

Stability of Motion and Mobility Analysis of a 4x4 Hybrid-Electric Vehicle with Passive Drivelines

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ABSTRACT

This paper presents an analysis of coupled longitudinal and lateral dynamics of a 4×4 hybrid-electric off-road vehicle (HEV) with two passive driveline systems, including drivelines with (i) an interaxle open symmetrical differential in the transfer case and (ii) a locked transfer case, i.e., positive engagement of two axles. The axle differentials are open. As the study proved, lateral dynamics of the 4×4 HEV, characterized by the tire side forces, vehicle lateral acceleration, yaw rate and tire gripping factors can be impacted by the tire longitudinal forces, whose magnitudes and directions (positive-negative) strongly depend on the driveline characteristics. At the same time, the tire side forces impact the relation between the longitudinal forces and tire slippages. Thus, coupled dynamics of the vehicle in its longitudinal and lateral directions results in a strong interdependence between the vehicle's mobility, estimated in the longitudinal direction, and the vehicle's stability in the lateral direction. The paper presents mathematical models of the 4×4 HEV vehicle and its driveline systems. A detailed mathematical apparatus describes an analysis of a new vehicle mobility index as a function of the longitudinal force distribution between the wheels and the tires' gripping factors that characterize the lateral skid of the tires and vehicle. It is shown that in certain conditions, the vehicle can be immobilized due to increased tire slippages caused by the tire side forces. Vice versa, the vehicle can go into a lateral skid due to increased tire side forces that are caused by the longitudinal force distribution between the wheels even when the vehicle velocity is small. Computational results illustrate the coupled dynamics and provide recommendations for decoupling the impact of the driveline systems on vehicle mobility and stability of motion.

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INTRODUCTION

A vehicle's wheel power distribution (determined by the driveline's method of dividing power among the axles and driving wheels) has a strong effect on the circumferential forces of the front wheels. In regards to vehicle mobility, the circumferential forces are tied to the slippage at each wheel which limits its forward motion capability. Lateral forces in turn are connected with the side slip angles which affect the stability in the lateral direction. Much research has gone into mathematical and experimental studies on vehicle longitudinal and lateral dynamics and their effect on vehicle motion qualities. A goal of many of these research studies is to better control the interaction of forces using active control systems. Keller, Gorelov, and Anchukov [1] modeled the motion of a truck with a lockable power dividing unit moving on a sloped road. An active differential control algorithm was developed which evaluates the difference in output shaft rotational velocities in order to lock the differential at high slippage when at a steep slope angle. [2] (Oraby et al.) simulated the lateral and longitudinal forces on a vehicle using an active steering system which controls the rear steer angle for its improvement in the handling performance. In [3], Karogal and Ayalew studied four types of torque distribution strategies for stability control using different patterns of corrective torques added to or subtracted from the left or right wheels. K. Isa [4] developed a simulation of the lateral and longitudinal dynamics of a vehicle using

a vision-based autonomous control using an image-processing lane detection algorithm. J. Pytko in [5] presented experimental lateral dynamics results from a test vehicle instrumented with rotating wheel dynamometers and inertia GPS platform driven over deformable surfaces. In [6], Senatore and Sandu modeled the effect of torque distribution on the mobility, handling, and tractive efficiency of an off-road vehicle on moist loam and dry sand. It has been shown that the circumferential force distribution significantly influences the magnitude and direction of the front wheels' wheel lateral forces [7]. Therefore, lateral dynamics properties (forces and side slip) affect the mobility estimate in the longitudinal direction while, at the same time, the circumferential forces contribute to changes in the lateral stability. This means that there is a coupling effect in the vehicle's lateral and longitudinal forces, the driveline power split, and the vehicle's mobility and stability characteristics. In this paper, a mathematical model is derived for a simulation study of the coupled effect of longitudinal and lateral forces on a 4×4 vehicle's mobility and stability of motion. The paper is organized as follows: first, a mathematical model of the 4×4 vehicle in planar motion is described, by which the determination of tire forces and slippage is made. The torque distribution from the transfer case to the tires is then modeled for the two types of passive drivelines. Next, the indices for

estimating the vehicle's mobility and stability is provided. Lastly, computational results are provided for the vehicle model performed with two steering maneuvers and changing terrain conditions.

COUPLED DYNAMICS MODEL

The mathematical model of the 4×4 vehicle models forces at each of the vehicle's four driving wheels. Figure 1 demonstrates the vehicle's forces, steering angles, and geometry [7].

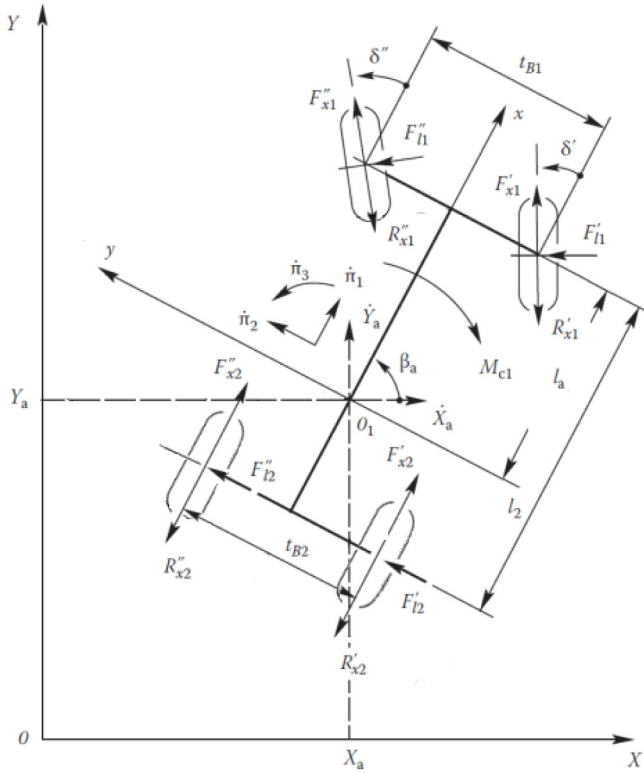


Figure 1. Vehicle planar motion model [7]

The motion of the vehicle can be described using Appel's equations [8]:

$$\frac{\delta S}{\delta \pi_i} = Q_i, \quad i = 1, k \quad (1)$$

where S is the acceleration energy, $\ddot{\pi}_i$ is the quasi-acceleration, Q_i is a generalized force corresponding to the i th quasi-coordinate, and k is the number of quasi-coordinates. In Fig. 1, the vehicle motion is described by three generalized coordinates of the center of mass: X_a , Y_a and β_a . Three quasi-velocities $\dot{\pi}_1$, $\dot{\pi}_2$, and $\dot{\pi}_3$ related to X_a , Y_a and β_a are introduced using the expressions in Equations (2), (3), (4) [8].

$$\dot{X}_a = \dot{\pi}_1 \cos \beta_a - \dot{\pi}_2 \sin \beta_a \quad (2)$$

$$\dot{Y}_a = \dot{\pi}_1 \sin \beta_a + \dot{\pi}_2 \cos \beta_a \quad (3)$$

$$\dot{\beta}_a = \dot{\pi}_3 \quad (4)$$

Using Equations (2), (3), (4), the acceleration energy S is determined for the energy of acceleration that arises from translational motion of the center of mass and rotational motion about this center [8].

$$S = \frac{m_a}{2} (\dot{X}_a^2 + \dot{Y}_a^2 + \rho_a^2 \dot{\beta}_a^2) = \frac{m_a}{2} (\dot{\pi}_1^2 - 2\dot{\pi}_1\dot{\pi}_2\dot{\pi}_3 + \dot{\pi}_2^2 + 2\dot{\pi}_2\dot{\pi}_1\dot{\pi}_3 + \rho_a^2 \dot{\pi}_3^2) \quad (5)$$

where m_a and ρ_a are the vehicle mass and radius of inertia.

Generalized forces Q_i which correspond to the quasi-coordinates are determined from the expression of virtual work [8]:

$$\delta A_1 = Q_1 \delta \pi_i, \quad i = 1, 3 \quad (6)$$

Using Equations (1), (5), and (6), the planar motion equations of the vehicle become

$$m_a (\ddot{\pi}_1 - \dot{\pi}_2 \dot{\pi}_3) = F_{\Sigma x} \quad (7)$$

$$m_a (\ddot{\pi}_2 + \dot{\pi}_1 \dot{\pi}_3) = F_{\Sigma y} \quad (8)$$

$$I_a \ddot{\pi}_3 = M_{\Sigma} \quad (9)$$

The planar motion of the vehicle is described by these three equations (7), (8), (9): these equations model forces and acceleration along the x-axis (longitudinal), forces and acceleration along the y-axis (lateral), and moments and rotational acceleration (yaw). $F_{\Sigma x}$ is the sum of all forces projected on the longitudinal axis and $F_{\Sigma y}$ is the sum of all forces projected on the lateral axis. M_{Σ} is the sum of moments. t_{bi} is the lateral distance between left and right tire centers. l_a is the longitudinal distance from the front axle to the center of gravity. l_i are the distance between axles ($l_1 = 0$ and l_2 is the wheelbase).

$\dot{\pi}_1$, $\dot{\pi}_2$, and $\dot{\pi}_3$ are quasi-velocities of the vehicle's center of gravity: $\dot{\pi}_1$ in the longitudinal direction, $\dot{\pi}_2$ in the lateral direction, and yaw rate $\dot{\pi}_3$. $\ddot{\pi}_i$ ($i = 1, 3$) are quasi-accelerations. $\delta_1^{(n)}$ are the wheel steer angles. The front axle's steer angle, δ_1 , is set by the vehicle steering system. The individual steer angles of the front wheels to give each wheel the same turn center, $\delta_1^{(n)}$, are derived from Fig. 2, showing the separate wheel steer angles and turn radii.

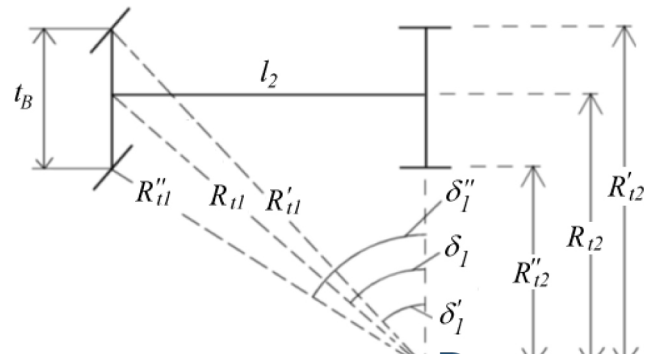


Figure 2. Wheel and axle turn radii

Three equations derive from the geometry in Fig. 2:

$$\cot \delta'_1 - \cot \delta''_1 = \frac{t_{b1}}{l_2} \quad (10)$$

$$R_t + 0.5t_{b1} = l_2 \cot \delta'_1 \quad (11)$$

$$R_t - 0.5t_{b1} = l_2 \cot \delta''_1 \quad (12)$$

Equation (13) is the result of summing Equations (11) and (12):

$$\cot \delta'_1 + \cot \delta''_1 = \frac{2}{\tan \delta_1} \quad (13)$$

By solving Equation (10) for $\cot \delta'_1$ or $\cot \delta''_1$, then substituting it in Equation (13), δ'_1 and δ''_1 can then be solved for resulting in

$$\delta'_1 = \tan^{-1} \left(\frac{l_2}{\frac{l_2}{\tan \delta_1} \mp 0.5t_{b1}} \right) \quad (14)$$

The rear wheels are not steered, making $\delta_2 = \delta_3 = 0$. Forces contributing to the vehicle motion are the rolling resistance forces $R_{xi}^{(n)}$, lateral forces $F_{li}^{(n)}$, and circumferential forces $F_{xi}^{(n)}$. Rolling resistance forces are proportional to the normal load on the tire by a rolling resistance coefficient, f [8]. This coefficient is a property of the tire and terrain. The tire model is the same for all tires and does not change throughout the tests, however the terrain condition changes as the vehicle moves. To model the effects of changing ground conditions, the terrain properties are modeled as a continuous random process using a method described in [9]. Normal reactions are modeled using a vibration multi-mass model which takes into account dynamic shifting in the vertical and lateral directions. The loads without vertical vibrations are first calculated as

$$R_{st,zi}^{(n)} = \frac{R_{st,zi}}{2} \pm F_{yi} \frac{h_y}{t_{bi}} \quad (15)$$

$R_{st,zi}$ are the static loads on each axle, calculated according to the vehicle's front/rear mass distribution. h_y is the height of the wheel gravity center. Forces F_{yi} act at the height of the gravity center for the front and rear wheel, loading the outer wheels and reducing the load of the inner wheels during a curvilinear maneuver. The forces are computed proportionally to the front and rear masses m_i and the lateral acceleration with Equation (16).

$$F_{yi} = \frac{m_i}{2} a_{yi} \quad (16)$$

a_{yi} is the lateral acceleration projected to each axle, calculated with

$$a_{yi} = \ddot{r}_2 + (l_a - l_i) \ddot{\pi}_3 \quad (17)$$

The displacements of the sprung and unsprung masses are used to calculate new values of the normal loads with dynamic through Equation (18) [8].

$$R_{zi}^{(n)} = R_{st,zi}^{(n)} + K_t (z_{ri}^{(n)} - z_{ui}^{(n)}) + C_t (\dot{z}_{ri}^{(n)} - \dot{z}_{ui}^{(n)}) \quad (18)$$

The next set of forces in the planar motion equations are the lateral forces. Lateral forces are proportional to the tire's side slip angles α_i and the tire's cornering stiffness, $C_{ai}^{(n)}$, which is dependent on the tire normal reaction, tire traction force, and surface of motion properties [8].

$$F_{li}^{(n)} = C_{ai}^{(n)} \alpha_i^{(n)} \quad (19)$$

The angles of the side slip can be calculated from Equation (20) by comparing the projection of the velocity of the wheels on the vehicle's longitudinal axis, $V_{xi}^{(n)}$, and the velocity projected on the lateral axis, $V_{yi}^{(n)}$. Tire sideslip angles are computed with

$$\alpha_i^{(n)} = \delta_i^{(n)} - \arctan(V_{yi}^{(n)} / V_{xi}^{(n)}) \quad (20)$$

Where $V_{yi}^{(n)}$ and $V_{xi}^{(n)}$ are velocity projections on the vehicle's longitudinal and lateral axes:

$$V_{yi}^{(n)} = \dot{r}_2 + (l_a - l_i) \dot{\pi}_3 \quad (21)$$

$$V_{xi}^{(n)} = \dot{\pi}_1 \pm 0.5t_{bi} \dot{\pi}_3 \quad (22)$$

The last force, $F_{xi}^{(n)}$ is the circumferential wheel force developed by the wheel torque. The tire radius which relates the torque and circumferential force is rolling radius in the driven mode $r_{wi}^{0(n)}$.

$$F_{xi}^{(n)} = \frac{T_{wi}^{(n)}}{r_{wi}^{0(n)}} \quad (23)$$

$r_{wi}^{0(n)}$ is accepted as a reference radius for determining theoretical velocity at zero slip ratio under any type of road and terrain condition [8]. This tire slip ratio $s_{di}^{(n)}$ is modeled using an exponential function [8]:

$$F_{xi}^{(n)} = \frac{R_{zi}^{(n)} \mu_{pxi}^{(n)} s_{di}^{(n)}}{\sqrt{(|s_{di}^{(n)}|)^2 + (1 - |s_{di}^{(n)}|)^2 \tan^2 \alpha_i^{(n)}}} \left[1 - e^{-k_i^{(n)} \sqrt{(|s_{di}^{(n)}|)^2 + (1 - |s_{di}^{(n)}|)^2 \tan^2 \alpha_i^{(n)}}} \right] \quad (24)$$

$\mu_{pxi}^{(n)}$ is the peak coefficient of friction, which determines the maximum circumferential forces that can be developed under the current terrain condition. Equation (24) was derived mathematically based on experimental results showing that a tire with a higher side slip wheel has a higher slippage when it is running with a higher side slip