

Availability of Balanced Truncation for Reducing an Automotive Cold Start Engine Model

Satoshi Asami and Toshihiko Shiraishi
Yokohama National University

ABSTRACT

High hydrocarbon emissions during the cold start period is a wellrecognized challenge that has increasing importance in moving towards green vehicles. A model-based controller is a helpful way to reduce cold start hydrocarbon emissions. In our previous work, a model-based controller for a Spark-Ignition engine and an after-treatment system have been developed by using the balanced truncation technique. In this work, the objective was to determine whether balanced truncation surpasses the other model reduction methods. Guyan reduction was selected to compare with balanced truncation for reducing both a linear structural model and the linearized cold start model. An interpretation of the inner domain in Guyan reduction was determined for the linearized cold start model. The comparison indicates that the reduced model derived via balanced truncation replicates the dominant modes of the structural model and the dynamic behavior of the cold start model in the frequency range above 1 Hz better than the reduced model derived through Guyan reduction.

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INTRODUCTION

As far as the practical use of passenger vehicles is concerned, the traveling distance of a single continuous drive is relatively short. For example, the most frequent traveling distance of private vehicles is approximately 20 km per day in the Tokyo metropolitan area [1]. The daily vehicle miles per trip is approximately 10 miles in the US [2]. Cold start operations make up a significant proportion of such travel. More than 80% of total hydrocarbon (HC) emissions in the normal driving cycle of passenger automobile are emitted during the cold start period [3]. Various methodologies, including improving the adjusting air-fuel mixture and shortening the period until catalyst light-off, have been suggested to solve this cold start problem. These methodologies are commonly categorized into semi-active control systems and active control systems. An active control system requires an external energy supply. A semi-active control system adjusts the parameters of the engine to improve the combustion efficiencies and control the temperature and the flow rate of exhaust gas. Proper modeling of the engine dynamics during the cold start period and a proper design for an optimal controller, including robustness, are essential technologies to embed a semi-active control system.

To minimize the HC emissions during the cold start period, various control-oriented models have been developed [4] [5]. In addition, many model-based controllers have been designed [6] [7] [8]. The performance of a model-based controller depends critically on plant models. Thus, plant modeling is a major undertaking in the development of a model-based controller. There are two prominent requirements for the plant model in developing a model-based controller. One requirement is simplicity, and the other is accuracy. A

high-order model-based controller requires a significant computational effort for real-time implementation. Therefore, the model must be sufficiently simple to satisfy the controller hardware constraints. In addition, a properly simplified model can make both controller design and controller calibration easier. However, the model must be sufficiently accurate to capture the essential and specific characteristics in behaviors of dynamic system. Those two objectives lead to a trade-off problem. A good plant model contains a proper balance between accuracy and simplicity.

Model reduction that properly reduces the number of state variables plays an important role in the modeling process to achieve a reasonable approximation with good balance. Many model reduction techniques for the linear model are matured and well-known [9] [10]. Because of the nonlinearities of controlled plants, the development of nonlinear model reduction techniques has become an active research area in academia [11]. Such techniques require a complicated process compared to linear model reduction. Therefore, especially for the reduction of a plant model with weak nonlinearities, the combination method of linearization and linear model reduction is more common in industry.

Typically, the objective of model reduction is two-pronged. As previously mentioned, one objective is to reduce the plant model and understand its essential dynamics in the controller design. Balanced truncation [12], modal analysis [13], and singular perturbation [14] are popular methods to reduce the linear plant model when developing a controller. The other objective is from the structural perspective, such as response analysis, sensitivity analysis, and

optimization. Modal analysis and Guyan reduction are order reduction techniques from the structural view point [15]. Few structural model reduction techniques have been applied to the plant model for controller development [16].

In our previous study, the balanced truncation technique was applied to a nonlinear cold start engine model that predicts HC emission from the tailpipe during the cold start period [17]. In addition, a sliding mode controller derived from the reduced cold start engine model was found to be better at tracking more influential trajectories on the final control target than that derived from the full state engine model. However, the superiority of using the balanced truncation technique instead of the other reduction methodologies remains unclear.

Balanced truncation is a numerical approach for designing a controller. In contrast, Guyan reduction is an asymptotic approach for structural analysis and is the polar opposite of balanced truncation. In this paper, Guyan reduction is chosen as a target for comparison to review the superiority of balanced truncation.

This paper is organized as follows. First, the general ideas of balanced truncation and Guyan reduction are presented. A linear mass-damperspring model example is reduced by using the two reduction techniques, and the reduced models are compared. Second, a nonlinear cold start engine model is normalized, linearized, and reduced by balanced truncation and by Guyan reduction. The idea of Guyan reduction is extended to reduce the plant model. Next, open loop simulations of the reduced models are compared to reach a conclusion.

BALANCED TRUNCATION

Consider a continuous-time time-invariant linear model in the state space form.

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \quad (1)$$

$$x = [x_1 \quad x_2 \quad \cdots \quad x_n]^T \quad (2)$$

where x is the state vector, u is the input signal vector, and y is the output signal vector. In addition, A , B , C , and D are the realization where $A \in \mathbb{R}^{n \times n}$ is the n^{th} -order system matrix. The original state space is transformed to a new non-physical state space of balanced realization, in which the controllability gramian and observability gramian are diagonal and equal to the Hankel singular value (HSV) matrix. HSVs express the magnitude of dominance between the input and the output of the model.

The HSVs are defined as the square roots of the eigenvalues of PQ , where P and Q are the controllability and observability gramians of the linear model, respectively.

$$\sigma_i \triangleq \sqrt{\lambda_i(PQ)} \quad i = 1, 2, \dots, n \quad (3)$$

where $\lambda_i(PQ)$ is one of the eigenvalues of PQ . The HSV matrix Σ is defined as the diagonal matrix that consists of HSVs placed in a descending order.

$$\Sigma \triangleq \begin{bmatrix} \sigma_1 & \cdots & 0 \\ & \sigma_2 & \\ \vdots & & \ddots \\ 0 & \cdots & & \sigma_n \end{bmatrix} \quad \text{where } \sigma_1 > \sigma_2 > \cdots > \sigma_n \quad (4)$$

A similarity transformation for the model is found such that the new controllability and observability gramians are diagonal and equal to Σ in the balanced realization. Because the model is observable, the observability gramian Q is non-singular, and there exists a square root matrix, R , to the observability gramian Q . A Hermitian eigenvalue decomposition on RPR^* is performed to find the unitary matrix U . The similarity transformation matrix that achieves balanced realization can be calculated.

$$Q = RR^* \quad (5)$$

$$RPR^* = U\Sigma^2U^* \quad (6)$$

$$T_B = R^{-1}U\Sigma^{1/2} \quad (7)$$

The balanced realization is calculated using the similarity transformation $x = T_B z$. The controllability gramian and the observability gramian of balanced realization are equal to the HSV matrix.

$$\begin{bmatrix} \dot{z} \\ y \end{bmatrix} = \begin{bmatrix} T_B^{-1}AT_B & T_B^{-1}B \\ CT_B & D \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} \quad (8)$$

$$z = [z_1 \quad z_2 \quad \cdots \quad z_n]^T \quad (9)$$

$$\bar{P} = \bar{Q} = \Sigma \quad (10)$$

The order of the model may be reduced with little loss in accuracy if a significant difference exists between the large-magnitude HSV set and the small-magnitude HSV set in the Σ matrix. The states that correspond to the large magnitude HSV set remain, and those states that correspond to the small magnitude HSV set are truncated in the balanced truncation. Thus, the new transformation matrix $\tilde{T}_B$ is formed by removing the last columns of T_B that correspond to the small-magnitude HSV set. A new transformation matrix to obtain the original states $\tilde{T}_B^{-1}$ is formed by removing the last rows of T_B^{-1} in the same fashion. Note the abuse in notation, $\tilde{T}_B^{-1} \neq (\tilde{T}_B)^{-1}$ and that $\tilde{T}_B^{-1}$ is not square.

The truncated balanced realization is shown below.

$$\begin{bmatrix} \dot{z} \\ y \end{bmatrix} = \begin{bmatrix} \tilde{T}_B^{-1}A\tilde{T}_B & \tilde{T}_B^{-1}B \\ C\tilde{T}_B & D \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} \quad (11)$$

$$\tilde{z} = [z_1 \quad z_2 \quad \cdots \quad z_l]^T \quad \text{where } l < n \quad (12)$$

z is the reduced state vector. Further $\tilde{A} \in \mathbb{R}^{l \times l}$ is the reduced system matrix.

In this paper, the calculations described above are performed using MATLAB/SIMULINK.

GUYAN REDUCTION

Guyan reduction is often used to reduce the degree of freedom (DOF) of an unmodified substructure in the component mode synthesis method [18]. The basic idea is an approximation to ignore dynamic behavior and retain the quasi-static behavior of the inner domain on which no external force acts in the substructure. Ignoring dynamic behavior leads to regarding the inertial force and the damping force as zero [19], whereby each characteristic matrix of the mass-spring model is reduced.

The structural model is separated into four types of domains: external force acting domain, response domain (where the response is observed by sensors), coupled domain (which interacts with other adjacent substructures), and inner domain (which includes the parts other than those listed above). The definitions of these domains are presented in Figure 1.

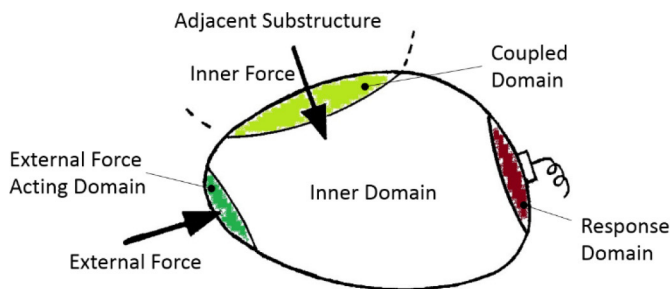


Figure 1. Definition of the inner domain in the Guyan reduction of the substructure.

A structural model is commonly presented as a mass-damper-spring model.

$$M\ddot{X} + C_d\dot{X} + KX = F \quad (13)$$

where M , C_d , and K are the mass matrix, damping matrix, and stiffness matrix, respectively. F is the external force vector. X is the displacement vector. The differential equation is partitioned into two domains. The inner domain is mathematically determined as the part where no external force acts on. Domain A corresponds to the inner domain. In addition, domain B is the other part of the inner domain. x_a and x_b are the displacement vectors of domains A and B, respectively.

$$X = \begin{bmatrix} x_a \\ x_b \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} m_a & 0 \\ 0 & m_b \end{bmatrix} \begin{bmatrix} \ddot{x}_a \\ \ddot{x}_b \end{bmatrix} + \begin{bmatrix} c_a & c_{ab} \\ c_{ab}^T & c_b \end{bmatrix} \begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} + \begin{bmatrix} k_a & k_{ab} \\ k_{ab}^T & k_b \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} = \begin{bmatrix} 0 \\ f_b \end{bmatrix} \quad (15)$$

The differential equation of domain A is extracted.

$$m_a\ddot{x}_a + c_a\dot{x}_a + c_{ab}\dot{x}_b + k_a x_a + k_{ab}x_b = 0 \quad (16)$$

In the domain A, the inertial force and damping force are asymptotically assumed to be zero for the approximation in the reduced model. In other words, the quasi-static effect of reduced part is considered only on the remaining part. Next, differential terms are deleted in the reduction, and an algebraic equation is derived.

$$m_a\ddot{x}_a + c_a\dot{x}_a + c_{ab}\dot{x}_b \approx 0 \quad (17)$$

$$x_a \approx -k_a^{-1}k_{ab}x_b \triangleq T_{ba}x_b \quad (18)$$

The full state space vector is transformed to the reduced space vector by the transformation matrix $\tilde{T}_G$, which is not square.

$$X = \begin{bmatrix} x_a \\ x_b \end{bmatrix} \approx \begin{bmatrix} T_{ba} \\ I \end{bmatrix} x_b = \tilde{T}_G \cdot x_b \quad (19)$$

Therefore, the reduced model is

$$\tilde{M}\ddot{x}_b + \tilde{C}_d\dot{x}_b + \tilde{K}x_b = f_b \quad (20)$$

where the reduced characteristic matrices are

$$\tilde{M} = \begin{bmatrix} T_{ba}^T & I \end{bmatrix} \begin{bmatrix} m_a & 0 \\ 0 & m_b \end{bmatrix} \begin{bmatrix} T_{ba} \\ I \end{bmatrix} = T_{ba}^T m_a T_{ba} + m_b \quad (21)$$

$$\begin{aligned} \tilde{C}_d = \begin{bmatrix} T_{ba}^T & I \end{bmatrix} \begin{bmatrix} c_a & c_{ab} \\ c_{ab}^T & c_b \end{bmatrix} \begin{bmatrix} T_{ba} \\ I \end{bmatrix} &= T_{ba}^T c_a T_{ba} + T_{ba}^T c_{ab} \\ &+ c_{ab}^T T_{ba} + c_b \end{aligned} \quad (22)$$

$$\begin{aligned} \tilde{K} = \begin{bmatrix} T_{ba}^T & I \end{bmatrix} \begin{bmatrix} k_a & k_{ab} \\ k_{ab}^T & k_b \end{bmatrix} \begin{bmatrix} T_{ba} \\ I \end{bmatrix} &= T_{ba}^T k_a T_{ba} + T_{ba}^T k_{ab} \\ &+ k_{ab}^T T_{ba} + k_b \end{aligned} \quad (23)$$

$$f_b = \begin{bmatrix} T_{ba}^T & I \end{bmatrix} F = \begin{bmatrix} T_{ba}^T & I \end{bmatrix} \begin{bmatrix} 0 \\ f_b \end{bmatrix} \quad (24)$$

STRUCTURAL MODEL REDUCTION EXAMPLE

Because Guyan reduction was originally developed for the reduction of structural models, the comparison of Guyan reduction with balanced truncation should be based on both structural models and control-oriented plant models. In this section, the DOF in a structural model example is reduced by balanced truncation and Guyan reduction to compare the frequency responses.

The example of a 5DOF mass-spring-damper model is shown in Figure 2. That model is identified as a flat plate of a vibrator table [20]. All masses except M_5 are supported by actuators, which include internal damping factors. Each DOF has two state variables: its displacement and its velocity. In other words, the model turns out to be a 10th-order state-space model.

The input for the model is an external force f applied on M_1 . The output from the model is the displacement of M_2 . Each displacement of x_i , $i = 1, 2, \dots, 5$ corresponds to M_i , $i = 1, 2, \dots, 5$, respectively.

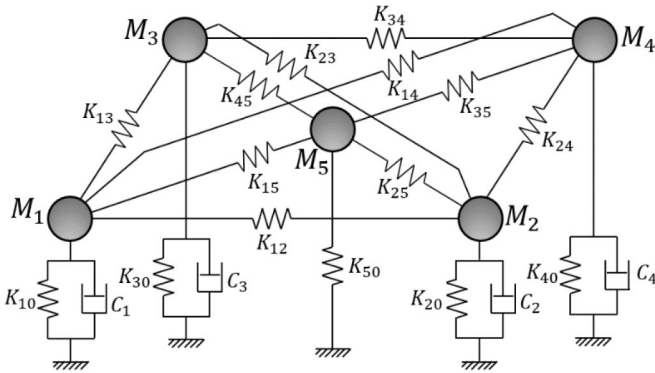


Figure 2. 5DOF structural model.

The characteristic matrices are as follows.

$$M_5 \ddot{X} + C_5 \dot{X} + K_5 X = F \cdot f \quad (25)$$

$$X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T \quad (26)$$

$$F = [1 \ 0 \ 0 \ 0 \ 0]^T \quad (27)$$

$$M_5 = \begin{bmatrix} 0.7864 & 0 & 0 & 0 & 0 \\ 0 & 0.7463 & 0 & 0 & 0 \\ 0 & 0 & 0.7479 & 0 & 0 \\ 0 & 0 & 0 & 0.7910 & 0 \\ 0 & 0 & 0 & 0 & 1.1651 \end{bmatrix} \quad (28)$$

$$C_5 = \begin{bmatrix} 3.5 & 0 & 0 & 0 & 0 \\ 0 & 3.5 & 0 & 0 & 0 \\ 0 & 0 & 3.5 & 0 & 0 \\ 0 & 0 & 0 & 3.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (29)$$

$$K_5 = \begin{bmatrix} 7219 & -312 & -545 & 3466 & -7141 \\ -312 & 6934 & 3284 & -503 & -6872 \\ -545 & 3284 & 6835 & -463 & -6549 \\ 3466 & -503 & -463 & 7314 & -7118 \\ -7141 & -6872 & -6549 & -7118 & 28729 \end{bmatrix} \quad (30)$$

Balanced Truncation

The spring-mass model is presented in a state space form.

$$\begin{bmatrix} \dot{x}_s \\ y_s \end{bmatrix} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} \quad (31)$$

$$A_s = \begin{bmatrix} -M_5^{-1}C_5 & -M_5^{-1}K_5 \\ I_{5 \times 5} & O_{5 \times 5} \end{bmatrix} \quad (32)$$

$$B_s = \begin{bmatrix} -M_5^{-1}F \\ O_{5 \times 1} \end{bmatrix} \quad (33)$$

$$C_s = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] \quad (34)$$

$$D_s = [0] \quad (35)$$

$$x_s = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T \quad (36)$$

$$u_s = f \quad (37)$$

The HSVs of this model are described in Figure 3.

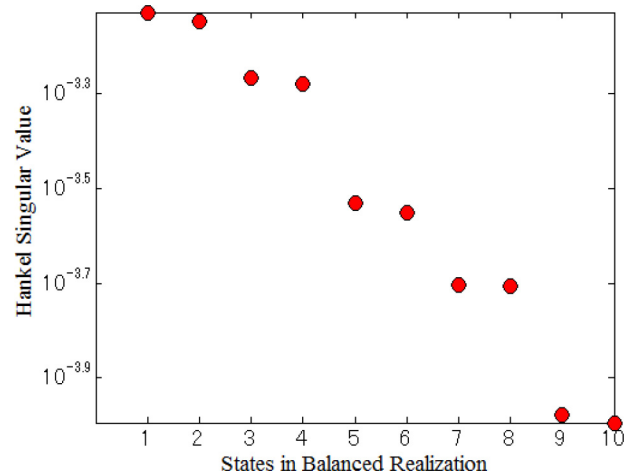


Figure 3. HSVs of the 5DOF structural model.

There is a significant difference of 1.6 times in the HSVs between the 4th and 5th states. Thus, the last six states are considered to be truncated. The truncated transformation matrix $\tilde{T}_{sB}$ is formed by removing the last six columns of the original transformation matrix T_{sB} . The result is a 4th-order reduced model.