

A Computational Multiaxial Model for Stress-Strain Analysis of Ground Vehicle Notched Components

Ayhan Ince
 Purdue University

ABSTRACT

Driveline and suspension notched components of off-road ground vehicles often experience multiaxial fatigue failures along notch locations. Large nominal load histories may induce local elasto-plastic stress and strain responses at the critical notch locations. Fatigue life prediction of such notched components requires detailed knowledge of local stresses and strains at notch regions. The notched components that are often subject to multiaxial loadings in services, experience complex stress and strain responses. Fatigue life assessment of the components utilizing non-linear Finite Element Analysis (FEA) require unfeasibly inefficient computation times and large data. The lack of more efficient and effective methods of elasto-plastic stress-strain calculation may lead to the overdesign or earlier failures of the components or costly experiments and inefficient non-linear FEA. An analytical computational model is developed to calculate actual elasto-plastic stresses and strains of notched components based linear-elastic FEA solution. The model is based on the integration of the material constitutive theory and the multiaxial notch correction method along with computational modeling techniques. The computation model is validated using non-linear FEA data of 1045 steel notched bar/shaft subjected to non-proportional multiaxial loadings. The comparison showed that calculated notch elasto-plastic stresses and strains from the model are in good agreement with the non-linear FEA data. The model is further assessed against experimental fatigue data of SAE 1045 steel notched shaft under multiaxial loadings. The notch stresses and strains estimated by the model have been subsequently used to perform fatigue life predictions and predicted lives showed good correlations with experimental fatigue data of the SAE 1045 steel notched shaft.

CITATION: Ince, A., "A Computational Multiaxial Model for Stress-Strain Analysis of Ground Vehicle Notched Components," *SAE Int. J. Engines* 10(2):2017, doi:10.4271/2017-01-0329.

INTRODUCTION

Most powertrain and chassis mounted driveline and suspension components for ground vehicles experience severe multiaxial loads (combined bending and torsional loadings) during service operations. These combined multiaxial cyclic loadings induce complex stress and strain paths during a loading cycle. In addition, many of vehicle structural and mechanical components include notch regions because of design requirements and significant stress concentrations occur at those notch locations. Multiaxial loadings generate complex stress and strain states at notches and may result in a fatigue failure of safety critical notched components. The combination of multiaxial loadings and complex geometries of notched components may be necessities in practical design applications and field based experiments conducting durability test are often expensive and time consuming. Therefore, analytical and numerical methods has become a desired approach to perform fatigue and durability analyses for design of notched components.

The reliable fatigue assessment of notched components subjected to multiaxial loadings require accurate stress and strain histories determined at local notch regions. Although non-linear FEA using software packages can be utilized to compute local stress and strain

histories induced by short loading histories, however the non-linear FEA is presently not practical and computationally efficient for real notched components experiencing long load histories. Therefore, the non-linear elasto-plastic FEA for long loading histories would be considered as an impractical approach in terms of lengthy computation times. For these reasons, more efficient and simpler analysis methods of elastic-plastic stress-strain calculations and fatigue life prediction methods are needed for engineering notched structures and components subjected to long cyclic multiaxial load histories in the design process.

Early research studies mainly concentrated on proposing theoretical stress concentration factors using elasticity theory. Neuber [1] proposed the most well-known notch stress-strain approximation method that formulates the theoretical stress concentration factor to the product of the elasto-plastic stress and strain concentration factors. As an alternative approach to Neuber's rule, Molski and Glinka [2] proposed the equivalent strain energy density (ESD) method. The ESD method suggested that the actual elasto-plastic strain energy density at the notch root is related to the pseudo strain energy density as if a body hypothetically showed elastic behavior. Recent research works [3,4,5,6,7,8] have presented that the notch correction method and material constitutive equations (a cyclic plasticity model) can be

integrated to compute the local elasto-plastic stresses and strains from pseudo linear elastic stresses at notches. These approximation methods to compute the actual local stress and strain components for critical notched components provides a great advantage over non-linear elasto-plastic FEA and durability tests due to low cost, computational efficiency and reasonable accuracy. The multiaxial stress-strain notch correction method based on incremental distortional strain energy proposed by Ince and Glinka [5] is adapted to develop a computational analytical model [6] to compute local stress-strain histories using linear elastic FEA results of notched components.

The multiaxial fatigue assessment requires accurate estimations of complex stress and strain responses and a good prediction capability of a fatigue damage model. In recent decades, a large number of research studies have been conducted to develop a successful multiaxial fatigue damage model to accurately predict the multiaxial fatigue failure. However, the most of proposed damage models have certain limitations in terms of specific load types and a type of material. Therefore, no a broadly accepted fatigue damage model has been developed yet. The fatigue damage models are generally classified as stress-based, strain-based and energy-based fatigue damage models. Readers can find a number of good review papers of existing multiaxial fatigue damage models in literature [9,10], therefore only a short introduction is provided here to present some understanding of shortcomings and good features of the existing fatigue damage models. The stress-based damage model is generally used for high cycle fatigue regime where the plastic deformation is negligibly small. The stress-based fatigue damage models do not take into account the effects of the material cyclic hardening/softening behavior, since the effects of the cyclic hardening and softening can only be seen in material strain responses. Strain-based fatigue damage models for multiaxial fatigue have generally successful applications in low cycle fatigue regime where plasticity deformation is considered to be significant. Like stress-based damage models, strain-based damage models are based on a non-linear function of the strain components on the specific material plane. Energy-based damage parameters came from the assumption that strain energy may be considered as a fatigue damage quantity of a material. Ince and Glinka [11] recently proposed a generalized multiaxial fatigue damage model in the form of both the strain-based and energy-based models to overcome some of the limitations of the existing fatigue damage models. The proposed fatigue damage model [11] is employed in this paper to perform fatigue life predictions of a SAE 1045 notched component under multiaxial loadings. The multiaxial fatigue life prediction remains a challenging problem due to a mount of complexity involved in the material deformation and fatigue characteristics. Therefore additional research works should be further needed for development of more accurate multiaxial fatigue assessment methods.

COMPUTATIONAL ANALYTICAL MODEL

FEA is often used for the stress and strain analysis of engineering components. Linear elastic stresses/strains for a notched component can be easily obtained from a linear elastic FEA solution. Once linear

elastic stresses/strains are determined from the linear elastic FEA solution, the elasto-plastic stress-strain approximation method based on the combined solution of the multiaxial notch correction method and the cyclic plasticity model can be used to calculate actual notch elastic-plastic stress and strain responses. Axle and shaft components usually are subjected to combined bending and torsion cyclic loads. When a notched body is subjected to a general multiaxial loading, the stress state at the notch root is bi-axial because of the notch root stress for a free surface as shown in Figure 1. Since equilibrium of the infinitesimal element at the notch root must be maintained, i.e. $\sigma_{23}^e = \sigma_{32}^e$ and $\varepsilon_{23}^e = \varepsilon_{32}^e$, there are three non-zero stress components and four non-zero strain components. Seven pseudo linear elastic stress and strain components ($\sigma_{ij}^e, \varepsilon_{ij}^e$) are obtained from the linear elastic FEA solution, however the actual elastic-plastic stress and strain components ($\sigma_{ij}^a, \varepsilon_{ij}^a$) at the notch root are considered as unknowns. Therefore, there are a total of seven independent equations needed for the complete solution of all actual elastic-plastic stress and strain components at the notch root. The incremental form of the multiaxial Neuber rule and the Garud cyclic plasticity model [12] are integrated as shown in Figure 2 to provide needed a set of seven equations for simultaneous solutions of all unknown stresses and strains. The cyclic plasticity model (or the material constitutive equations) provides four equations and the incremental form of Neuber notch correction rule provides remaining three equations.

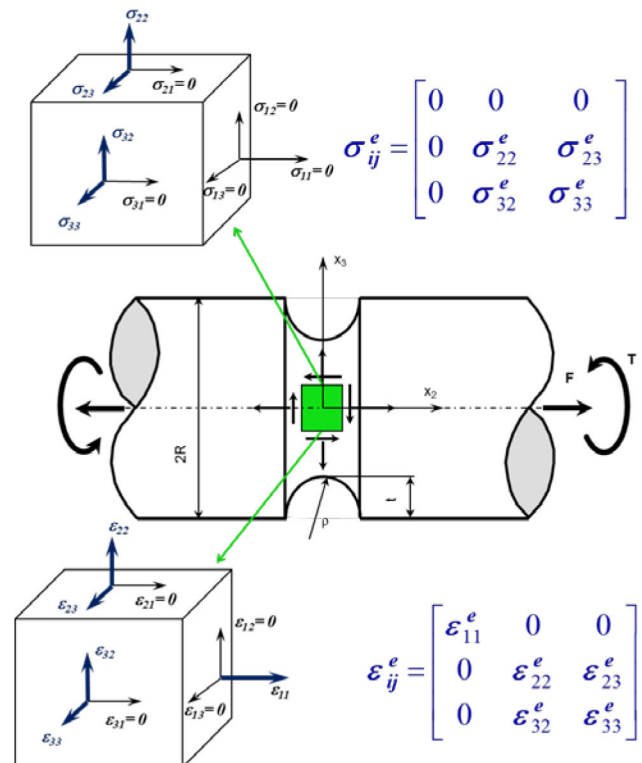


Figure 1. Stress and strain state at a notch root for a notched body under multiaxial loadings.

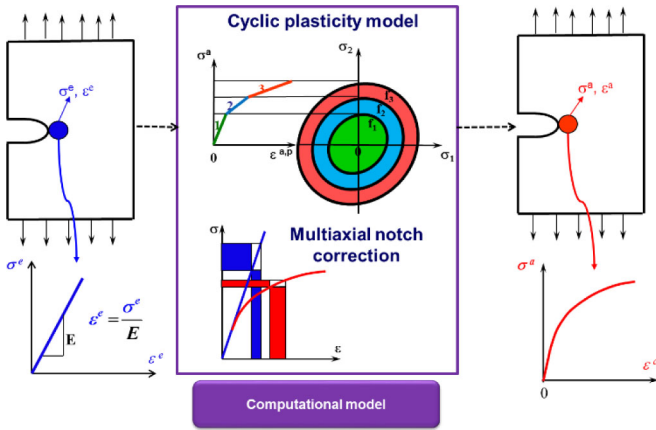


Figure 2. A schematic view of analytical computation model.

The material constitutive equations based on the Prandtl-Reuss flow rule can be presented as:

$$\Delta \varepsilon_{ij}^a = \frac{1+\nu}{E} \Delta \sigma_{ij}^a - \frac{\nu}{E} \Delta \sigma_{kk}^a \delta_{ij} + \frac{3}{2} \frac{\Delta \varepsilon_{eq}^{pa}}{\sigma_{eq}^a} \cdot S_{ij}^a \quad (1)$$

where:

$$\sigma_{eq}^a = \frac{3}{2} S_{ij}^a S_{ij}^a, \quad \Delta \varepsilon_{eq}^{pa} = \frac{\Delta \sigma_{eq}^a}{E_T^p}$$

where $\Delta \varepsilon_{ij}^a$ are increments of the actual strain components, $\Delta \sigma_{ij}^a$ are increments of the actual stress components and δ_{ij} is the Kronecker delta, $\Delta \varepsilon_{eq}^{pa}$ is the equivalent plastic strain increment, $\Delta \sigma_{eq}^a$ is the equivalent stress increment, where ν is Poisson's ratio, E is elastic modulus, G is shear modulus, E_T^p is current value of generalized plastic modulus and S_{ij}^a are deviatoric stress components.

When notched bodies experience a cyclic loading, the elastic stresses and strains at the notch root can be associated to the function of actual elastic-plastic stresses and strains at the notch root using the Neuber rule [1] or the ESED method [2].

Singh [11] proposed a generalized version of Neuber's rule. Singh proposed that the increment of total strain energy density at the notch root in an elastic-plastic body can be related to the increment of total strain energy density at the notch root in a hypothetical elastic body under same and boundary conditions. Based on his assumption, the Neuber rule [2] can be formulated in the incremental form in Equation (2).

$$\sigma_{ij}^e \Delta \varepsilon_{ij}^e + \varepsilon_{ij}^e \Delta \sigma_{ij}^e = \sigma_{ij}^a \Delta \varepsilon_{ij}^a + \varepsilon_{ij}^a \Delta \sigma_{ij}^a \quad (2)$$

Ince and Glinka [5] proposed the incremental deviatoric version of the total strain energy density for the multiaxial stress state at the notch root, i.e.

$$S_{ij}^e \Delta e_{ij}^e + e_{ij}^e \Delta S_{ij}^e = S_{ij}^a \Delta e_{ij}^a + e_{ij}^a \Delta S_{ij}^a \quad (3)$$

Ince and Glinka [5] proposed the both the material constitutive equations and incremental form of the Neuber notch correction provided in deviatoric space because deviatoric stresses are considered to induce the material deformation and hydrostatic stresses are only responsible for the material volume change. Material constitutive equations, Equation (1) gives four equations the rest of three equations can be determined from the equivalence of increments of the total distortional strain energy density, Equation (3) to obtain the set of seven independent equations necessary to completely solve elastic-plastic notch-tip strain and stress responses for a notched component subjected to multiaxial non-proportional cyclic loads in Equation (4).

$$\begin{aligned} \Delta e_{ij}^a &= \frac{1+\nu}{E} \Delta S_{ij}^a + \frac{3}{2} \frac{\Delta \sigma_{eq}^a}{E_T^p \sigma_{eq}^a} \cdot S_{ij}^a \\ S_{ij}^e \Delta e_{ij}^e + e_{ij}^e \Delta S_{ij}^e &= S_{ij}^a \Delta e_{ij}^a + e_{ij}^a \Delta S_{ij}^a \end{aligned} \quad (4)$$

Equation (4) from which all unknown deviatoric strain, Δe_{ij}^a and stress, ΔS_{ij}^a increments can be calculated from the linear pseudo elastic notch tip stress history, i.e., increments of the stress and strain, $\Delta \sigma_{ij}^e$ and $\Delta \varepsilon_{ij}^e$ which are known from the linear-elastic FEA. The calculated deviatoric stress increments, ΔS_{ij}^a , and strain increments, Δe_{ij}^a can subsequently be converted into the actual stress and strain $\Delta \sigma_{ij}^a$ and $\Delta \varepsilon_{ij}^a$ using Equation (5).

$$\begin{aligned} \Delta \sigma_{ij}^a &= \Delta S_{ij}^a + \frac{1}{3} \Delta \sigma_{kk}^a \delta_{ij} \\ \Delta \varepsilon_{ij}^a &= \Delta e_{ij}^a + \frac{1}{3} \Delta \varepsilon_{kk}^a \delta_{ij} \end{aligned} \quad (5)$$

The analytical computation model proposed by Ince and Glinka contains a general numerical algorithm and subroutines to implement equations described here. The detailed information for the algorithm and subroutines can be found in the literature [6]. The analytical model was previously verified using experimental strain data of the notched bar [5,6] and the limited numerical FEA data sets [14]. This paper aims to further assess the model validity by comparing model predicted results with non-linear elasto-plastic FEA results for SAE 1045 steel notched bar/shaft subjected to multiaxial loadings.

Correlation of Computational Analytical Model with FEA Elastic-Plastic Stress-Strain Data

Accurate estimations of stress and strain responses at the critical notch areas are quite essential for reliable fatigue life assessment of a component. Ince and Glinka proposed model is used to estimate the notch root stress and strain histories for a notched bar subjected to axial-torsion non-proportional loadings. The notch root elastic stresses were calculated on the basis of the linear elastic FE solution to compute actual elastic-plastic stress and strain histories at the notch root.

The cylindrical notched bar made of SAE 1045 steel has a radius of $R=42$ mm was used to assess the prediction capability of the computational model. The FE model of the notched bar is shown in Figure 3. The notched bar was modeled using 3-D 20-node solid.