

Off-Road Tire-Terrain Interaction: An Analytical Solution

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ABSTRACT

A novel semi-analytical solution has been developed for the calculation of the static and dynamic response of an off road tire interacting with a deformable terrain, which utilizes soil parameters independent of the size of the contact patch (size-independent). The models involved in the solution presented, can be categorized in rigid and/or pneumatic tires, with or without tread pattern. After a concise literature review of related methods, a detailed presentation of the semi-analytical solution is presented, along with assumptions and limitations. A flowchart is provided, showing the main steps of the numerical implementation, and various test cases have been examined, characterized in terms of vertical load, tire dimensions, soil properties, deformability of the tire, and tread pattern. It has been found that the proposed model can qualitatively capture the response of a rolling wheel on deformable terrain.

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INTRODUCTION

Tire-road interaction is a highly complex dynamic phenomenon, which has been subject to extensive research and development within the automotive industry. The tire is the dominant link between a vehicle body and road surface dynamic interaction in terms of accelerating, braking and steering forces. On-road tires have attracted significant attention with regards to reducing rolling resistance and fuel consumption following several European and/or world guidelines. However, off-road tires-soil interaction occupies an equally important position within the tire industry not only for military purposes but also for the growth of agricultural-based countries' economies.

The ability of tracked and wheeled vehicles to transverse certain types of soft soils is a complex, multivariable phenomenon and because of this, several assumptions have to be made in order to create accurate and yet computationally efficient off-road tire models. Finite element models can produce highly accurate time-domain solutions, taking into account the exact properties of the tire and soil

surface, however, they are typically unsuitable as inputs for interactive, dynamic vehicle simulations as they operate far from real time. Semi-analytical and/or numerical simplified models have been created in the past where the soil is being regarded as an elastic-perfectly plastic material [1] and the tire is being considered as a rigid wheel model, representative of a highly inflated tire rolling on a very soft soil [2]. To the best of the authors' knowledge, regardless of the assumptions involved, in most of the models published in the literature the off-road tire-soil interaction is studied in terms of two main effects. The first tries to capture the relationship between the normal load and vertical displacement (sinkage) of the wheel into the soil; the second has to do with the shear stress-shear displacement developed on the tire-soil interface.

In a pioneering paper representative of the former effect [3], based on the observation that the main resistance in a tire's movement is due to the effort to create a rut in order to transverse, Bernstein proposed eq. (1), which was later extended in a more generalized version, i.e. eq. (2), presented in [4]:

$$p = k \cdot z^{0.5} \quad (1)$$

$$p = k \cdot z^n \quad (2)$$

Many researchers subsequently attempted to improve or even establish their own semi-analytical relationships based on experimental data and in situ measurements. The most known and widely used equations until now are eq. (3) developed in [5] and eq. (4) developed in [6]. Both the aforementioned studies try to divide the global k soil factor of eq. (2) into a cohesive and frictional component, which would permit a more accurate and realistic soil response for a variety of materials. A significant contribution on the terramechanics literature was the inclusion of the wheel width b , on the overall pressure-sinkage response [6]. Some of the assumptions and limitations, common for both models are: (a) the use of a constant radial pressure distribution along the width of the tire and (b) the utilization of non-invariant soil parameters, which would necessitate case-dependent experimental soil measurements in order to extract the frictional and cohesive soil components.

$$p = \left(\frac{k_c}{b} + k_\phi \right) \cdot z^n \quad (3)$$

$$p = \left(ck'_c + \gamma bk'_\phi \right) \left(\frac{z}{b} \right)^n \quad (4)$$

Wong and Reece studied both driven and towed rigid wheels rolling on sand in a series of papers, and divided the contact patch of the wheel-soil interface into two failure zones [7]. The first zone is from the point where the wheel comes into contact with the soil up to the point where the maximum radial pressure occurs, θ_M , and the second zone is from the latter point up to the point where the wheel loses its contact with the soil, θ_r , illustrated in Fig.1. Following this work, and based on eq. (4) they proposed eq. (5).

In [8], which was based on [9], the indentation of a rigid plate on various soft soils was studied and a Load Sinkage Analytical (LSA) model was developed, described by eq. (6), which is based on four invariant soil parameters (cohesion, friction angle, density and modulus of elasticity of the soil) which can be given or measured for any terrain using classical soil mechanics or routine test methods (through hand held instruments). In this study a novel approach is developed, which is based on an extension of the work presented in [8], with the following modifications: (a) it implements the two failure zones defined in [7], and (b) it applies eq. (6) to infinitesimal segments resulting from the discretization of the footprint of a rigid or deformable wheel.

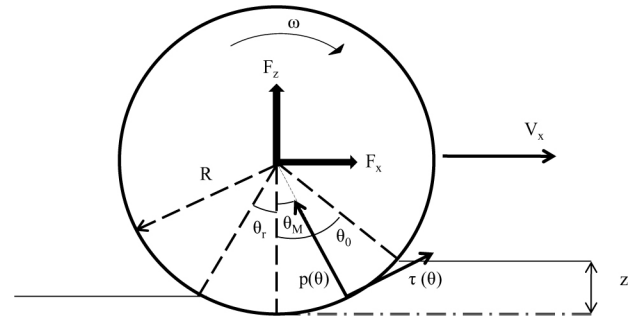


Figure 1. Schematic representation of a rigid wheel rolling on soft soil with two failure zones.

$$p(\theta) = \begin{cases} \left(ck'_c + \gamma bk'_\phi \right) \left(\frac{r}{b} \right)^n (\cos\theta - \cos\theta_0)^n, & \theta_M \leq \theta < \theta_0 \\ \left[\cos\left(\theta_0 - \left(\frac{\theta - \theta_r}{\theta_M - \theta_r} \right) (\theta_0 - \theta_M) \right) - \cos\theta_0 \right]^n, & \theta_r \leq \theta < \theta_M \end{cases} \quad (5)$$

$$p = \frac{1}{\frac{D_1}{B_1} + \frac{D_2}{E \cdot z} \cdot \omega_1 \cdot B \cdot \xi} \quad (6)$$

With regards to the second main effect which is related to shear stress-shear displacement relationship, three main approaches have been developed [10, 11]. In the first approach presented in [10], eq. (7) has been presented, which is representative of soils not exhibiting a hump in their shear stress - shear displacement diagrams, such as loose sand, saturated clay, dry fresh snow and most of the disturbed soils. In these types of soils, with increasing the shear displacement, the shear stress increases monotonically up to a certain value, where it stabilizes after a certain value of the shear displacement.

$$\tau = (c + p \cdot \tan \phi) \left[1 - e^{-j/K} \right] \quad (7)$$

In the second and third approaches presented in [11], relations are presented which are representative of materials exhibiting a “hump” near the maximum shear stress, and after passing this limit, the shear stress may decrease continuously (second approach), or stabilize to a certain value (third approach). For the second approach, presented in [11], eq. (8) has been proposed, representative of soils which exhibit a “hump” of maximum shear stress and then by further increasing the shear displacement the shear stress continuously decreases.

$$\tau = (c + p \cdot \tan \phi) (j / K_w) e^{(1-j/K_w)} \quad (8)$$

With regards to the third category of shear stress response, herein the shear stress exhibits a “hump” and then by further increasing the shear displacement it decreases to a constant value. The equation describing this response was established in its final form as in eq. (9).

However, as stated in [18], K_r and K_w used in eqs. (8) and (9) are non-invariant soil plate parameters and can be determined only experimentally. In this study eq. (7) is used to describe the shear stress developed at the tyre-soil interface.

$$\tau = (c + p \cdot \tan \phi) K_r \cdot \left\{ 1 + \left[\frac{1}{K_r \left(1 - \frac{1}{e} \right)} - 1 \right] e^{\left(\frac{j}{K_w} \right)} \right\} \left(1 - e^{\left(-\frac{j}{K_w} \right)} \right) \quad (9)$$

In [7] eq. (10) is proposed for a driven wheel, which is the most widely adopted relationship for the description of the shear displacement as a function of slip (used in the present study):

$$j = R \left[(\theta_0 - \theta) - (1 - i)(\sin \theta_0 - \sin \theta) \right] \quad (10)$$

Based on the semi-empirical equations presented so far, which are restricted to the most fundamental, various researchers have developed semi-analytical models applied either on rigid or flexible tires. It should be highlighted that use of the aforementioned equations, including their inherent limitations such as, the inability to predict the additional sinkage caused by the slipping conditions of the wheel, may lead to large errors and unrealistic response of the rolling wheel. Following this limitation, in [12] a new model is proposed where the initially constant exponent of deformation is replaced by a function of slip, as in eq. (11). The accuracy of the latter equation is assessed though a series of experiments and close agreement with experimental results is observed in the dynamic sinkage predictions.

$$N = n_0 + n_1 i \quad (11)$$

Among the solutions targeted for flexible tires, a first direction was given by Bekker, who expressed the basic idea of the replacement of a deformable wheel by a larger substitute circle. Following that, in [13], the aforementioned idea was implemented and satisfactory results close to experimental data were obtained. Moreover, equations were given for a flexible tire with a more solid theoretical approach, where the tire properties were related to its vertical deformation [14]. Furthermore, in [15], the equations proposed in [14] were used along with additional modifications to propose an enhanced tire-soil interaction model, capable of accurately predicting the traction, the slip sinkage and the multipass effect.

Regarding treaded wheels, in [16] two different pressure distributions were considered, the first being applied at the lug tips, and the other being applied on the carcass between the lugs. Following this the overall pressure distribution was given as the sum of the two aforementioned pressures, weighted with the tread and void ratios respectively. In [13], a new approach (which was initially proposed in [17]), to consider the negative (void) and positive (tread) portions of the tire is presented where the soil is assumed to be interlocked between the tread blocks and acting only in a shear mode. Based on

the soil cutting theory of soil mechanics, the forces normal to the sides of the treads were estimated and their contribution on the overall tractive performance was considered in [18].

TIRE-SOIL INTERACTION MODELING

In this section, the mechanics of a rigid or deformable wheel (with or without tread pattern) in contact with loose soil is addressed. A novel semi-analytical solution is described, as well as its assumptions and limitations are outlined. This solution can capture the static and dynamic response of four different models (rigid-treadless, deformable-treadless, rigid-treaded and deformable-treaded). Two types of sinkage may be identified for each model: static sinkage (z_{static}) and dynamic sinkage (z_{dynamic}), where z_{dynamic} is a function of slip and for increasing slip, the so-called digging effect is more pronounced, leading thus to increased z_{total} . This section is organized in two parts: firstly, the assumptions of the new solution are mentioned; afterwards, a flowchart of the calculations is presented and explained.

Assumptions

The tire is assumed to interact with homogeneous soil, described by invariant parameters. It is considered that no stress concentrations are present, and the vertical load does not exceed the bearing capacity of the soil, as described by the Terzaghi theory. The tire is assumed to move only in the longitudinal direction with no side forces affecting the overall traction. The pressures acting on the tire are assumed to be in the radial direction and constant along the width of the tire. Regarding the treaded wheels, interlocking action is assumed for the soil inside the region between the successive tread blocks; this means that the soil acts only in a shear mode. For the external surface of the tread blocks, the minimum force given by either the friction force (stemming from the vertical load and given by the Coulomb friction law) or the maximum shear force (which develops due to the stresses in the soil and is given by the Mohr-Coulomb law) is selected, since the shear strength of the tire-soil system is determined by the weaker of the above two types of shear strengths.

The contact patch of any deformable tire is considered to be described by the geometry of a larger substitute circle, as initially proposed by Bekker. As far as the tread pattern is concerned, the void inbetween the tread blocks is assumed to be fully filled with soil, and the contribution of the forces acting normal to the surfaces of the tread pattern which are perpendicular to the carcass to the tire response is neglected.

Static Sinkage

The configuration of the wheel model is shown in Fig. 2. The angle θ_s is the static entry angle and becomes zero at the point where the maximum pressure occurs. The pressure distribution is symmetric; only the part involving the positive angle θ is considered in the integration and then the result is doubled. The tire footprint is discretized into a large number of segments (typically ≥ 1000), at each of which the soil pressures are calculated according to eq. (12):