

Time-Dependent Reliability-Based Design Optimization of Vibratory Systems

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ABSTRACT

A methodology for time-dependent reliability-based design optimization of vibratory systems with random parameters under stationary excitation is presented. The time-dependent probability of failure is computed using an integral equation which involves up-crossing and joint up-crossing rates. The total probability theorem addresses the presence of the system random parameters and a sparse grid quadrature method calculates the integral of the total probability theorem efficiently. The sensitivity derivatives of the time-dependent probability of failure with respect to the design variables are computed using finite differences. The Modified Combined Approximations (MCA) reanalysis method is used to reduce the overall computational cost from repeated evaluations of the system frequency response or equivalently impulse response function. The method is applied to the shape optimization of a vehicle frame under stochastic loading. The novelty of the proposed work lies in the integration of the joint up-crossing rate method for time-dependent reliability of vibratory systems, sparse grid approach for efficient evaluation of multi-dimensional integrals, reanalysis methodology for efficient vibratory analysis, and optimization tools for time-dependent Reliability-based Design Optimization (RBDO).

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1. INTRODUCTION

Finite element analysis (FEA) has become an integral part of product design lifecycle for many industries including automotive, aerospace and heavy machinery. Dynamic analysis allows engineers to accurately predict the structural response under different loading conditions. The state-of-the-art FEA software have greatly improved in terms of speed and capacity, but the size and complexity of FEA models have also increased significantly. Thus, it is still expensive and time-consuming to use FEA in design optimization where the involved design evaluations are too many.

The response of a linear vibratory system excited by a stationary Gaussian process is also a stationary and Gaussian process. A time-dependent reliability analysis is thus needed to calculate the probability that the system will perform its intended functions successfully for a specified time. This probabilistic analysis requires even more design evaluations which makes time dependent reliability analysis prohibitively expensive for large FE models.

A deterministic design optimization (DDO) problem in FEA is usually defined by the minimization of the weight of a structure such that the maximum stress for example, does not exceed a specific value defined by the yield stress of the material. In order to consider variability and uncertainty in the random parameters and loading, one should apply Reliability-Based Design Optimization (RBDO) to

minimize the weight such that the probability of failure, i.e., the probability the maximum stress exceeds the material yield strength, is less than a small target value. If the problem is time dependent, as in random vibrations, a Time-Dependent Reliability-Based Design Optimization (TD-RBDO) should be applied where the constraints are expressed in terms of a time dependent probability of failure.

In structural analysis, RBDO has been widely used for gauge and shape optimization. However, there are only few references related to TD-RBDO. A time dependent reliability analysis and design optimization method using a nested extreme response surface approach has been presented in [1]. In [2], TD-RBDO has been implemented using polynomial chaos and RBDO under stationary stochastic loads has been presented in [3]. Time-dependent reliability analysis has also been applied to lifecycle cost design [4, 5].

The challenge in TD-RBDO is usually the high computational cost and complexity to compute the time-dependent probability of failure. Monte Carlo simulation (MCS) can be used but it is computationally very demanding, especially if the probability of failure is low. The up-crossing rate approach has been developed as an alternative. It was first introduced by Rice [6] and further extended in [7–10]. The PHI2 method [6] is an early attempt to estimate the time-dependent probability of failure using two successive time-invariant analyses based on FORM and the binomial cumulative distribution to calculate the probability of the joint event. It has been shown [9, 10] that the

PHI2 approach is inaccurate for vibratory systems. Other analytical approaches to estimate the time-dependent probability of failure have also been proposed [11, 12].

A MCS approach was proposed in [13] to estimate the time-dependent failure rate over the product lifecycle using an importance sampling method considering a decorrelation length [14]. Subset simulation [15] has also been proposed as an efficient simulation method for computing small failure probabilities for general reliability problems. An extreme value method [16] has used the distribution of the extreme value of the response to compute time-dependent probability. Also, a time-dependent reliability analysis was proposed [17] using the total probability theorem and the composite limit state concept.

The computation of time-dependent probability of failure based on the joint up-crossing rate method has been proposed in [18]. It is adopted here to calculate the time-dependent probability of failure required in TD-RBDO of random vibratory systems.

To consider the effect of input random variables ($\mathbf{X}$ in Figure 1) the total probability theorem [17] may be used because it is easier and more efficient to calculate a conditional time-dependent probability of failure; i.e., a probability of failure considering a realization of $\mathbf{X}$. In this case, the time-dependent probability of failure is the mean value of all conditional probabilities which is expressed by a multi-dimensional integral involving the joint PDF of $\mathbf{X}$. This integral may be computed using sampling methods, metamodeling techniques, grid-based quadrature approaches or gradient-based methodologies. In this study, the sparse grid approach [19–21] is used based on a Gauss-Hermite quadrature rule because the input random variables are assumed Gaussian.

In RBDO, the system dynamics, expressed by its Frequency Response Function (FRF), change every time the optimizer modifies the values of the design variables. For large-scale systems with many thousands or even millions of DOFs, the FRF calculation is computationally intensive. To reduce this cost substantially, we use a combination of reduced-order modeling (modal model) and reanalysis [22–24]. Reanalysis methods are based on local and global approximations. Local approximations, such as the Rayleigh-Ritz reanalysis procedure [25–26], are very efficient but are accurate only for small structural changes. On the other hand, global approximations can be accurate for large changes, but they are computationally more expensive.

The Combined Approximations (CA) method [27–29] merges the advantages of both local and global approximations and can be used for large design changes. The CA method is based on a combination of binomial series (local) approximations and reduced basis (global) approximations. CA methods have been developed for linear static reanalysis and eigen-problem reanalysis [30–34], where large computational cost reductions have been shown (factor of 5 to 10). The Modified Combined Approximation (MCA) method used in this work, further improves accuracy and computational efficiency.

The objective of this paper is to combine state-of-the-art methodologies to implement time-dependent reliability-based optimization of large-scale vibratory systems. For that, the time-dependent probability of failure is computed using the joint up-crossing method [18], conditional probabilities of failure and the total probability theorem. To reduce the computational effort, we use sparse grid quadrature integration, and the MCA reanalysis method. The proposed methodology is applied to the TD-RBDO of a cantilever beam and a vehicle frame under stochastic loading. To the authors' knowledge, this is the first study concerned with TD-RBDO of large-scale random vibratory systems. The proposed method is applicable to linear vibratory systems excited by Gaussian processes. Future research will address linear or nonlinear systems excited by non-Gaussian processes.

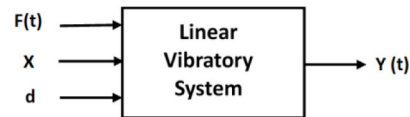


Figure 1. Schematic of linear vibratory system

2. TIME-DEPENDENT RELIABILITY-BASED DESIGN OPTIMIZATION (TD-RBDO)

2.1 Problem Statement

The TD-RBDO problem is defined as

$$\min_{\mathbf{d}, \mu_{\mathbf{X}_R}} f(\mathbf{d}, \mu_{\mathbf{X}_R}, \mu_{\mathbf{X}_p}) \quad (1a)$$

such that

$$P(\exists \tau \in [0, T] \text{ so that } g_i(\mathbf{d}, \mathbf{X}, \mathbf{F}(\tau)) \leq 0) \leq p_{f_i}^t, \quad i = 1, \dots, n \quad (1b)$$

$$\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \quad (1c)$$

$$\mu_{\mathbf{X}_R}^L \leq \mu_{\mathbf{X}_R} \leq \mu_{\mathbf{X}_R}^U \quad (1d)$$

where $f(\mathbf{d}, \mu_{\mathbf{X}_R}, \mu_{\mathbf{X}_p})$ is the objective function (e.g., total mass of the structure), and $g_i(\mathbf{d}, \mathbf{X}, \mathbf{F}(\tau))$ is the i^{th} limit state function, indicating failure if $g_i \leq 0$. $\mathbf{X} = [\mathbf{X}_R, \mathbf{X}_p] \in R^m$ is the vector of random variables where $\mathbf{X}_R$ are the random design variables and $\mathbf{X}_p$ are the random design parameters. $\mathbf{d} \in R^q$ is the vector of deterministic design variables. $\mathbf{F}(t) \in R^l$ is the vector of l random processes and $p_{f_i}^t$ is the target probability of failure corresponding to $g_i(\mathbf{d}, \mathbf{X}, \mathbf{F}(\tau))$.

The time-dependent probability of failure $P(g_i(\mathbf{d}, \mathbf{X}, \mathbf{F}(\tau)) \geq 0, \exists \tau \in [0, t])$ in Equation (1b) may be denoted as $P_f(0, T) = P(E)$ where E is the failure event, and can be computed in terms of the conditional probability of failure $P(E/\mathbf{X})$, using the total probability theorem. The latter is expressed by the following integral

$$P_f(0, T) = P(E) = \int_{\Omega} P\left(\frac{E}{\mathbf{X}}\right) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (2)$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of $\mathbf{X}$. An efficient and accurate method to calculate the conditional probability of failure involves an integral equation in terms of up-crossing and joint up-crossing rates [18]. It is presented briefly in Section 2.2 for completeness since it is part of the proposed TD-RBDO approach. To calculate the integral in Equation (2) efficiently, we use a sparse grid methodology [19–21]. Section 2.3 provides an overview of the sparse grid approach and compares it to the standard tensor grid method. To reduce the dominant computational effort from repeated structural analyses during the optimization, we use the MCA reanalysis [35] to estimate the eigenvalues and eigenvectors of a new design based on those of a baseline design (see Section 2.4). The optimization is performed using a standard gradient-based optimizer, whereas the sensitivity derivatives of the time-dependent probability of failure with respect to the design variables are estimated using finite-differences.

2.2 Conditional Time-Dependent Probability of Failure

This section presents an overview of how to compute the conditional time-dependent probability of failure $P\left(\frac{E}{\mathbf{X}}\right)$ of Equation (2). It is a summary of the time-dependent reliability analysis in [18] and is used herein for the TD-RBDO of large-scale vibratory problems with many degrees of freedom, modeled using FEA tools.

Neglecting the deterministic design parameters for simplicity, the failure event is defined as

$$E = \{\exists t \in [0, T]: g(\mathbf{X}, \mathbf{F}(t), t) = \alpha(t) - Y(t) \leq 0\}. \quad (3)$$

Failure occurs if the output $Y(t)$ exceeds a time-dependent level crossing threshold $\alpha(t)$ at any time in $[0, T]$ where T defines the time horizon (time of interest). The time-dependent probability of failure, or cumulative probability of failure [1, 3], is then equal to

$$P_f(0, T) = P(E) = P\{\exists t \in [0, T]: g(\mathbf{X}, \mathbf{F}(t), t) \leq 0\} \quad (4)$$

with failure occurring if $g(\cdot) \leq 0$ at any time $t \in [0, T]$.

The time-dependent probability of failure $P_f[0, t]$, $0 \leq t \leq T$ can be computed by the expression

$$P_f[0, t] = P_f^0 + \left(1 - P_f^0\right) \int_0^t f(\tau) d\tau \quad (5)$$

where $f(t)$ is the PDF of the first time to failure and

$$P_f^0 = P\{Y(0) > \alpha(0)\} \quad (6)$$

is the instantaneous probability of failure at $t = 0$.

An algorithm to compute $f(t)$ accurately is presented in detail in [18]. The key points of the algorithm are briefly repeated here for completeness. The PDF $f(t)$ is computed by solving the integral equation

$$v^+(t) = f(t) + \int_0^t \frac{v^{++}(t, \tau)}{v^+(\tau)} f(\tau) d\tau \quad (7)$$

where $v^+(t)$ is the up-crossing rate and $v^{++}(t, \tau)$ is the joint up-crossing rate, defined as

$$v^+(t) = \lim_{\Delta t \rightarrow 0} \frac{P[g(\cdot, t - \Delta t) > 0, g(\cdot, t) \leq 0]}{\Delta t} \quad (8)$$

and

$$v^{++}(t, t_1) = \lim_{\Delta t \rightarrow 0} \frac{P[g(\cdot, t - \Delta t) > 0, g(\cdot, t) \leq 0, g(\cdot, t_1 - \Delta t) > 0, g(\cdot, t_1) \leq 0]}{(\Delta t)^2} \quad (9)$$

Based on Figure 2, the up-crossing and joint up-crossing rates are computed as follows. The up-crossing rate is given by

$$v^+(t) = \frac{P(E_t) - P(E_{t-\Delta t} \cap E_t)}{\Delta t} \quad (10)$$

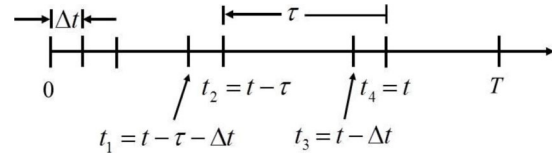


Figure 2. Notation for computation of up-crossing rates

where E_t is the failure event at time t . $P(E_t)$ is equal to $\Phi(-\beta_t)$ where β_t is the instantaneous reliability index at time t and Φ is the Cumulative Distribution Function (CDF). $P(E_{t-\Delta t} \cap E_t)$ depends on the correlation coefficient function $\rho(t - \Delta t, t)$ of the response. Specifically, $P(E_{t-\Delta t} \cap E_t) = 0$ for $\rho(t - \Delta t, t) = -1$, $P(E_{t-\Delta t} \cap E_t) = \Phi(-\beta_{t-\Delta t}) \Phi(-\beta_t)$ for $\rho(t - \Delta t, t) = 0$ and $P(E_{t-\Delta t} \cap E_t)$ is expressed as

$$\begin{aligned} P(E_{t-\Delta t} \cap E_t) &= \Phi_2(-\beta_{t-\Delta t}, -\beta_t, \rho_{t-\Delta t, t}) = \\ &= \frac{1}{2\pi} \int_{\arccos(\rho_{t-\Delta t, t})}^{\pi} \exp\left[-\frac{1}{2}(\beta_{t-\Delta t}^2 + \beta_t^2 - 2\beta_{t-\Delta t}\beta_t \cos \theta)\right] \sec^2 \theta d\theta \end{aligned} \quad (11)$$

for $-1 \leq \rho(t - \Delta t, t) \leq 1$. In Equation (11), Φ_2 stands for the bivariate standard normal CDF.

The joint up-crossing rate $v^{++}(t, \tau)$ using the notation of Figure 2, is expressed as