P. Box wrote "essentially, all models are wrong, but some are useful" [19]. Which begs the question how useful? Through validation one can begin to understand how useful a model is. A considerable amount of modeling work validate models simply by doing a single comparison and making the claim the model is validated (i.e., correct). In fact, the model is "wrong" and requires a quantitative assessment of the accuracy of the model. Model validation defined by MIL-STD 3022 is "the process of determining the degree to which a model, simulation, or federation of models and simulations, and their associated data are accurate representations of the real world from the perspective of the intended use(s)" (i.e., "did you build the right model") [20]. The term model validation is typically well defined, but processes and tools for model validation are still being developed. There are a number of standards that have discussed how one might perform and/or document model validation [20, 21, 22, 23, 24, 25]. The references [21] and [22] are from the American Society of Mechanical Engineers (ASME) and were developed out of the computational solid mechanics group and primarily discuss means of applying p-box type techniques to the validation problem. The reference [23] is from ASME computational fluid dynamics and heat transfer group and deals with identification of parameters of an item or experiment by examining all the sources of uncertainty. The reference [24] discusses the idea of model validation in the context of computational fluid dynamics. The references [20] and [25] are similar in the focus on the documentation that should be included in the model validation process.

This paper uses acceptance sampling as a means to validate the models. In this paper a single-sampling plan from statistical quality control was used [26], [27], and [28] that ensures Type I and Type II risk levels are met.

The acceptance sampling technique was based on meeting the following two probabilities

$$\begin{aligned} P_a(p_1) &\geq 1 - \alpha, \\ P_a(p_2) &\leq \beta, \end{aligned}$$

where p_1 is called the acceptable quality level (AQL), and p_2 is the rejectable quality level (RQL) or lot tolerance percent defective (LTPD). In words, the sampling plan ensures the probability of acceptance at the AQL level is higher than $1 - \alpha$, and lower than β at RQL. The binomial distribution was used here as the population of possible computations (i.e., simulations) is large and effectively samples can be pulled with replacement. These probabilities reference the binomial distribution such that

$$P_a(p) = \sum_{d=0}^c \left\{ \left[\frac{n!}{d!(n-d)!} \right] p^d (1-p)^{n-d} \right\}.$$

The two probabilities can then be optimized over number of test runs (n) and number of allowed bad points/rejects (c) (see [26] for more information on the optimization).

The confirmation tests, the number of tests executed, and the critical number not only protect us from not identifying a good model but also identifying a bad model as good. Each confirmation test is randomly chosen (i.e., the operating conditions are randomly chosen) and evaluated against a pre-determined metric. For example, each confirmation test may measure the amount of error in a given output and if that error exceeds some threshold then the individual confirmation test fails. If the number of confirmation test fails exceeds the critical number then the validation test fails.

MODEL AND CONTROL UNCERTAINTY ANALYSIS

This section presents the generalized solution to the model and control uncertainty analysis that starts with a description of the assumptions that will be used throughout this work, then discusses how the estimation of future uncertainty will be performed. In this section, different estimations will be presented that will be used for comparison.

UNCERTAINTY DEVELOPMENT

The class of systems to be considered for control are ones in strict feedback form [29] (though other forms could be used). In this form, the differential equations are in a form such that each subsystem (i.e., first state equation) can be solved before including the second state equations and so on. The following system is demonstrative of strict feedback form (it includes the model uncertainty and is only second order)

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + \delta_1(X, P, s, u) \quad (1)$$

$$\dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)u + \delta_2(X, P, s, u). \quad (2)$$

In the equation above, $X = [x_1, x_2]^T \in \mathbb{R}^2$. The uncertainty functions, δ_1 and δ_2 , can be functions of parameters (P), states (X), disturbances (s), or input (u). The uncertainties can be time varying functions though constant uncertainties are considerably easier to measure as will be discussed shortly. It is assumed for this work that the disturbance s is measurable and differentiable. For simplicity, dependencies on X, P, s , and u are replaced with Φ_{ji} . Define a domain G that represents the the set of values that Φ_{ji} can take. Thus, $\Phi_{ji} \in G$ where i corresponds to the state, and j will be defined below. The functions $f_1, g_1, f_2, g_2, \delta_1$, and δ_2 are required to be differentiable and Lipschitz in Φ_{ji} within the domain G , for purposes of system identification and uncertainty estimation. It is assumed that the Lipschitz constant is consistent throughout G (i.e., if one calculates the Lipschitz constant over a subset of G it will be similar to other subsets of G). The functions f_1, g_1, f_2 , and g_2 are known dynamics of the plant and will be used for control. For use within the backstepping control the functions $f_1, g_1, f_2, g_2, \delta_1$, and δ_2 are required to also be sufficiently smooth [29]. For purposes of identification of the uncertainty, there is no requirement on the boundedness of the functions. For control purposes this assumes that $|\delta_1(X, P, s, u)| \leq \Delta_1$ and $|\delta_2(X, P, s, u)| \leq \Delta_2$ within G where Δ_1 and Δ_2 are constant, but unknown. These unknown constants will be found through

identification of models of the uncertainties. The requirement of boundedness and domain can be relaxed, but a different control modification would then be needed. See [29] or [30] for approaches to handle unbounded uncertainties.

To determine the total uncertainty in each of the above state equations, simple algebraic manipulation was used to find

$$\delta_1(\Phi_{j1}) = -f_1 - g_1x_2 + \dot{x}_1 \equiv \dot{x}_1 - \dot{x}_{c1}, \quad (3)$$

$$\delta_2(\Phi_{j2}) = -f_2 - g_2u + \dot{x}_2 \equiv \dot{x}_2 - \dot{x}_{c2}, \quad (4)$$

where $\dot{x}_{ci}$ will be known as the control differential. The control differential is used in the development of the controller (i.e., $\dot{x}_{c1} = f_1 + g_1x_2$ and $\dot{x}_{c2} = f_2 + g_2u$).

With the definition of uncertainty determined, there is a desire to de-compose the problem to make estimation of the uncertainties δ_1 and δ_2 tractable. It will be shown that the total uncertainty limit is equal to the model uncertainty limit added to the control uncertainty limit. To start, within domain G , there exist data sets D_j over which the uncertainty estimations are to be performed, when the subscript j can take on different names, as listed next:

- D_t is an experimental data set used for tuning and system identification. The initial use for the data set is system identification of the model, then it will be used to identify the model uncertainty and controller uncertainty. The controller uncertainty here will be used to construct an augmented controller if necessary.
- D_{sv} is a simulated data set of the intended validation set. It will be used in the prediction of the data set D_v .
- D_v is an experimental data set used for model validation. This data set will be used to update the model uncertainty for the future data set and calculate a control uncertainty based on the augmented control laws.
- D_s is a simulated data set using the augmented control. The data set is constructed from the validation data set D_v using the non-controller inputs and disturbances and modifies the control signal u per the augmented controller. This data set will be used for control validation.
- D_f is the experimental future data set to be taken. This data uses the modified control and similar inputs as the validation data set. The goal of this work will be to predict behavior of this data set. Validation of the model and control and their corresponding uncertainty limits will occur with data sets D_v and D_s .

It is important that the validation data set, D_v , is different from the tuning data set, D_t . A larger difference between the two data sets increases the likelihood one will have robust validation. In [31], Oberkampf discusses the need for evaluating a tuned model on data quite different than what was used for tuning. One item proposed here is a metric to differentiate between strong validation and weak validation data sets to ensure robust validation.

Before beginning the determination of the control and model uncertainty a few definitions need to be established. For this work the signal norm will be defined as

$$\|z\|_{\infty} = \sup_{t \geq 0} \|z(t)\|.$$

The vector norm used is a 2-norm,

$$\|Z\|_2 = \sqrt{|Z^2|}.$$

The following describes how one might de-compose the problem so that model uncertainty and control uncertainty can be calculated separately and then combined to find the total uncertainty.

$$\begin{aligned}\delta_i(\Phi_{ji}, t) &= \dot{x}_i(\Phi_{ji}, t) - \dot{x}_{ci}(\Phi_{ji}, t), \\ \delta_{mi}(\Phi_{ji}, t) &\equiv \dot{x}_i(\Phi_{ji}, t) - \dot{x}_{mi}(\Phi_{ji}, t), \\ \delta_{ci}(\Phi_{ji}, t) &\equiv \dot{x}_{mi}(\Phi_{ji}, t) - \dot{x}_{ci}(\Phi_{ji}, t), \\ \delta_i(\Phi_{ji}, t) &= \delta_{mi}(\Phi_{ji}, t) + \delta_{ci}(\Phi_{ji}, t)\end{aligned}$$

where $\dot{x}_{mi}$ is the model differential. These derivatives are computed using state variable filtering [32],[8]. In cases where the model differential is in strict feedback form, the controller uncertainty will be zero. However, in cases such as the dc-dc boost circuit problem, that may not be the case.

Assume that the following bounds exist for each of the five data sets D_j

$$\begin{aligned}\Delta_{mi}(D_j) &\geq \|\delta_{mi}(\Phi_{ji}, t)\|_{\infty} \quad \forall \Phi_{ji} \in D_j \subset G, \\ \Delta_{ci}(D_j) &\geq \|\delta_{ci}(\Phi_{ji}, t)\|_{\infty} \quad \forall \Phi_{ji} \in D_j \subset G, \\ \Delta_i(D_j) &\equiv \Delta_{mi}(D_j) + \Delta_{ci}(D_j) \geq \|\delta_i(\Phi_{ji}, t)\|_{\infty} \\ &\quad \forall \Phi_{ji} \in D_j \subset G.\end{aligned}$$

To find an upper limit on $\|\delta_i(\Phi_{ji}, t)\|$ one needs to estimate $\Delta_{mi}(D_j)$ and $\Delta_{ci}(D_j)$. Since the data set D_j is a future data set, it will need to be estimated. The following is a method for calculating those variables. First by using the negative triangle inequality, one obtains the following

$$\begin{aligned}\|\delta_{ci}(\Phi_{fi})\| - \|\delta_{ci}(\Phi_{si})\| &\leq \|\delta_{ci}(\Phi_{fi}) - \delta_{ci}(\Phi_{si})\| \\ \|\delta_{ci}(\Phi_{fi})\| &\leq \|\delta_{ci}(\Phi_{fi}) - \delta_{ci}(\Phi_{si})\| + \dots \\ &\quad \|\delta_{ci}(\Phi_{si})\|\end{aligned}\tag{5}$$

Because of the Lipschitz property of the uncertainty, one can then show the first element on the right hand side in Eq.(5) is

$$\|\delta_{ci}(\Phi_{fi}) - \delta_{ci}(\Phi_{si})\| \leq L_{ci} \|\Phi_{fi} - \Phi_{si}\|.$$

To calculate $L_{ci} \|\Phi_{fi} - \Phi_{si}\|$ it will be assumed that $\|\Phi_{fi} - \Phi_{si}\| \leq \|\Phi_{vi} - \Phi_{ti}\|$. This assumption is based on the fact the control validation data is typically more similar to the future data set than to the tuning data set, and thus the “distance” is shorter between the control validation data to the future data set than the control validation to the tuning data sets. The more different the control validation data set is from

the tuning data set, the more likely this assumption will hold. In this work, the goal is to predict the performance of the future data set D_f (i.e., design a controller that will meet the performance requirements). The control engineer can thus pick the control validation data set first (i.e., so that it is similar to the future data) and then pick the tuning data set (i.e., different from the validation data), and perform the necessary validation and tuning to understand the model uncertainty and control uncertainty. That may not always be possible (e.g., the future data set is at operating conditions that cannot be emulated in the lab and can only be created during flight). In the situation where the validation data set is different than the future data set, the future data set can be treated as another validation data set and the ideas that will be discussed in the next section can still be applied.

A similar property can be associated with the model uncertainty such that

$$\|\delta_{mi}(\Phi_{fi})\| \leq L_{mi} \|\Phi_{fi} - \Phi_{vi}\| + \|\delta_{mi}(\Phi_{si})\|.$$

It is assumed that $\|\Phi_{vi} - \Phi_{ti}\| \geq \|\Phi_{fi} - \Phi_{vi}\|$ for model purposes. Again it is assumed that the distance from data set D_v to D_t is larger than D_f to D_v . So that

$$\begin{aligned}\delta_i(\Phi_{fi}, t) &= \delta_{mi}(\Phi_{fi}, t) + \delta_{ci}(\Phi_{fi}, t), \\ \delta_i(\Phi_{fi}, t) &\leq L_{mi} \|\Phi_{vi} - \Phi_{ti}\| + \|\delta_{mi}(\Phi_{vi}, t)\| + \dots \\ &\quad L_{ci} \|\Phi_{si} - \Phi_{ti}\| + \|\delta_{ci}(\Phi_{si}, t)\|, \\ \delta_i(\Phi_{fi}, t) &\leq L_{mi} \|\Phi_{vi} - \Phi_{ti}\| + \Delta_{mi}(\Phi_{vi}) + \dots \\ &\quad L_{ci} \|\Phi_{si} - \Phi_{ti}\| + \Delta_{ci}(\Phi_{si}).\end{aligned}\tag{6}$$

LIPSCHITZ CONSTANT ESTIMATION AND DISTANCE EVALUATIONS

In [33] an estimate of the Lipschitz condition was derived for a univariate black box problem. The authors stated “We remark that we have converted the Lipschitz constant estimation problem into a routine curve fitting problem.” In [34], Lipschitz estimation of multivariate problems was solved that required more points to find an estimate than what is used here. Here, a similar approach is taken as in [33] but for a multivariate problem, and making use of a polynomial based function estimate. Obviously, more complicated basis functions could be used, but issues of over-fitting of models can be problematic. The other key take-away from this section is that the desire is to calculate $L\|x - y\|$ and not just the Lipschitz constant. Because of this distinction it will be demonstrated it is important to normalize the problem, so that overestimation of the entire quantity does not occur.

The first step is to estimate the uncertainty function δ for the existing data sets. Note, the uncertainty function δ here can either be a model or control uncertainty function. This estimate at times can be good, but typically is limited over the region on which it is estimated. Once an estimate of δ is created, the calculation of the partial derivative of the function could be performed to estimate the Lipschitz constant.

Rather than calculating the Lipschitz constant directly from this equation, the set of equations will be transformed to be of the form $\delta(Z_a) = z_{1a} + z_{2a} + \dots + z_{na}$ where each z_{ia} is a placeholder for each polynomial and parameter. For example, if the uncertainty equation was of the form

$$\delta(X_a) = a_1 x_{1a} + a_2 x_{2a} + \dots + a_n x_{na}$$

then the transformation would be

$$z_{1a} = a_1 x_{1a},$$

$$z_{2a} = a_2 x_{2a},$$

...

$$z_{na} = a_n x_{na}.$$

With the above transformation, the Lipschitz constant, L , is $\sqrt{n}$, and the distance component, $\|X_1 - X_2\|$, becomes $\|Z_1 - Z_2\|$ where $Z_1 = [z_{11}, z_{21}, \dots, z_{n1}]^T$ and $Z_2 = [z_{12}, z_{22}, \dots, z_{n2}]^T$.

Although by this transformation it is not guaranteed that the answer will be less conservative, typically it is. For a two state system, (i.e., $n = 2$), early work showed the value for $L\|X_1 - X_2\|$ to be quite large, and it was primarily due to $a_1 \gg a_2$ and $x_{2a} \gg x_{1a}$. Although this is not guaranteed and in the case where $a_1 \gg a_2$ and $x_{1a} \gg x_{2a}$ this transformation may overestimate $L\|X_1 - X_2\|$ by $\sqrt{2}$.

Further, this method removes the step of having to perform another optimization problem in the case where the function $\delta(X_a)$ is higher than order one. If $\delta(X_a)$ is higher than order one, then the Lipschitz constant becomes a function of X_a and an optimization is needed to determine the largest value of L .

As mentioned earlier there was a desire to determine weak versus strong validation data sets. The distance calculation of the transformed system $\|Z_1 - Z_2\|$ presented above is one such comparison. The distance function used throughout calculates the maximum distance over all the points in the first data set between its closest neighbor in the second data set by the Euclidean distance function

$$\|Z_1 - Z_2\| = \max_j \dots \left[\min_i ((Z_1(j) - Z_2(i)) \cdot (Z_1(j) - Z_2(i))) \right]. \quad (7)$$

where ' $\cdot$ ' is the vector dot product. Figure 1 depicts what is considered relatively weak validation data, where the tuning data set and the validation data set is intermixed. Figure 2 demonstrates the notion of a strong validation data as the tuning data set is "farther" away from the validation data set than in the previous figure (i.e., distance of 1.5308 compared to 0.1684). The distance difference between the two sets is about an order of magnitude. Ideally, a strong validation data set would encompass the area of intended use. A strong validation data set will be defined such that

$$\|\Phi_{vi} - \Phi_{ti}\| \gg \|\Phi_{vi} - \Phi_{svi}\|.$$

Re-stated in words, if the distance between the validation and the tuning data set is much greater than the distance between the validation data set and its predicted simulation, then that validation data set is a strong validation data set. If the data set is a strong validation data set, then the following will likely hold true

$$\|\Phi_{vi} - \Phi_{ti}\| \approx \|\Phi_{svi} - \Phi_{ti}\|.$$

This property will be used later to predict the validation data set. To maximize the likelihood of the aforementioned occurring, one needs to change the behavior of the test vectors. In this work, the voltage command will not vary in the tuning data set where as the future data set and the validation data set will have a varying voltage output command. It was possible in this work to use similar validation data as the future data set. In that case one would then select tuning data that is quite different from the validation data set. In situations where selection of the validation data set is not fixed to the future data set (e.g., not able to obtain flight data for validation of a component), one could maximize the uncertainty functions to select validation data "farthest" away from the tuning data set. The future data set could then be estimated (i.e., the future data set becomes a new validation data set, and the tuning and validation data set become the new tuning data set).

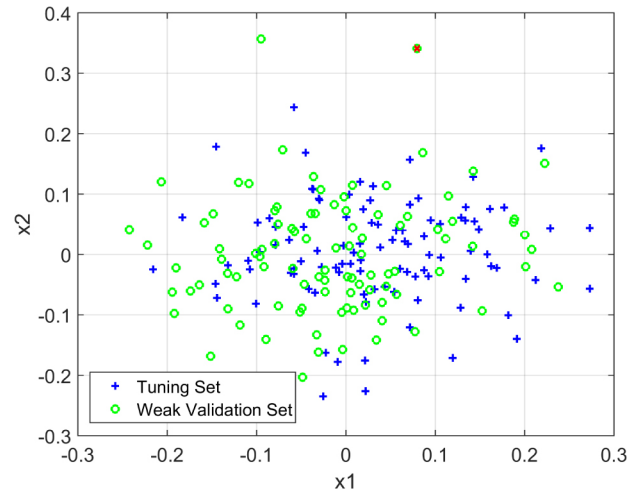


Figure 1. Example of weak validation data (red x indicates farthest point), distance = 0.1684.

Note, because of the nature of the minimum and maximum calculation, the distance function specified is dependent on the order of the vector specified, so the notation from Eq.(7) will be followed throughout. Also note, that since the distance function is dependent on the functions used for construction there will be a superscript assigned to each indicating the set of data used for tuning and the type of uncertainty (either model or control), so $\|\Phi_{svi} - \Phi_{ti}\|^{mt}$ is based on the model uncertainty functions created from the data set D_c . Also note, the quantity $\|\Phi_{svi} - \Phi_{ti}\|^{mv}$ is tuned from the tuning data set and the validation data set.