

Analysis of Low-Cost MEMS Accelerometer and Gyroscope Characteristics for Stochastic Sensor Simulation within Motorcycle Models

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ABSTRACT

Vehicle dynamics control (VDC) for motorcycles had a fast growth during the last 10 years. The available technologies comprise curve-safe ABS and traction control (TC) systems, anti-wheelie control, right up to comprehensive motorcycle stability systems including even more control functions. VDC systems rely on real-time information about the current motorcycle dynamic state. Thus motorcycles are equipped with additional sensor units, namely MEMS inertial measurement devices, capable of gathering accelerations and angular rates. The application of model-based estimation theory enables the determination of the necessary information about the in-plane and out-of-plane motion, e.g. the motorcycle lean angle. Since VDC systems include safety critical control functions, the validation within simulations including sensor characteristics is mandatory. The MEMS accelerometer and gyroscope features include low-cost and small footprint, however there are considerable stochastic sensor errors to cope with. In this study the characteristic of different MEMS sensors and their noise models are investigated. The sensor noise terms are identified by analyzing measurement data using the Allan variance method. Different sensors are compared and the stochastic noise coefficients are quantified. The sensor noises are modeled with according random processes defined by linear time-invariant systems and white-noise inputs. As a result, the obtained stochastic sensor models can be used for model-based estimation and control algorithm design, as well as verification within simulation environments.

CITATION: Winkler, A. and Grabmair, G., "Analysis of Low-Cost MEMS Accelerometer and Gyroscope Characteristics for Stochastic Sensor Simulation within Motorcycle Models," *SAE Int. J. Veh. Dyn., Stab., and NVH* 1(1):2017, doi:10.4271/2016-32-0027.

INTRODUCTION

Vehicle dynamics control (VDC) for motorcycles contributes towards a safety enhancement of powered two-wheelers. In contrast to the automotive sector, the development of motorcycle VDC started with some delay, but had a fast growth more recently. Starting with 2016, antilock braking systems are mandatory for all motorcycle sold in Europe. One decade ago, only a few manufacturers offered motorcycles equipped with traction control systems. These days all well-known brands in the field provide traction control systems, wheelie control and even more comprehensive stability control systems. Recently a wide-ranging VDC system was presented, contributing to further increase the safety for motorcycle riders by extending the acceleration and braking control functions to cover more riding situations [1]. In [2] a survey of the state of the art of motorcycle VDC systems is presented. Control functions which contribute most to the safety enhancement, are those which work in a wide range of riding situations, e.g. curve-safe ABS and traction control systems. The control algorithms require detailed information about the current motorcycle state, including wheels, powertrain and especially the attitude. The dynamic state is acquired by a combination of sensor measurements and model-based estimation algorithms. Many quantities are not measureable directly, thus available measurements are used to calculate the states of interest, based on mathematical descriptions of the dynamic behavior of the motorcycle. An important set of sensors for VDC systems are accelerometers and

gyroscopes based on MEMS (micro-electromechanical systems) technology. In Automotive control systems MEMS sensors are well established, since they provide the basis for e.g. the ESC (electronic stability control). Contrary to probable expectations the state estimation algorithms of four wheeled vehicles cannot be adopted in a straight forward manner for the use in motorcycle applications. The motorcycle dynamics are considerably more complex and different compared to cars. The tilting behavior of two wheeled vehicles (roll dynamics) represents a challenge for state estimation, especially for attitude estimation. Due to the integration of the sensor measurements within the necessary estimation algorithms, the sensor error characteristics are critical especially for precise attitude calculation. Taking the sensor noise characteristics into account allows an accuracy increase of the calculated motorcycle state.

The above mentioned MEMS sensors distinguish itself with very small footprint and low-cost. On the downside the measurements are corrupted by deterministic and stochastic error sources, which have a considerable disadvantageous influence. Consequently, a main task is the determination and analysis of the sensor behavior. Since the behavior of each sensor is different, the identification is mandatory for every type which is used. The sensor characteristic is reasonable for two different tasks. Considering the sensor noise model during the development of model-based estimation algorithms can improve the performance significantly. On the other hand, if a sensor noise model

is available, it can be included in comprehensive simulation environments, providing the possibility to test control algorithms with more realistic conditions. The basic structure of algorithms based on MEMS accelerometers and gyroscopes are depicted in Figure 1.

In this article the focus is on the design of the stochastic sensor model and its identification for different inertial sensors, as preparation for designing attitude estimation and acceleration based control algorithms. While deterministic error sources are characterized by repeatability and thus can be measured and compensated, the stochastic error sources include influences which are neither repeatable nor predictable, because of their random character. In contrast to deterministic errors, a static compensation using parameters obtained by calibration is not achievable. However, they can be modeled using random processes. Assuming a specific random noise characteristic can be achieved with a linear-time-invariant (LTI) system excited by a white-noise input, it is possible to obtain a noise model. First of all the stochastic errors must be identified and quantified. The concept of Allan variance is used for the analysis of random-drift errors of accelerometers and gyroscopes. It is a time-domain analysis method for the identification of various random-drift errors in measurement data. As a result the stochastic error coefficients and the according noise models of three different sensors are determined.

The paper is organized as follows. First of all the theory of the sensor analysis methodology is presented. Starting with a description of error sources which are supposed to corrupt the sensor signals, next some stochastic terms are introduced. As a main part of the analysis methodology the Allan variance is presented. After an introduction to the calculation of the Allan variance, the appearance and identification of specific noise terms in the Allan variance plot is described. In the next section a short overview of motorcycle models is given and the detailed modeling of the sensor stochastic noise is presented. Finally the discussed modeling and identification methodology is applied to three different MEMS sensors. In an experimental analysis of measurement data the stochastic behavior of the sensors is identified and the noise coefficients are determined. Last but not least, simulation data obtained with the developed sensor model is compared to measurement data and a short conclusion is given.

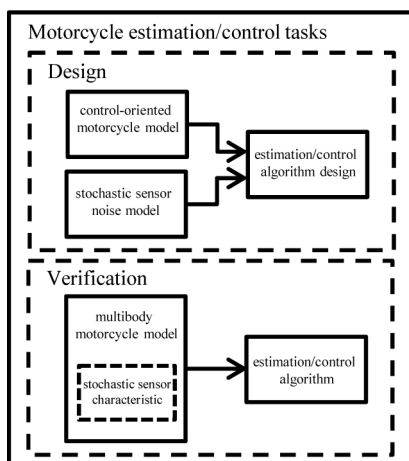


Figure 1. Structure of model-based estimation and control algorithms based on MEMS sensors

SENSOR ANALYSIS METHODOLOGY

Inertial measurement units (IMU) typically consist of accelerometers and gyroscopes, which measure 3 dimensional dynamical acceleration and turn rate of a body, respectively. Additionally temperature sensors are often built-in, in order to have the possibility to compensate for temperature dependence of the above mentioned sensors. Utilizing an IMU for attitude or velocity calculation requires the integration of the sensor acceleration and turn rate measurements. Since the measurements of sensors based on the MEMS technology are likely corrupted by different errors, both deterministic and stochastic nature, the integration leads to an accumulation of the errors and thus to a substantial drift of attitude and velocity results.

Consequently, knowledge about the errors present in the measurement outputs is obligatory in order to design a sensor noise model. For the development of an accurate sensor model the main error sources have to be identified and the noise components have to be quantified.

Error Sources

The errors of MEMS acceleration and turn rate sensors can be categorized according to their behavior. The corruption of the signals can have deterministic character on the one hand and stochastic behavior on the other hand. While the former can be handled in a straight forward manner, on the latter have to be but special emphasis. The according models as well as the parameters of stochastic influences are obtained using specific identification methods, as explained further below.

Deterministic Errors

Deterministic error sources are characterized by repeatability and thus can be measured and compensated. The components with most influence to the measurements are comprised of

- scale factor error (linear/nonlinear)
This is the relation of the sensor output change to the true turn rate/acceleration change. The scale factor error consists of a linear part and eventually a nonlinearity of the characteristic curve (commonly defined as a polynomial).
- static offset (bias)
Constant sensor output which is present even if the sensor is in a static condition. In the case of accelerometer, the constant bias is seen if the measurement axis is oriented perpendicular to the gravity vector.
- misalignment of sensing axis
The system of sensor axis does not match exactly a orthonormal dextral coordinate system due to tolerances of the production process. Furthermore there is an unknown misalignment between the sensitive axis and the package as well as the casing of the measurement unit.
- drift ramp
Component which increases linearly with time, after turning the sensor on.
- temperature dependent bias

As common for semiconductor devices, the output varies with temperature.

A comprehensive listing of accelerometer and gyroscope deterministic error sources can be found in [3]. Acquiring sensor data within defined calibration procedures with known expected outputs, allows calculating the calibration parameters. Due to the repeatability of the deterministic errors the components can be compensated according to the calibration accuracy. In the special case of temperature compensation, an additional built-in temperature sensor is necessary. The true body accelerations and turn rates given in a Cartesian coordinate system $\{x, y, z\}$

$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \quad \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (1)$$

and the measured values are related considering the described influences. The gyroscope sensor output $\tilde{\omega} \in \mathbb{R}^3$ incorporating deterministic error sources can be written as

$$\tilde{\omega} = M_G S_G \omega + B_G + R_G t + B_{G,T}(T) \quad (2)$$

where $\omega \in \mathbb{R}^3$ is the real turn rate, $S_G \in \mathbb{R}^{3 \times 3}$ is the scale factor, $M_G \in \mathbb{R}^{3 \times 3}$ is the misalignment matrix, $B_G \in \mathbb{R}^3$ is the static bias, $R_G \in \mathbb{R}^3$ is the drift ramp coefficient, t states the time and $B_{G,T}(T) \in \mathbb{R}^3$ is the temperature (T) dependent bias. The relationship between the measured acceleration $\tilde{a} \in \mathbb{R}^3$ and the true value $a \in \mathbb{R}^3$ yields the same equation as for the gyroscope

$$\tilde{a} = M_A S_A a + B_A + R_A t + B_{A,T}(T) \quad (3)$$

where again $S_A \in \mathbb{R}^{3 \times 3}$ is the scale factor, $M_A \in \mathbb{R}^{3 \times 3}$ is the misalignment matrix, $B_A \in \mathbb{R}^3$ is the static bias, $R_A \in \mathbb{R}^3$ is the drift ramp coefficient, t states the time and $B_{A,T}(T) \in \mathbb{R}^3$ is the temperature (T) dependent bias. The determination of the above described calibration parameters is commonly done by means of least squares estimation. Although, calibration procedures are not subject of this article.

Stochastic Errors

The stochastic error sources include influences which are neither repeatable nor predictable, because of their random character. In contrast to deterministic errors, a static compensation using parameters obtained by calibration is not achievable. Stochastic error sources include [4]

- quantization noise
The voltage output of the sensor is sampled using an analog-to-digital converter. Due to the finite bit resolution there is a difference between the actual signal amplitude and the sampled bit value.
- angle/velocity random walk
Comprises high-frequency noise terms, resulting in angle (or in case of an accelerometer in velocity) random walk. The arising noise in the rate (acceleration) output corresponds to white-noise.

- bias instability
A low-frequency noise component causing bias fluctuation in the sensor output. Electronics random flickering produces the bias instability.
- rate/acceleration random walk
Appears as a random process describing an uncertain origin.
- turn-on to turn-on bias
Every time the sensor is turned-on a different bias value occurs. Once turned-on the bias is constant over time.

Random errors cannot be compensated directly, but they can be modeled using random process models. Assume that a specific random noise characteristic can be achieved with a linear-time-invariant (LTI) system excited by a white-noise input. Subsequently, if the desired output statistics can be identified and the white-noise is assumed to be the input, the transfer function of the LTI system can be determined.

Linear Systems with Random Input

Consider a linear time-invariant system with input $w(t)$, then the output $y(t)$ is given as

$$y(t) = \int_{-\infty}^t g(t-\lambda)w(\lambda)d\lambda \quad (4)$$

where g is the impulse response of the linear system. If $w(t)$ is a random process, consequently $y(t)$ has also random behavior. Applying the Fourier transform to the impulse response

$$G(j\omega) = \int_{-\infty}^{+\infty} g(t)e^{-i\omega t}dt \quad (5)$$

results in the system transfer function $G(i\omega)$. The Fourier transform of the cross-correlation function $r_{wy}(\tau)$ of the system input $w(t)$ and system output $y(t)$ can be written as

$$S_{wy}(i\omega) = S_w(i\omega)G(i\omega)^* \quad (6)$$

The Fourier transform of the auto-correlation function $r_{yy}(\tau)$ of $y(t)$ yields

$$S_y(i\omega) = G(i\omega)S_{wy}(i\omega) \quad (7)$$

$$S_y(i\omega) = G(i\omega)S_w(i\omega)G(i\omega)^* \quad (8)$$

with $S_{wy}(i\omega)$ as the cross power spectral density of input and output. $S_y(i\omega)$ is the power spectral density of the output $y(t)$, $G(i\omega)^*$ the complex-conjugate of the transfer function and $\tau = t_2 - t_1$. Using equation (8) a linear system realization $G(i\omega)$ can be found, proper for producing a noise with power spectral density $S_y(i\omega)$ when driven by white noise with PSD $S_w(i\omega)$. For a detailed derivation the reader is referred to [5].



White-Noise

A basic stochastic process, which is often used for the modeling of more complex random processes, is white-noise. A continuous white-noise process $w(t)$ has constant power over all frequencies, thus the power spectral density is constant

$$S_w(i\omega) = \sigma_w^2. \quad (9)$$

As a reason of the constant power spectral density white-noise has infinite power. $r_{ww}(\tau)$ is the auto-correlation function of the random process given as

$$r_{ww}(\tau) = \sigma_w^2 \delta(\tau). \quad (10)$$

Due to the infinite power, continuous white-noise is not realizable. Generally a white-noise process can exhibit any random distribution, however often the Gaussian distribution $w(t) \sim N(0, \sigma_w^2)$ is used with the expectation

$$E[w(t)] = 0. \quad (11)$$

Colored-Noise

Any random process, which does not have equal power over the whole frequency range, is referred to as colored. A convenient method for modeling colored noise processes is utilizing [Equation \(8\)](#) assuming a white-noise input with power spectral density $S_w(i\omega) = \sigma_w^2$. If the power spectral density of the colored-noise process $S_y(i\omega)$ or rather the auto-correlation function can be identified, the transfer function $G(i\omega)$ can be directly calculated. Note that there are more degrees of freedom than design constraints, which requires the jointly selection of the input noise process and the transfer function. $G(i\omega)$ is chosen so that all poles and zeros lie in left half of the complex s-plane, which results in a stable linear system. Colored-noise processes are for example Gauss-Markov processes or random-walk processes. Further details can be found in [\[5\]](#) and [\[6\]](#).

Identification of Stochastic Parameters using Allan Variance

The Allan variance is a time-domain analysis method, developed in the 1960s for the frequency stability analysis of high precision oscillators [\[7\]](#). The Allan variance calculation can be directly performed on time-domain measurement data, thus is easier to handle than frequency-domain approaches. It is a method for the identification of various random-drift errors in measurement data. More precisely, the underlying random process characteristics associated with specific noise terms are revealed and the magnitude can be determined. Since random drift errors are not only present in high precision oscillators, but in a variety of measurement devices, the Allan Variance method can be adapted for use with different sensors. In [\[4\]](#) the concept of Allan variance is used for the analysis of random-drift errors of accelerometers and gyroscopes, which is presented below.

The Allan variance states the root mean squares (RMS) random-drift error as a function of averaging time. Suppose a measurement data with sample time t_s and N consecutive data points. The data is divided into time clusters with a length of n samples, accordingly each cluster is related to a time $T = nt_s$. The cluster sizes are restricted to $1 \leq n < \frac{N}{2}$. Given a finite number of data points, subsequently the number of clusters with a fixed length N is also finite. Assume an inertial sensor output $\Omega(t)$, then the average of a cluster $\bar{\Omega}_k(T)$ yields

$$\bar{\Omega}_k(T) = \frac{1}{T} \int_{t_k}^{t_k+T} \Omega(t) dt \quad (12)$$

with index k stating the k -th data point, associated with the time t_k . The average is also calculated for the subsequent cluster with the same length, but shifted n samples

$$\bar{\Omega}_{next}(T) = \frac{1}{T} \int_{t_{k+1}}^{t_{k+1}+T} \Omega(t) dt \quad (13)$$

where $t_{k+1} = t_k + T$. Of each two adjacent clusters with the same cluster time T , the difference

$$\Delta_{k+1,k}(T) = \bar{\Omega}_{next}(T) - \bar{\Omega}_k(T) \quad (14)$$

is calculated. According to the definitions in [\(12\)](#) and [\(13\)](#) for each cluster time T a number of $N - 2n$ differences $\Delta_{k+1,k}(T)$ given in [equation \(14\)](#) can be determined. $\Delta(T)$ is a set of realizations of a random variable for each cluster time. The Allan variance $\sigma^2(T)$ represents the variance of $\Delta(T)$ over the entire data for a specific cluster time, thus defined as a function of T

$$\sigma^2(T) = \frac{1}{2(N - 2n)} \sum_{k=1}^{N-2n} [\Delta_{k+1,k}(T)]^2. \quad (15)$$

The estimation accuracy of $\sigma^2(T)$ depends on the number of clusters with the same length, which fit into the entire data. This implies that especially for long cluster times a large set of data points is required.

Usually the square root of the Allan variance $\sigma(T)$ is plotted versus T in a log-log plot. A convenient choice for the cluster lengths is $n = 2^j, j = 0, 1, 2, \dots, \log_2 \left(\frac{N}{2} \right)$ assumed an even amount of data points.

Characteristic Noise Components

The power spectral density (PSD) of a random process is derived as the Fourier transform of its auto-correlation function and thus represents the characteristic in the frequency domain. The PSD is uniquely related to the Allan variance by

$$\sigma^2(T) = 4 \int_0^{\infty} S_{\Omega}(f) \frac{\sin^4(\pi f T)}{(\pi f T)^2} df$$