

A Novel Kinematic Model of a Steerable Tire for Examining Kingpin Moment during Low-Speed-Large-Steering-Angle Cornering

Dai Quoc Vo, Hormoz Marzbani, Mohammad Fard, and Reza N. Jazar
 RMIT University

ABSTRACT

As long as a tire steers about a titled kingpin pivot, the point coming in contact with the road moves along its perimeter. This movement affects the determination of kingpin moments caused by the tire forces, especially for large steering angles. The movement, however, has been neglected in the literature on the steerable-tire-kinematics-related topics. In this investigation, the homogeneous transformation is employed to develop a kinematic model of a steering tire in which the instantaneous ground-contact point on the tire is considered. The moments about the kingpin axis caused by tire forces are then computed based on the kinematics. A four-wheel-car model is constructed for determining the kingpin moment of steering system during the low-speed cornering maneuver. The result shows that the displacement of the ground-contact point along the tire perimeter is significant for large steering angles. It also indicates that, at the low-speed cornering, the moment about the kingpin axis caused by vertical forces benefits self-centering while those caused by longitudinal forces, lateral forces, and 'aligning' torques facilitate the turn. A parameter study is also conducted to visualize the effects of wheel alignment parameters on the kingpin moment. It appears that an increase in kingpin offset or a decrease in caster angle affects the kingpin moment such that it supports the self-centering. The effects of kingpin inclination and caster trail on the moment, however, depend on the amplitude of steering angle. The result is compared to those derived from a multi-body model for validation.

KEYWORDS: caster, kingpin inclination, caster trail, kingpin offset, tire kinematics, kingpin moment, self-centering, low-speed cornering

CITATION: Vo, D., Marzbani, H., Fard, M., and Jazar, R., "A Novel Kinematic Model of a Steerable Tire for Examining Kingpin Moment during Low-Speed-Large-Steering-Angle Cornering," *SAE Int. J. Passeng. Cars - Mech. Syst.* 10(1):2017, doi:10.4271/2016-01-8030.

1. INTRODUCTION

The self-centring ability of road steering wheels in low speed cornering is an important aspect in the design of vehicle steering system. In the low speed cornering maneuver, the driver steers the hand wheel with a large angle to take the car out of parking lot, for example. After being released, the steering wheel is expected to return automatically towards the straight position by self-centring moment. This moment is greatly influenced by forces, torques acting on the steering tires, the suspension and steering geometry. This topic has long attracted researchers' attention [1, 2, 3, 4, 5, 6]. More specifically, the effects of steering geometry and frictional force on the maximum steering torque of a stationary vehicle was analysed by Gough [2]; Sharp and Granger [5] conducted a research in which measurements made on a static rig, and on a car are complemented by a mathematical system model; Cho [1] proposed an analytical model together with a multi-body model which shows how the steering response can be interpreted in terms of vertical force, lateral force with aligning torque, and longitudinal force. In those investigations, the kinematics of the road steering wheel was developed by implicitly accepting the assumption that the ground-tire contact is a fixed point on the tire perimeter. It was also assumed that 'the tire-to-ground contact point is the wheel radius below the wheel centre; therefore, its vertical displacement is the same as that of the wheel centre' [5]. However, when a wheel is steering around a titled

kingpin axis with a large steering angle, the two assumptions are no longer reasonable. The former will be proven inexact in [Section 2.2](#). The latter is not true as the wheel gains camber when it is steered about the tilted axis. Therefore, a kinematic model that can take these effects into account is essential for determining kingpin moment during the low-speed cornering maneuver.

In this investigation, the homogeneous transformation is utilized to develop the kinematics and dynamics of steering tires. Specifically, the moment acting on the steering tire around the kingpin axis is computed with the effect of the ground-contact point change is taken into account. A four-wheel planar vehicle model is used to determine the total restoring moment of the steering system. A parameter study is also carried out to show the effect of each parameter on the self-centring ability of the car. The analytical is then validated using a multi-body simulation.

2. DETERMINATION OF KINGPIN MOMENT

In this section, the homogeneous transformation is employed as a tool to develop the steering tire kinematics from which the ground-contact point is derived. Having the point enables us to compute the kingpin axis moment caused by forces and torques. In order to determine the net kingpin moment for the steering system during the maneuver a

four-wheel car model is developed and solved using Simulink platform. A multi-body model is then constructed in CarSim software to validate the analytical results.

2.1. Homogeneous Transformation

In this analysis, we accept the assumption that the road wheel (there is no difference between road wheel and tire in this paper) is a flat and rigid disk. Without loss of generality, we also assume that the wheel is upright when it is not steering, so it has zero initial camber.

In order to establish the kinematics of the road steering wheel the following coordinate frames, as shown in Figure 1, are utilized. The sign convention in [7] is followed here.

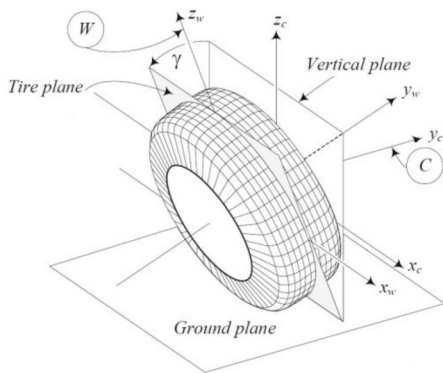


Figure 1. The wheel coordinate frame W and the wheel-body coordinate frame C .

Wheel coordinate frame $W(x_w, y_w, z_w)$ is the frame that its origin W is attached to the wheel centre. Its x_w axis is initially parallel to the ground plane and directed forward; the z_w axis is vertical to the ground and upward, as long as the wheel is not steering. The (x_w, Wz_w) plane coincides with tire plane which is made by centrally narrowing the tire into a flat disk. The y_w is spin axis and directed to the left (of the driver). The W -frame is fixed to the wheel so it follows every motion of the wheel except the spin.

When the wheel is not steering, a wheel-body coordinate frame $C(x_c, y_c, z_c)$ that initially coincides with the wheel coordinate frame is attached to the car body. The wheel-body coordinate frame is motionless with the car body, so it does not follow any motion of the wheel when steering.

As defined, the orientations and locations of the two coordinate frames are not affected by the spin motion of the wheel. Therefore, the spin motion is excluded from this kinematic analysis.

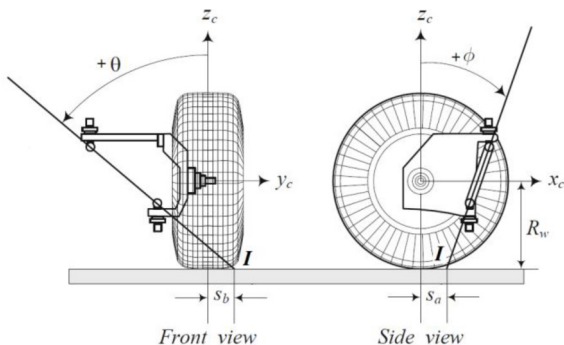


Figure 2. The orientation and location of the kingpin axis with sign convention for left front wheel.

Steering a tire can be considered to be the relative motion of the W -frame with respect to the C -frame. Here, the orientation and the location of the steering axis are expressed in the C -frame. The orientation can be determined by caster angle ϕ and kingpin inclination (KPI) angle θ while the location can be the intersection of the axis and the ground I , as shown in Figure 2. As presented in [8], the steering axis is expressed by its direction unit vector $\hat{u}$, and a position vector r_I :

$${}^C \hat{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \begin{bmatrix} \cos \theta \sin \phi \\ -\sin \theta \cos \phi \\ \cos \theta \cos \phi \end{bmatrix} \quad (1)$$

$${}^C r_I = \begin{bmatrix} s_a \\ s_b \\ -R_w \end{bmatrix} \quad (2)$$

With the previously defined coordinate frames and steering axis, a steering motion can now be mathematically equivalent to a δ -angle - rotation of the W -frame about the steering axis, with respect to the C -frame [8].

The kinematics of the steering tire is then developed by transforming the coordinates of any point from the W -frame into the C -frame during steering process. The transformation is governed by the following equation [8, 9]:

$${}^C r = {}^C T_W {}^W r \quad (3)$$

where ${}^C r$ and ${}^W r$ are the homogeneous representations of the position vector of point P in the C -frame and the W -frame, respectively.

$${}^C r = \begin{bmatrix} {}^C x_P \\ {}^C y_P \\ {}^C z_P \\ I \end{bmatrix}; {}^W r = \begin{bmatrix} {}^W x_P \\ {}^W y_P \\ {}^W z_P \\ I \end{bmatrix} \quad (4)$$

and ${}^C T_W$ is a 4×4 homogeneous matrix:

$${}^C T_W = \begin{bmatrix} R_{\hat{u}, \delta} & d_I - R_{\hat{u}, \delta} d_I \\ 0 & I \end{bmatrix} \quad (5)$$

where $R_{\hat{u}, \delta}$ is called Rodriguez rotation formula, a 3×3 matrix for transforming between the two initially coincident frames if the rotation axis passes through the common origin of the frames; and, d_I is position vector of the point I expressed in the C -frame.

$$R_{\hat{a},\delta} = \begin{bmatrix} u_1^2 \text{vers}\delta + \cos\delta & u_1 u_2 \text{vers}\delta - u_3 \sin\delta & u_1 u_3 \text{vers}\delta + u_2 \sin\delta \\ u_1 u_2 \text{vers}\delta + u_3 \sin\delta & u_2^2 \text{vers}\delta + \cos\delta & u_2 u_3 \text{vers}\delta - u_1 \sin\delta \\ u_1 u_3 \text{vers}\delta - u_2 \sin\delta & u_2 u_3 \text{vers}\delta + u_1 \sin\delta & u_3^2 \text{vers}\delta + \cos\delta \end{bmatrix} \quad (6)$$

$$d_I = r_I = \begin{bmatrix} s_a \\ s_b \\ -R_w \end{bmatrix} \quad (7)$$

where $\text{vers}\delta = 1 - \cos\delta$

Using the homogeneous representation is just for simplifying numerical calculations. The first three coordinates of a position vector are still the same as physical coordinates of the vector. Therefore, in this investigation, we still use the regular vectors and their homogeneous representation equivalently.

2.2. The Change of Ground-Contact Point Along the Tire Perimeter

In this section, we will prove that the point on the tire being in contact with the ground moves along its perimeter when steering, and that the movement is significant for large steering angles.

Here, T_0 and T respectively denote the ground-contact points (on the tire perimeter) associated with zero and δ steering angles. The homogeneous representation of T_0 in the W -frame for any steering angle is:

$${}^W r_{T_0} = \begin{bmatrix} 0 \\ 0 \\ -R_w \\ 1 \end{bmatrix} \quad (8)$$

Its expression in the C -frame is written as:

$${}^C r_{T_0} = \begin{bmatrix} {}^C x_{T_0} \\ {}^C y_{T_0} \\ {}^C z_{T_0} \\ 1 \end{bmatrix} = {}^C T_W {}^W r_{T_0} \quad (9)$$

Substituting (1) in (6); (7) and (6) in (5); and then, (5) and (8) in (9) yields:

$${}^C z_{T_0} = -\frac{1}{2} \frac{s_a \cos^2 \theta \sin 2\phi - s_b \sin 2\theta \cos^2 \phi}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \text{vers}\delta - \frac{s_a \cos \phi \sin \theta + s_b \cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta - R_w \quad (10)$$

${}^C z_{T_0}$ shows the vertical distance between point T_0 on the tire and point C fixed to the car body when the wheel is steering. It can be verified that when caster and KPI angles are zero, T_0 is the tire radius below C .

According to [8], the z -coordinate of the point T in the C -frame after being steered of δ angle is determined through the camber angle:

$${}^C z_T = {}^C z_W - R_w \cos \gamma \quad (11)$$

where

$${}^C z_W = -\frac{s_a \cos \phi \sin \theta + s_b \cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta - R_w + \frac{1}{2} \frac{(2R_w \cos^2 \theta - s_b \sin 2\theta) \cos^2 \phi - s_a \cos^2 \theta \sin 2\phi}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \text{vers}\delta \quad (12)$$

and

$$\gamma = \frac{\pi}{2} - a \cos \left[\frac{\cos \theta \sin \phi}{\sqrt{\cos^2 \phi + \cos^2 \theta \sin^2 \phi}} \sin \delta - \frac{\cos^2 \phi \sin \theta \cos \theta}{\cos^2 \phi + \cos^2 \theta \sin^2 \phi} \text{vers}\delta \right] \quad (13)$$

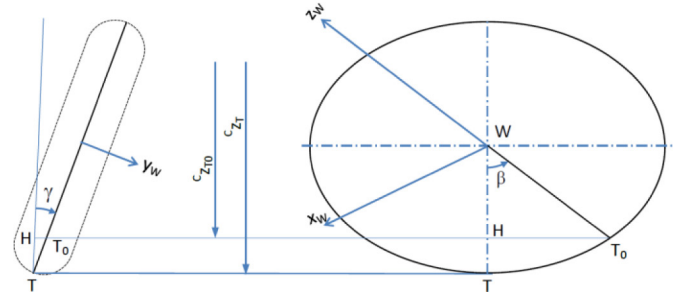


Figure 3. The angular motion of the ground-contact point on the steering tire perimeter.

We can determine the angle β that the ground-contact point on the tire makes when it moves along the tire perimeter. As shown in Figure 3, we have:

$$\begin{aligned} \cos \beta &= \frac{WT - HT}{WT_0} = \frac{R_w - HT}{R_w} \\ &= 1 - \frac{HT}{R_w} = 1 - \frac{{}^C z_{T_0} - {}^C z_T}{R_w \cos \gamma} \end{aligned} \quad (14)$$

Substituting (10) and (11) in (14) yields:

$$\cos \beta = \frac{\cos \delta}{\cos \gamma} + \frac{\cos^2 \theta \cos^2 \phi}{(\cos^2 \phi + \cos^2 \theta \sin^2 \phi)} (1 - \cos \delta) \quad (15)$$

As indicated by (15), β depends only on the orientation of the steering axis and the steering angle. The angular motion that the ground contact point makes along the tire perimeter, for an exemplary