

A Corrected Surrogate Model Based Multidisciplinary Design Optimization Method under Uncertainty

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ABSTRACT

Vehicle weight reduction has become one of the most crucial problems in the automotive industry because that increasingly stringent regulatory requirements, such as fuel economy and environmental protection, must be met. The lightweight design needs to consider various vehicle attributes, including crashworthiness and stiffness. Therefore, in essence, the vehicle weight reduction is a typical Multidisciplinary Design Optimization problem. To improve the computational efficiency, meta-models have been widely used as the surrogate of FE model in the multidisciplinary optimization of large structures. However, these surrogate models introduce additional sources of uncertainties, such as model uncertainty, which may lead to the poor accuracy in prediction.

In this paper, a method of corrected surrogate model based multidisciplinary design optimization under uncertainty is proposed to incorporate the uncertainties introduced by both meta-models and design variables. Firstly, various meta-models are constructed and the meta-models with the highest accuracy are selected to serve as the surrogates of FE model. Followed by the evaluation of corresponding model bias of the selected models. Then the Gaussian Process model for the bias function is built and employed to correct the previously built low fidelity meta-model for the reason that Gaussian Process Regression, a kind of nonparametric probabilistic model, can quantify the uncertainties introduced by the usage of the meta-model. Finally, based on the non-dominated sorting genetic algorithm II (NSGA-II), robust solution which can meet the performance requirements of different subjects is found efficiently. The proposed method is demonstrated through a vehicle weight reduction problem while satisfying the safety and NVH performance requirements.

CITATION: Wu, X., Fang, Y., Zhan, Z., Liu, X. et al., "A Corrected Surrogate Model Based Multidisciplinary Design Optimization Method under Uncertainty," *SAE Int. J. Commer. Veh.* 10(1):2017, doi:10.4271/2017-01-0256.

INTRODUCTION

Nowadays, vehicle weight reduction research has attracted increasing efforts as it is a viable solution for both fuel economy and environmental protection. Vehicle weight reduction is a technology which can achieve lightweight design for structures, and meet the requirements of NVH, safety, reliability, steering and ride comfort. Therefore, in essence, vehicle weight reduction is a typical Multidisciplinary Design Optimization (MDO) problem.

Aiming to solve MDO problems, researchers have investigated optimization methods and several typical MDO methods such as multidisciplinary feasible method, individual discipline feasible design optimization [1], collaborative optimization [2], concurrent subspace optimization [3], etc, over the past few decades. Recently, MDO methods has been widely investigated and utilized in the vehicle design. Yang and Gu [4] employed the MDO method and achieved about 16-kilogram weight reduction in the design of a full vehicle while meeting the constraints of NVH, roof crash, full front impact and 50% front offset crash. Chuang and Yang [5] combined

the advanced MDO methodology with the TRB manufacturing technology to optimize the vehicle underbody. Compared with the original design, the optimized underbody TRB parts is lighter and the functional requirements of all constraints are met. Xu and Chuang [6] developed a data mining-based strategy to improve the efficiency of heuristic optimization algorithms for the multi-objective MDO of side impact problem. There are many literatures about the application of MDO methods in vehicle design which can be referred. In the majority of MDO problems, especially those taking the uncertainties of design variables into account, the FE simulation is too time consuming to be directly used for design optimization. To improve the computational efficiency, various metamodel techniques have been used as surrogates of FE models [7]. And therefore, the surrogate models play a key role in the MDO. However, no matter which metamodel technique is used, there is always some discrepancies between the result of the true FE model and that of the metamodel [8]. Xu and Majcher [9] conducted a comparative study of response Surface model-based MDO and linear MDO through a vehicle structure MDO problem. Their research shows that the

surrogate models introduce additional sources of uncertainties, such as the model uncertainty, which may lead to the poor accuracy in prediction. This is especially true when the RSMs are used to predict new designs in terms of interpolation and extrapolation. In such situations, the solution of MDO is untrusted. Fortunately, the recently developed bias correction methods based on theories such as the Bayesian inference [10,11, 12, 13], the Gaussian process modeling [14, 15] and the Copula [16, 17,18] provide effective approaches to solve and quantify the additional sources of uncertainties introduced by the surrogate models in the MDO. Among the three bias correction methods, the Gaussian process method is regarded as a potential method since it shows superiority in capturing the mean of the responses when limited training data is available.

This paper aims to propose a corrected surrogate model based MDO method under uncertainty to incorporate both of the uncertainties introduced by meta-models and design variables. Firstly, various meta-models are constructed and the meta-models with the highest accuracy are selected to serve as the surrogates of FE model. Followed by the evaluation of corresponding model bias of the selected models. Then the Gaussian Process method for the bias function is built and employed to correct the previously built low fidelity meta-model. Finally, the robust solution which can meet the performance requirements of different subjects is found efficiently through the NSGA-II.

The rest of the paper is organized as follows: Gaussian Process model and Gaussian process based bias correction method is briefly introduced, followed by the framework of the proposed multidisciplinary design optimization method is constructed. Next, a case study of vehicle design is employed to demonstrate the effectiveness of proposed MDO method in practical application. Finally, discussion and summary are given in the end.

GAUSSIAN PROCESS

GP model is widely used to predict the unknown responses based on noisy observations [19, 20]. On the basis of existing sample information, it can completely extract the relation property of variables and construct the approximation relationships between input variables and responses. We define $y(\mathbf{x})$ as the response of the random variable $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$. $X_s = (\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(s)})^T$ represents the sample set with s samples and $Y_s = (y^{(1)}, y^{(2)}, \dots, y^{(s)})^T$ is a set of corresponding responses, where $y^{(i)} = y(\mathbf{x}^{(i)})$. $y(\mathbf{x})$ can be regarded as the realization of Gaussian process in which the both the mean and covariance functions respectively satisfy the conditions that $E[G(\mathbf{x})] = \mathbf{h}^T(\mathbf{x})\boldsymbol{\beta}$ and $Cov[G(\mathbf{x}), G(\mathbf{x}')] = \alpha^2 R(\mathbf{x}, \mathbf{x}')$. The Gaussian process is expressed as follows:

$$y(\mathbf{x}) \sim GP(E[G(\mathbf{x})], Cov[G(\mathbf{x}), G(\mathbf{x}')]) \quad (1)$$

where $\mathbf{h}^T(\mathbf{x})$ is the polynomial of variable $\mathbf{x}$ defined by user. $\boldsymbol{\beta}$ is the coefficients of $\mathbf{h}^T(\mathbf{x})$ and α^2 is constant. $R(\mathbf{x}, \mathbf{x}')$ is the correlation function between two arbitrary sample points of the training set and it stands for the correlation of the two sample points. There are various

types of correlation function and the most commonly used correlation function is Gaussian correlation function, which is expressed as follows:

$$R(\mathbf{x}, \mathbf{x}') = \prod_k^d \exp \left[w_k (x_k - x'_k)^2 \right] \quad (2)$$

where $\mathbf{w} = [w_1, w_2, \dots, w_d]$ contains the smoothing parameters and determines the form of the covariance function. The smaller the smoothing parameter is, the less the correlation of the two sample points will be. It can be concluded from equation (1) and equation (2) that the unknown parameters $\boldsymbol{\beta}$, α^2 and $\mathbf{w}$ can comprehensively describe the Gaussian process model. Therefore, these parameters are the hyper-parameters of Gaussian random process. According to the observation sample data X_s and Y_s , the maximum estimation is commonly used to solve the hyper-parameters of Gaussian random process. Therefore, the mean and variance of the mean of the response on an unknown point $\mathbf{x}$ are predicted as:

$$E[y(\mathbf{x})|Y_s] = \mathbf{h}^T \hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{x}) \mathbf{R}^{-1}(\mathbf{Y}_s - \mathbf{H} \hat{\boldsymbol{\beta}}) \quad (3)$$

$$Cov[y(\mathbf{x}), y(\mathbf{x}')|Y_s] = \alpha^2 \{R(\mathbf{x}, \mathbf{x}') - \mathbf{r}^T(\mathbf{x}) \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}') + [\mathbf{h}^T(\mathbf{x}) - \mathbf{H}^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x})]^T [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}]^{-1} [\mathbf{h}^T(\mathbf{x}') - \mathbf{H}^T \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}')] \} \quad (4)$$

where $\mathbf{H} = [\mathbf{h}^T(\mathbf{x}_1), \mathbf{h}^T(\mathbf{x}_2), \dots, \mathbf{h}^T(\mathbf{x}_s)]$, $\mathbf{R}$ is $S \times S$ matrix and the element lies in i th row and j th column is $R(\mathbf{x}_i, \mathbf{x}_j)$, $\hat{\boldsymbol{\beta}} = [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{Y}_s$ and it is the parametric estimation for the parameter $\boldsymbol{\beta}$.

In the Gaussian Process, the variance function expresses the correlation between two arbitrary sample points. On the other hand, it is utilized to calculate the variance of the non-sampling points. The predicted variance makes the Gaussian Process model overcomes the difficulty to analyze the error of predicted response by quantifying the data uncertainty of the unknown points outside of the observed sample set.

BIAS CORRECTION METHOD BASED ON GAUSSIAN PROCESS

Model bias correction is a process of correcting or adjusting the predicted system response to meet the physical test in model validation [21, 22]. When a meta-model is applied to serve as the surrogate of high-fidelity FE model or physical system, additional sources of errors caused by the interpolation uncertainty are inevitable to be introduced. The model bias correction method is employed to quantify the interpolation uncertainty and improve the prediction capability of low-fidelity surrogate model.

For the convenience of formulation, design variables, FE model result and surrogate model result are represented by $\mathbf{x}$, $Y_f(\mathbf{x})$ and respectively. The relationship between $Y_f(\mathbf{x})$ and $Y_s(\mathbf{x})$ can often be generalized as follows:

$$Y_i(\mathbf{x}) = Y_m(\mathbf{x}) + \delta(\mathbf{x}) + \varepsilon(\mathbf{x}) \quad (5)$$

where $\varepsilon(\mathbf{x})$ is the random variable representing the test error that may depend on $\mathbf{x}$, and it may be caused by the uncertain factor in the prototype test such as measurement errors and manufacturing errors. $\delta(\mathbf{x})$ is the bias of the surrogate model. Hence, the "true value" of output, namely $Y_{true}(\mathbf{x})$, is equal to the test observation minus the test error, or the CAE model prediction plus the model bias, as shown in Equation (10).

$$Y_{true}(\mathbf{x}) = Y_i(\mathbf{x}) - \varepsilon(\mathbf{x}) = Y_m(\mathbf{x}) + \delta(\mathbf{x}) \quad (6)$$

Then, we assume $\tau(\mathbf{x}) = \delta(\mathbf{x}) + \varepsilon(\mathbf{x})$. It is noted that the prediction bias is an essential step for estimating the value of $Y_{true}(\mathbf{x})$. For the convenience of formulation and method demonstration, it is reasonable to assume that the bias $\tau(\mathbf{x}) = \delta(\mathbf{x}) + \varepsilon(\mathbf{x})$ obeys the Gaussian distribution with unknown parameters $\beta_\tau, \alpha_\tau^2, \mathbf{w}_\tau$, which can be expressed as

$$\tau(\mathbf{x}) \sim \text{GP}(\mathbf{h}_\tau(\mathbf{x})^T \beta_\tau, \alpha_\tau^2 \{\exp[\sum_{k=1}^p w_{\tau,k}(\mathbf{x}_k - \mathbf{x}'_k)]\}) \quad (7)$$

Where the unknown parameters $\beta_\tau, \alpha_\tau^2, \mathbf{w}_\tau$ of the equation (11) can be obtained through the previous mentioned maximum likelihood method based parameter calculation. In this paper, various metamodels are built as the alternative surrogate models, among which the model with highest accuracy for each disciplinary is chosen to serve as the primary surrogate of the FE models in the process of MDO. If the accuracy of the rough selected model can't meet the requirement for usage, the Gaussian based correction would be introduced to improve the prediction capacity of the low-fidelity models.

FRAMEWORK OF THE CORRECTED SURROGATE MODEL BASED MULTIDISCIPLINARY DESIGN OPTIMIZATION METHOD UNDER UNCERTAINTY

In this section, a corrected surrogate model based MDO method under uncertainty is proposed. In this method, the Gaussian process modeling (GPM)-based bias correction method is employed to quantify the uncertainty introduced by the usage of surrogate model and improve the prediction capability. The corrected surrogate model serves as the surrogate of the FE model to accelerate the MDO. Since the corrected surrogate model takes the interpolation uncertainty into account and transmits uncertainty from design variables to system responses, the proposed method can solve the problem of MDO under uncertainty. For the convenience of illustration, the framework of the proposed method is given in Fig. 1.

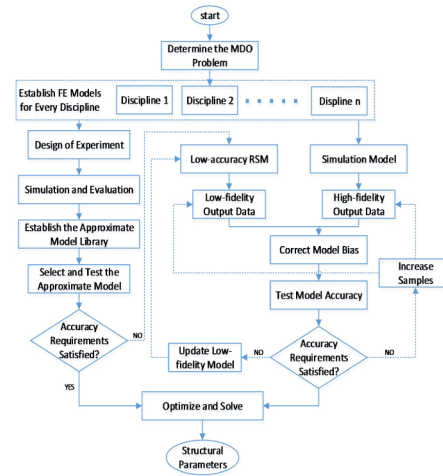


Fig. 1. Framework of the Corrected Surrogate Model Based Multidisciplinary Design Optimization Method under Uncertainty

Like the common process of MDO, the design variables and design objects are determined and the corresponding FE model for each disciplinary are constructed without firstly conducting physical test. Then the Optimal Latin Hypercube (OLH) sampling method is utilized to generate the inputs of the training set and test set, and corresponding responses are evaluated through finite element (FE) simulations. Several commonly used metamodels such as Kriging interpolation model, second-order polynomial regression model (PR) and RBF are constructed. The metamodels with the highest accuracy are selected to serve as the primary surrogates of FE models for each disciplinary. If the primary surrogate models meet the accuracy requirement, they will be directly utilized in the MDO. Otherwise, the prediction capacity of selected surrogate model need to be improved through the previous mentioned bias correction method. In order to evaluate the metamodels objectively some evaluation metrics such as RSME, SME, RPE, R square etc have been widely used. Among these metrics, the R square is within [0, 1], which can give an evaluation for the accuracy of metamodel intuitively. Therefore, the R square is selected as the metric to determine the whether the model belongs to low-fidelity surrogate model or high-fidelity surrogate model. The R square is calculated as the following equation:

$$r^2 = 1 - \frac{\sum_{i=1}^n (y_i - y_i^*)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (8)$$

In the equation, $\bar{y}$ is the mean value for $y_1, y_2, \dots, y_n$. A higher value of R square indicates a more accurate model. In this paper, only the model with R square more than 0.9 is regarded as the high-fidelity surrogate model.

In the bias correction, several sample points are generated and the corresponding responses are evaluated by FE model and low-fidelity surrogate model. With the low-fidelity data and high-fidelity data, the Gaussian process model is constructed for the bias