

A Stress-Based Non-Proportionality Parameter for Considering the Resistance of Slip Systems of Shear Failure Mode Materials

Sean A. McKelvey and Yung-Li Lee
FCA US LLC

ABSTRACT

Multiaxial loading on mechanical products is very common in the automotive industry, and how to design and analyze these products for durability becomes an important, urgent task for the engineering community. Due to the complex nature of the fatigue damage mechanism for a product under multiaxial state of stresses/strains which are dependent upon the modes of loading, materials, and life, modeling this behavior has always been a challenging task for fatigue scientists and engineers around the world. As a result, many multiaxial fatigue theories have been developed. Among all the theories, an existing equivalent stress theory is considered for use for the automotive components that are typically designed to prevent Case B cracks in the high cycle fatigue regime. The focus of this paper is on the introduction of the equivalent stress model with an emphasis on characterizing a non-proportionality parameter to account for material dependent resistance of interactions between slip systems under nonproportional loading. This parameter is determined experimentally by comparing stress-life curves for two extreme loading cases, in-phase and 90° out-of-phase axial-torsional fatigue tests. However, multiaxial testing is very expensive and time consuming. As a result, an empirical formula was developed for the non-proportionality parameter, α_{LTP} , based on readily available material properties.

CITATION: McKelvey, S. and Lee, Y., "A Stress-Based Non-Proportionality Parameter for Considering the Resistance of Slip Systems of Shear Failure Mode Materials," *SAE Int. J. Mater. Manf.* 9(2):2016, doi:10.4271/2016-01-9081.

INTRODUCTION

All the stress-based multiaxial fatigue models were derived based on the experimental data and observation from load controlled fatigue tests. But recent research studies have found that multiaxial fatigue behavior for load and strain-controlled tests on smooth tubular specimens made of shear dominant failure mode materials is completely different [1]. The purpose of this paper is to evaluate some of the popular stress-based multiaxial fatigue models using strain-controlled experimental fatigue data for shear failure mode materials found in literature.

Several studies [2, 3, 4, 5, 6, 7] have shown that 90° out-of-phase axial-torsional straining results in fatigue life less than or equal to in-phase straining tests for the same von Mises strain amplitude, based on the equivalent von Mises strain amplitude versus fatigue life curves generated from strain-controlled axial-torsional fatigue testing on unnotched tubular specimens. This phenomenon of decreasing fatigue life under 90° out-of-phase straining (also known as the nonproportional hardening effect) can be explained by the additional stresses induced to overcome the interactions between slip systems from the continuous rotation or change of the maximum shear strain axis [1, 3, 4, 8, 9, 10, 11, 12]. Therefore, the magnitude of the stress increase depends on the frequency of the slip interactions encountered (i.e. loading path dependent) and the ease to overcome the resistance of the slip systems (i.e. material dependent). A typical material such as Type 304 stainless steel [13] has shown a more damaging effect than an aluminum alloy [14] while subjected to

identical out-of-phase loading paths because it was found that a stronger interaction occurs in Type 304 stainless steel compared to the aluminum alloy.

A recent study by Sonsino [1] indicated that based on the normal stress amplitude versus fatigue life (SN) curves from load controlled axial-torsional fatigue testing on unnotched specimens made of a wrought steel (SAE 1040), SN curves for in-phase and 90° out-of-phase loading are not parallel, but intersect at a fatigue strength at 10⁶ cycles, named "the branching point" of the uniaxial cyclic and monotonic stress-strain curves. It was noted that for any equivalent stress level below "the branching point", the load controlled testing is the same as the strain-controlled testing due to elastic material behavior and therefore same fatigue behavior (i.e. decrease of fatigue life under out-of-phase loading) was observed under both 90° out-of-phase loading and straining in comparison to the in-phase loading case. However, the opposite fatigue behavior (i.e. increase of fatigue life under 90° out-of-phase loading) was found for any equivalent stress level exceeding "the branching point", attributable to the nonproportional hardening effect.

It is also interesting to evaluate the stress-based multiaxial fatigue models [15, 16, 17, 18] based on the strain-controlled experimental fatigue data, because these theories have been criticized for being insensitive to the material nonproportional hardening effect. For computational efficiency, the critical plane approach [16, 17, 18] is preferable for use in estimating life of Case A cracks which propagate

along the surface, while the equivalent stress approach [15] is simple to compute the life of Case B cracks which propagate from the surface into the depth. Note the Case B type crack involves a more computationally intense critical plane search.

The focus of this paper is on an existing stress-based model developed by Lee, Tana, and Jordan [15], abbreviated as the LTJ model in this paper, with an emphasis on characterizing a non-proportionality parameter, α_{LTJ} to account for material dependence. This model is defined as:

$$\sigma_{LTJ,a} = \sigma_{VM} (1 + \alpha_{LTJ} f_{NP}) \quad (1)$$

where $\sigma_{LTJ,a}$ is the LTJ stress amplitude, σ_{VM} is the equivalent von Mises stress and α_{LTJ} and f_{NP} are the non-proportionality parameter and the non-proportionality factor, respectively. The non-proportionality parameter, α_{LTJ} is a material dependent parameter that accounts for material dependent resistance of interactions between slip systems under nonproportional loading. The 90° out-of-phase loading path is understood to be the worst case loading path resulting the greatest nonproportional loading effects [19]. For all other loading paths, the effect of the nonproportional loading is reduced. The non-proportionality factor, f_{NP} , is used to account for the loading path taken and ranges from a value of 0 for in-phase loading to a value of 1 for 90° out-of-phase loading. This factor acts to reduce the value of α_{LTJ} i.e. nonproportional loading effect, which is in agreement with material behavior. Several models (such as the integral approach by Itoh et al. [13], the rectangular moment of inertia method by Bishop [20] and the moment of inertia method by Meggiolaro and de Castro's moment of inertia [21, 22, 23]) have been developed to account for the load path dependent non-proportionality factor, f_{NP} , and are not discussed in this paper.

It should be noted that the LTJ equivalent stress-based multiaxial fatigue model was based on the strain-based model by Itoh et al [13]. Unlike the model by Itoh, a tensile failure model, the LTJ model is intended for shear dominant failure materials. This is important as cracks tend to grow specific planes, usually on or near the maximum tensile plane or the maximum shear plane [8, 11]. Materials with cracks growing on the maximum tensile plane are called tensile failure mode materials. Likewise, shear failure mode materials will have cracks that grow on the maximum shear stress planes. When choosing a multiaxial fatigue model, the damage mechanism needs to be considered [8].

In this paper, some selected stress-based multiaxial fatigue models are reviewed. Then a method for determining the non-proportionality parameter, α_{LTJ} based on experimental data is described. Life estimations are then performed using the LTJ approach with experimentally determined values for α_{LTJ} . Next, life estimations made using the LTJ approach are compared to life estimations made using two other approaches, Findley [16] and Gaier-Dannbauer [18] critical plane approaches, and the pros and cons of each method are discussed. Finally, empirical equations for α_{LTJ} are derived based on commonly available material properties.

REVIEW OF SELECTED MULTIAXIAL FATIGUE CRITERIA

There is a need to provide a brief review of the stress-based multiaxial fatigue models in this section even though the detailed description of these models can be found elsewhere [11,24]. Stress-based multiaxial fatigue models are typically a good choice for analysis of powertrain components, which are generally designed for life in the high cycle fatigue (HCF) or infinite life regime. Two common types of stress-based multiaxial fatigue models are critical plane approaches and equivalent stress approaches. Findley's critical plane approach, utilized in FE Safe™ and n-Code™, and the Gaier-Dannbauer scaled normal stress approach, utilized in FEMFAT™, are two popular critical plane models. Since these methods are routinely used in commercial software packages, they will be used as a basis for comparison with the LTJ equivalent stress approach.

Findley's Critical Plane Approach

Findley's model [16] is a popular critical plane approach for shear failure mode materials and is defined as follows:

$$\tau_{a,Findley} = (\tau_a + k_F \sigma_n)_{max} \quad (2)$$

where τ_a is the shear stress on a plane, σ_n is the normal stress on a plane, and k_F is a material constant which reflects the sensitivity of shear stress damage to normal stress. With Findley's model a critical plane search is performed and the plane with the combination of normal and shear stress that produces the maximum value of $\tau_{a,Findley}$ is the plane used for the fatigue analysis. For design of component expected to survive infinite life, Findley's damage parameter should be less than the shear fatigue strength at 10^6 cycles.

The material constant k_F can be determined by considering two load cases, axial loading and pure shear loading. For the axial loading case, Findley's parameter would maximize at:

$$(\tau_a + k_F \sigma_n)_{max} = 1/2 (\sqrt{\sigma_a^2 + k_F^2 \sigma_{max}^2} + k_F \sigma_{max}) = \tau_{Findley,a} \quad (3)$$

where σ_a is the alternating stress and σ_{max} is the maximum stress. The maximum stress would be the same as the alternating stress for the axial loading case. For pure shear loading, the Findley parameter would maximize at:

$$(\tau_a + k_F \sigma_n)_{max} = \tau_a \sqrt{1 + k_F^2} = \tau_{Findley,a} \quad (4)$$

Equation (3) and Equation (4) are set equal to each other and rearranged in the following form:

$$\frac{\sigma_{E,R=-1}}{\tau_E} = \frac{2}{1 + \frac{k_F}{\sqrt{1 + k_F^2}}} \quad (5)$$

Note σ_a and σ_{max} in Equation (3) has been substituted with the fully reversed normal fatigue strength at 10^6 cycles ($\sigma_{E,R=-1}$) and τ_a in Equation (4) has been substituted with the shear fatigue strength at 10^6 cycles (τ_E) of the material. The fatigue strength values at 10^6 cycles are substituted here because k_F is determined at 10^6 cycles. The material constant k_F is then determined by solving Equation (5).

The Findley damage parameter can be extended to the finite life region and is related to fatigue life as follows:

$$\tau_{Findley,a} = \tau'_f \sqrt{1 + k_F^2} (2N_f)^{b_0} \quad (6)$$

where τ'_f is the shear fatigue strength coefficient, and b_0 is the shear fatigue exponent. It should be noted here that assuming a constant value for k_F calculated at 10^6 cycles can lead to errors in life estimations as fatigue life deviates further away from 10^6 cycles. Kallmeyer and Kurath have stated that the material parameter k_F varies depending on fatigue life region of interest [25, 26, 27]. However, all life estimations performed in this investigation were done with a constant k_F calculated at a fatigue life of 10^6 cycles.

Gaier-Dannbauer Scaled Normal Stress Approach

Gaier and Dannbauer [18] developed a critical plane approach, the scaled normal stress approach, which assumes a crack will initiate on a plane where the scaled normal stress is maximized and exceeds the failure criteria. The failure criterion is defined as:

$$(f_{GD}\sigma_n)_{max} = \sigma_{E,R=-1} \quad (7)$$

The scaled normal stress factor, f_{GD} , is defined as follows:

$$f_{GD} = 1 + (1 - k)V \quad (8)$$

where k is a material parameter relating normal stress to shear stress and is defined mathematically as:

$$k = \frac{\sigma_{E,R=-1}}{\tau_E} \quad (9)$$

and V is the ratio of minimum to maximum principal stress given by:

$$V = \frac{\sigma_3}{\sigma_1} \quad \text{for} \quad |\sigma_1| > |\sigma_3|$$

$$V = \frac{\sigma_1}{\sigma_3} \quad \text{for} \quad |\sigma_3| > |\sigma_1| \quad (10)$$

In general, ductile materials will have a $k = 2$ and brittle material will have $k = 1$. The value for V will range from -1 to 1, with $V = -1$ for shear loading (i.e. $\sigma_1 = -\sigma_3$), $V = 0$ for tension/bending loading (i.e. $\sigma_3 = 0$), and $V = 1$ for hydrostatic loading (i.e. $\sigma_1 = \sigma_2 = \sigma_3$). The damaging effect of torsion loading on ductile materials is considered by scaling up the stress tensor with a scaled normal stress factor $f_{GD} > 1$. In agreement with distortion energy criterion, the magnitude of the hydrostatic stress is scaled down. Furthermore, brittle materials with $k = 1$ will have a scaled normal stress factor, f_{GD} , equal to 1, which correlates with the normal stress hypothesis. The scaled normal stress, $(f_{GD}\sigma_n)_{max}$, has the same relationship with fatigue life as normal stress. Fatigue life estimations with the scaled normal stress can be made using the following equation:

$$(f_{GD}\sigma_n) = \sigma'_f (2N_f)^b \quad (11)$$

where σ'_f is the fatigue strength coefficient and b is the fatigue strength exponent.

LTJ Equivalent Stress Model

The LTJ approach uses modification factors that are applied to the equivalent von Mises stress to account for the nonproportional loading effects. The equivalent von Mises stress used in Equation (1), in a 2-dimensional plane stress condition, is calculated from the following equations:

$$\sigma_{VM} = \max \left\{ \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a}\sigma_{y,a} + 3\alpha_s^2 \tau_{xy,a}^2} \times \left(\frac{\sigma'_f}{\sigma'_f - \sigma_{eq,m}} \right) \right\} \quad (12)$$

$$\sigma_{eq,m} = \sigma_{x,m} + \sigma_{y,m} \quad (13)$$

where $\sigma_{x,a}$ and $\sigma_{y,a}$ are the normal stress amplitude acting in the x and y directions, respectively, $\tau_{xy,a}$ is the shear stress amplitude acting on the xy plane. The parameter α_s represents the sensitivity of the shear stress amplitude on the normal stress amplitude [15]. It should be noted that, in this study, $\alpha_s = 1$ has been used for all calculations which reduces the root of Equation (12) to the classical von Mises equation. The equivalent mean stress, $\sigma_{eq,m}$, in Equation (12) is calculated from Equation (13), where $\sigma_{x,m}$ and $\sigma_{y,m}$ are the mean stresses acting in the x and y directions respectively.

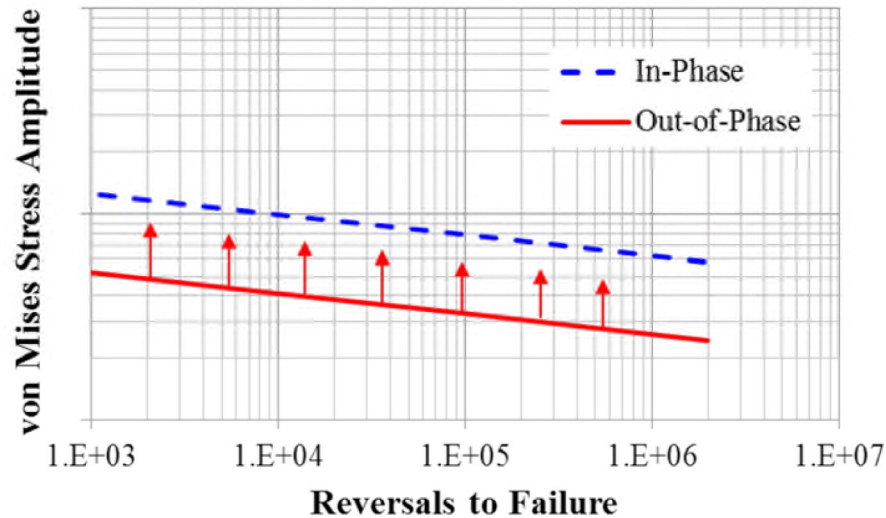


Figure 1. In-phase and out-of-phase SN curves based on von Mises equivalent stress

The non-proportionality parameter is defined as:

$$\alpha_{LTJ} = 1 - \frac{\sigma_{NP}}{\sigma_p} \quad (14)$$

where σ_{NP} is the equivalent von Mises stress under 90° out-of-phase sinusoidal loading and σ_p is the equivalent von Mises stress under proportional loading. The non-proportionality parameter is determined experimentally by comparing the SN curves for in-phase and 90° out-of-phase multiaxial loading. The values of σ_{NP} and σ_p to be used in Equation (14) are taken from the equivalent von Mises in-phase and 90° out-of-phase SN curves at life of 10^6 cycles. The non-proportionality parameter adjusts the out-of-phase data so that it lies on top of the in-phase proportional data (see Figure 1). It should be noted that the non-proportionality parameter is assumed to be constant (i.e. no dependence on fatigue life).

The non-proportionality factor, f_{NP} , takes into consideration the non-proportional load path taken. The stress-based non-proportionality factor is defined as follows:

$$f_{NP} = \frac{C}{T\sigma_{l,max}} \int_0^T (|\sin \xi(t)| \sigma_l(t)) dt \quad (15)$$

where C is a constant that makes f_{NP} equal to 1 under 90° out-of-phase sinusoidal loading, T is time for one cycle, $\sigma_{l,max}$ is the maximum principal stress for a cycle, and $\sigma_l(t)$ is the principal stress at time t . This paper only considers 90° out-of-phase loading where f_{NP} take a value of 1.

Although the LTJ method is intended for HCF analysis, it can be extended to the finite life region for life estimations as follows:

$$\sigma_{VM,NP,a} = \sigma'_f (2N_f)^b \quad (16)$$

Some strain-based models use a nonproportional hardening factor, α_{NP} , to account for the nonproportional loading effects [12, 13, 21, 22, 23]. This factor is based on von Mises equivalent stress-strain amplitude curves and is defined as:

$$\alpha_{NP} = \frac{\sigma_{NP}}{\sigma_p} - 1 \quad (17)$$

where σ_{NP} is the equivalent von Mises stress under 90° out-of-phase sinusoidal loading and σ_p is the equivalent von Mises stress under in-phase loading taken from the von Mises equivalent stress-strain amplitude curves for a given von Mises equivalent strain. Figure 2 shows a schematic illustration of the von Mises equivalent stress-strain curves for in-phase and 90° out-of-phase loading.

Previous authors have developed equations to estimate the nonproportional hardening factor [6, 28, 29]. Burodii and Strizhalo [28] developed the following relationship:

$$\log|\alpha_{NP}| = 0.705 \left(\frac{\sigma_u}{\sigma_y} - 1 \right) - 1.22 \quad (18)$$

where σ_u is the ultimate tensile strength and σ_y is the yield strength. Burodii derived Equation (19) based on experimentally calculated α_{NP} values derived from von Mises equivalent stress-strain curves for in-phase and 90° out-of-phase multiaxial loading. It should be noted that Equation (18) estimates a constant value for α_{NP} .

Shamsaei and Fatemi [6] developed an equation that uses monotonic and in-phase (or uniaxial) cyclic deformation behavior to estimate α_{NP} which is defined as follows:

$$\alpha_{NP} = 1.6 \left(\frac{K}{K'} \right)^2 \left(\frac{\Delta \varepsilon}{2} \right)^{2(n-n')} - 3.8 \left(\frac{K}{K'} \right)^2 \left(\frac{\Delta \varepsilon}{2} \right)^{(n-n')} + 2.2 \quad (19)$$

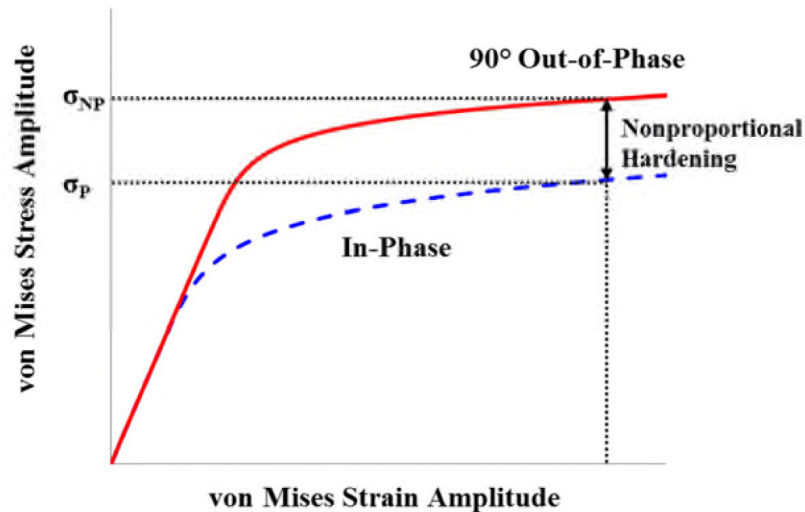


Figure 2. Schematic of von Mises equivalent stress-strain amplitude curves

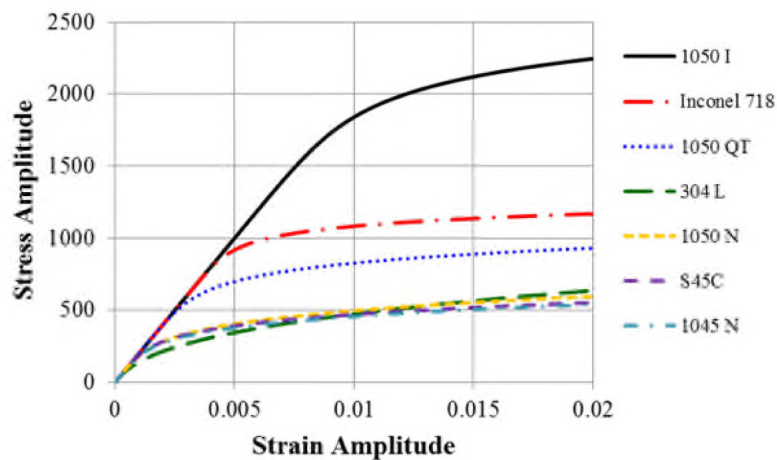


Figure 3. In-phase cyclic stress-strain curves for materials used in this study

where K is the monotonic strength coefficient, K' is the cyclic strength coefficient, n is the monotonic strain hardening exponent, n' is the cyclic strain hardening exponent, and $\Delta\epsilon/2$ is the equivalent von Mises strain amplitude. Equation (19) is also based on experimentally calculated α_{NP} values derived from von Mises equivalent stress-strain curves for in-phase and 90° out-of-phase multiaxial loading. Contrary to Equation (18), Equation (19) will estimate different values of α_{NP} depending on the equivalent strain level which agrees with material behavior.

The nonproportional hardening factor and its' approximation techniques have been introduced here so that its' differences from α_{LTJ} can be better explained. It should be mentioned here that the nonproportional hardening factor, α_{NP} , is not applicable for use in the LTJ model. The reasoning for this is discussed in the following section.

EXPERIMENTAL DATA

The experimental data used in this study was taken from available literature. A total of seven shear failure mode materials were used; 304L stainless steel [29], Inconel 718 super alloy [30], 1050 normalized steel [29], 1050 quenched and tempered steel [29], 1050 induction hardened steel [29], 1045 normalized steel [9], and S45C carbon steel [31]. All multiaxial fatigue data from referenced literature presented in this paper are based on strain-controlled axial-torsion testing of thin-walled tubular specimens. The in-phase data presented in this paper are based on combined axial-torsion loading and not pure axial or pure torsion loading. Table 1 lists the chemical properties of the materials investigated. Table 2 lists the monotonic and cyclic material properties for these materials. Approximations for additional properties not found in Table 2 can be found in the Metals Handbook [32]. It can be seen in Table 2 and Figure 3 that the materials used in this study have a wide range of material properties. It should be noted that most of the materials

investigated exhibited nonproportional cyclic hardening. However, the 1050 quenched and tempered steel and 1050 induction hardened steel showed no nonproportional cyclic hardening [29].

Figure 4 shows SN data for in-phase and 90° out-of-phase loading for the seven materials used in this study. Note an equivalent von Mises stress, as defined in Equation (12), is used in these plots. The stress amplitudes, calculated from experimentally measured loads, used to determine the von Mises equivalent stresses were taken from the literature sources [9, 29, 30, 31]. It can be seen in this figure that the out-of-phase data is above, below, or even on top of the in-phase curve depending on the material.

Figure 5 has the same data from Figure 4, but the out-of-phase data has been adjusted with the non-proportionality parameter determined from Equation (14) using the von Mises equivalent SN curves to determine the values of σ_{NP} and σ_p at 10^6 cycles. The experimentally determined non-proportionality parameter was then used in Equation

(1) to determine the LTJ stress amplitude. The values of the non-proportionality parameter used can be found in Table 2. It can be seen that the in-phase and out-of-phase data are now on top of each other.

The available strain-based nonproportional hardening parameters [6, 28, 29] produce only positive values. This is because the out-of-phase stress-strain curve is always above the in-phase stress-strain curve when there is non-proportional hardening. Applying a positive value for α_{LTJ} would make the out-of-phase data even further away from the in-phase data for materials where the out-of-phase SN data are above the in-phase data (see Figure 4(a)). Note the experimental values for α_{LTp} determined from Equation (14), are listed in Table 2. This suggests that the strain-based non-proportional hardening parameters are not applicable for use in the LTJ equation. It should be noted that the strain-life curves for out-of-phase loading for all materials investigated in this study are below the in-phase strain-life curves. Therefore, when doing a strain-based analysis the non-proportional hardening factor, α_{NP} , should be used as it models the material strain-life behavior.

Table 1. Chemical composition of materials used (wt., %)

Material	Inconel 718 [27]	1050 Steel [26]	304L Stainless Steel [26]	1045 Steel [9]	S45C Steel [28]
Carbon	0.034	0.48	0.029	0.44	0.427
Manganese	0.09	0.91	1.86	0.7	0.734
Silicon	0.06	0.22	0.37	0.23	0.023
Phosphorous	0.007	0.011	0.029	0.019	0.018
Sulfur	0.001	0.034	0.004	0.046	0.004
Chromium	18.23	0.04	18	0.05	0.037
Nickel	52.12	0.02	10	0.03	0.042
Molybdenum	2.89	0.02	0.04	-----	0.005
Columbium + Tantalum	5.03	-----	-----	0.008	-----
Titanium	1.01	-----	-----	0.002	0.002
Aluminum	0.54	0.04	-----	0.05	0.025
Cobalt	0.28	-----	-----	-----	-----
Boron	0.0035	-----	-----	-----	-----
Copper	0.05	0.13	0.02	0.03	0.131
Iron	19.48	bal	bal	bal	bal
Vanadium	-----	0.003	-----	0.002	-----
Tin	-----	0.01	-----	-----	-----
Calcium	-----	0.0001	-----	-----	-----
Nitrogen	-----	-----	0.056	-----	-----