

Warranty Forecasting of Repairable Systems for Different Production Patterns

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ABSTRACT

Warranty forecasting of repairable systems is very important for manufacturers of mass produced systems. It is desired to predict the Expected Number of Failures (ENF) after a censoring time using collected failure data before the censoring time. Moreover, systems may be produced with a defective component resulting in extensive warranty costs even after the defective component is detected and replaced with a new design. In this paper, we present a forecasting method to predict the ENF of a repairable system using observed data which is used to calibrate a Generalized Renewal Processes (GRP) model. Manufacturing of products may exhibit different production patterns with different failure statistics through time. For example, vehicles produced in different months may have different failure intensities because of supply chain differences or different skills of production workers, for example. In addition during the warranty period, there may be a time called “clean point” where a defective component or subsystem is detected and replaced with a new more reliable unit for all products produced thereafter. This introduces a different statistical behavior before and after the detection of the “clean point.” We will present a modified GRP model for warranty forecasting of systems with or without a “clean point” and different production patterns. The capabilities of the proposed approach will be demonstrated using vehicle production data.

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1. INTRODUCTION

For quality control and warranty purposes, manufacturers collect failure data for a component or system over a short interval of observation. It is then desirable to use this data to forecast future failures (Figure 1) for planning purposes and potential improvements of their production process. This paper proposes a statistical approach to do so. We assume that the components/systems are repairable, and the repairs are instantaneous or very short compared to the lifetime of the unit. We also assume that there is only one failure mode in the population. If there are more than one failure modes the proposed analysis can be applied for each mode separately and aggregate the results.

There are many papers in the literature on warranty forecasting [e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. However, most of them either use simplifying assumptions or address specialized cases of forecasting [8, 9, 10, 11, 12]. The proposed approach to warranty forecasting is general. It uses stochastic simulation of the Generalized Renewal Process (GRP) model [1, 3, 4, 6, 13, 14, 15] of repairable systems, and considers the commonly encountered Limited Failure Population (LFP) [12] effect and the concept of clean point.

For manufactured products, there is often a small proportion of the production volume with a higher propensity to failure compared to other units in the population. This may be due to users' behavior, environmental conditions, supply chain disruptions, different assembly locations, or difficult to detect flaws in design. It is desired to use observed failure patterns during a short time interval of observation after production to identify the size of the defective subpopulation, and the parameters of its assumed time-to-failure distribution. Such a population is called Limited Failure Population (LFP) [12]. The initial population is divided into two groups - a defective and a non-defective subpopulation. The number of units in the former is unknown and impossible to know a priori because only a portion of it fails during the observation time. The remaining units will eventually fail after the censoring time (end of observation window). The probability of failure of the non-defective subpopulation is assumed very small. The event of all units being defective is a special case of LFP.

The defective subpopulation is therefore, the main contributor to the overall failure rate. Detection of such “defective” units is not possible using conventional statistical tools such as Weibull analysis [2], for example. Moreover, even if we know the existence of a defective subpopulation, its proportion to the overall production volume is unknown. Two methods can be used to estimate the proportion of the defective subpopulation - one is based on censored data [2] and another is based on truncated data [2, 12, 16]. The censored data method applies when we know the number of defective units but we do not know when they will fail. On the other hand, the truncated data method can be used when we do not know either the number of defective units or when they will fail.

To our knowledge, the application of both methods in forecasting is very limited. In this paper, we use the method of censored data. We assume that the proportion of defective units is not known. The proposed warranty forecasting model detects the existence of a defective subpopulation and estimates its proportion by calibrating a GRP model using available failure data during a time interval of observation. The model uses a censored Maximum Likelihood Estimator (MLE) [2] to account for the limited time of observation. The calibration ensures that on average, the predicted ENF matches the observed ENF at each time during the interval of observation. The analysis is performed separately for each Month of Production (MOP) and the results are aggregated to estimate the ENF of the whole population. In doing so, we capture the effect of different production patterns such as changes or disruptions of supply chain for short periods, or quality differences due to different groups of production workers. The ENF is estimated by a proposed MCS which considers the principle of “clean point.”

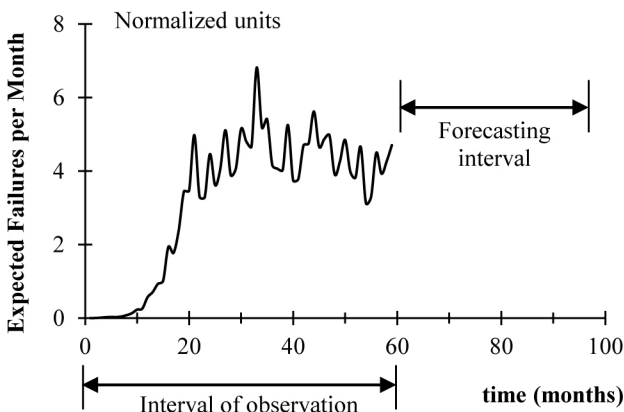


Figure 1. Definition of observation and forecasting intervals

Manufacturers collect data and use them to detect unusual failure rates which may indicate the existence of a defective subpopulation. Because of warranty cost issues they commonly intervene by redesigning the component in the defective subpopulation in order to eliminate or reduce the level of failure rate. We call this intervention “clean point.” It usually results in a failure rate level that is practically zero compared to the initial level. Thus, clean point is a fixed time after which a failed defective component is replaced with an improved redesigned component which will not fail again during the lifecycle of the product.

1.1. Forecasting Approach and Model Calibration

As we have mentioned, only the defective subpopulation contributes to the failure rate. A censored MLE approach estimates the CDF of Time-To-First-Failure (TTFF) which is then used in a GRP model to obtain the expected number of failures (ENF). It should be emphasized that if the size of the defective subpopulation is not known, we cannot estimate the CDF of TTFF.

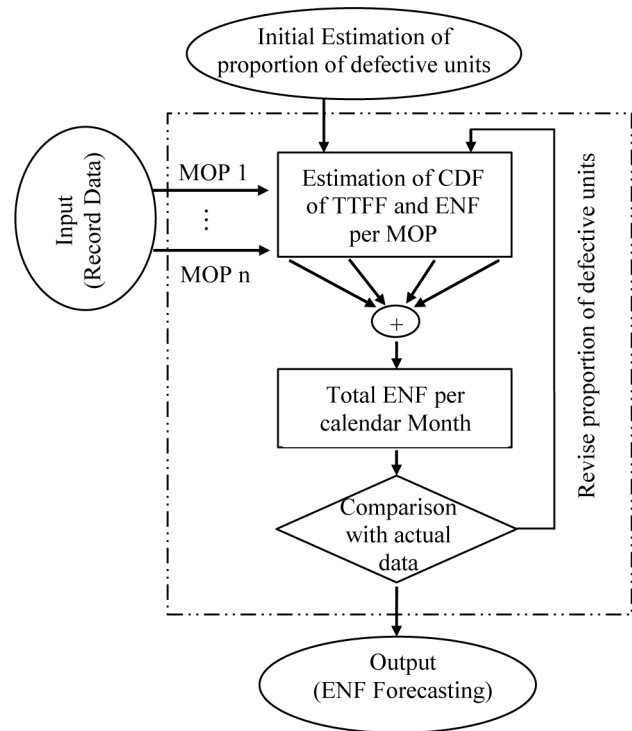


Figure 2. Overview of warranty forecasting approach

To overcome this issue we perform a model calibration. Figure 2 provides a schematic of our forecasting methodology including model calibration. We assume a proportion of defective subpopulation, and estimate the CDF of TTFF for each MOP (Section 2). Then a Generalized Renewal Process (GRP) model is used to estimate the ENF for each MOP (Section 3). A Monte Carlo Simulation (MCS) implements the GPR model (Section 4). Our MCS version uses an estimated useful life (Section 3.2) to avoid numerical issues as the virtual age approaches the support of the CDF of TTFF. Available MCS algorithms [3, 4, 5, 6, 7] do not consider this issue. The proposed MCS algorithm also implements the concept of clean point.

As shown in Figure 2, the ENF predictions for each MOP are aggregated to estimate the overall ENF with contributions from all MOPs. A RMSE measure is then formed between the predicted ENF and observed ENF during the interval of observation. The assumed proportion of the defective subpopulation is updated until a minimum RMSE is achieved. Details are provided in Section 5.

Figure 3 provides a schematic of sample paths in MCS and indicates some of our terminology. Assume we have a product population of n units. For each unit, the horizontal hashed line indicates calendar time. We will call it sample path hereafter. The symbol “ i ” for each sample path indicates the time t_i the i th unit is put in service. We call

it in-service time. The “x” symbol indicates failure with a subsequent repair. The censoring time, defining the interval of observation, is denoted by t_c . Because the in-service time is different for each unit, the simulation time up to a specific time t between t_c and t_{END} , the interval of forecast, is different for each sample path. The failure/repair events are repeated within the observation interval $(0, t_c)$. They also repeat during the interval of forecast, if there is no “clean point”. If $N(t)$ is the number of failures in $(0, t)$ where $t \in (t_c, t_{END}]$, the Cumulative Intensity Function (CIF) $W(t)$ provides the Expected Number of Failures $ENF(t)$ from 0 to time t as $ENF(t) = E[N(t)] = n \cdot W(t)$ where n is the number of units. The time derivative

$w(t) = \frac{dW(t)}{dt}$ is known as the Rate of Occurrence of Failures (ROCOF) or equivalently renewal density. It represents the mean number of failures in $(t, t + dt)$ over dt . It should be noted that the ROCOF $w(t)$ is not the same with the failure rate which is equal to $\lambda(t) = \frac{d[ENF(t)]}{dt} = n \cdot w(t)$.

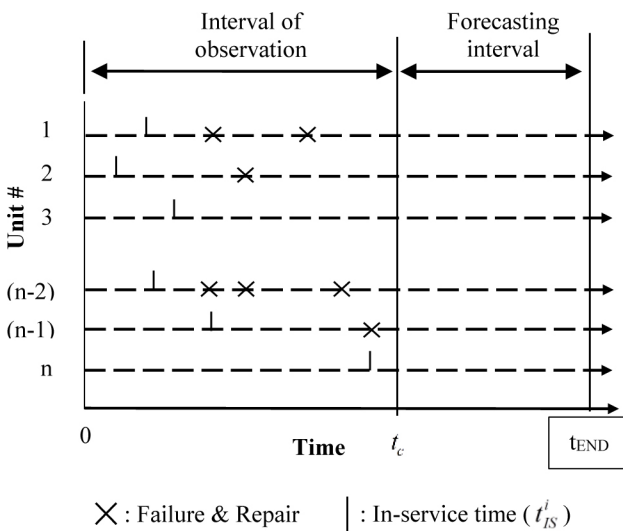


Figure 3. Notation for MCS and terminology

The paper is organized as follows. [Section 2](#) describes the estimation of the CDF of TTFF and provides details on how we estimate the proportion of defective units. [Section 3](#) first provides an overview of Kijima Type I GRP model of repairable systems and then describes the estimation of useful life. The proposed stochastic simulation of the GRP model is outlined in [Section 4](#) including details on how to estimate the ENF for each MOP. In [Section 5](#), we present an overview of the proposed forecasting approach and our calibration metric for estimating the size of a defective subpopulation if it exists. Two automotive applications are presented in [Section 6](#) demonstrating the benefits of the proposed method. Finally, we summarize and conclude in [Section 7](#).

2. ESTIMATION OF CDF OF TTFF AND PROPORTION OF DEFECTIVE UNITS

We assume that the TTFF of the defective subpopulation follows the Weibull distribution with the following CDF

$$F(t; \beta, \gamma) = 1 - \exp\left(-\left(\frac{t}{\beta}\right)^\gamma\right) \quad (1)$$

where β and γ are the scale and shape parameters. The Weibull distribution is commonly used in reliability engineering because of its flexibility to fit experimental data and its relative simplicity. We will use the support of the CDF to define the “useful life” of a component (see [Section 3.2](#)). Because the Weibull distribution is open ended from the right ($F(t; \beta, \gamma) = 1$ if $t \rightarrow \infty$) we consider as support the interval $[0, t_s]$ so that $F(t_s; \beta, \gamma) = 0.99$.

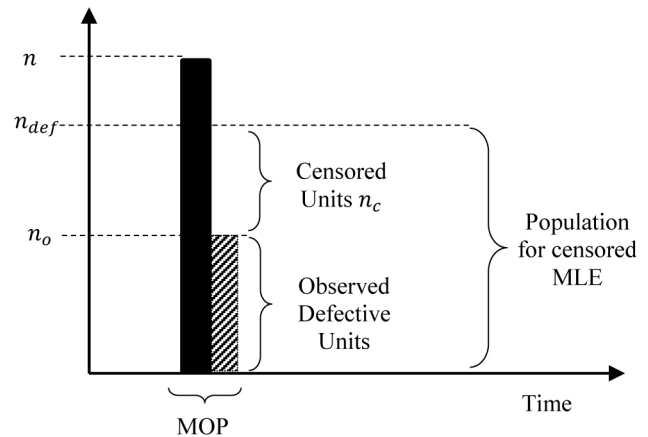


Figure 4. Notation to define defective subpopulation for a specific Month of Production (MOP)

Let n , n_{def} and n_o denote the number of all produced units, the number of defective units, and the number of observed defective units in the interval of observation respectively, during a particular MOP. We assume that only n_{def} units out of n will fail as time progresses (defective subpopulation). Thus, the number of censored defective units is $n_c = n_{def} - n_o$. [Figure 4](#) shows schematically these quantities. The following relationships hold

$$n_o \leq n_{def} \leq n \quad (2a)$$

and

$$n_c = n_{def} - n_o. \quad (2b)$$

The parameters β and γ of the Weibull CDF of TTFF for each MOP can be estimated using the Maximum Likelihood Estimation (MLE) method for censored data [2] only if we know n_{def} . [Section 2.1](#) describes how we estimate n_{def} using information from the observed data.

2.1. Proportion of Defective Units

A population consists of many units produced in different MOPs. We define the ratio