

A New Combined Isotropic, Kinematic and Cross Hardening Model for Advanced High Strength Steel under Non-Linear Strain Loading Path

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ABSTRACT

A fully modularized framework was established to combine isotropic, kinematic, and cross hardening behaviors under non-monotonic loading conditions for advanced high strength steels. Experiments under the following types of non-proportional loading conditions were conducted, 1) uniaxial tension-compression-tension/compression-tension-compression full cycle reversal loading, 2) uniaxial reversal loading with multiple cycles, and 3) reversal shear. The calibrated new model is decoupled between isotropic and kinematic hardening behaviors, and independent on both anisotropic yield criterion and fracture model. Nine materials were calibrated using the model, include: DP590, DP600, DP780, TRIP780, DP980LY, QP980, AK Steel DP980, TBF1180, and AK Steel DP1180. Good correlation was observed between experimental and modeled results.

KEYWORDS: Constitutive Modeling, Advanced High Strength Steel, Isotropic Hardening, Kinematic Hardening, Cross Hardening, Non-linear strain path

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INTRODUCTION

During a sheet metal forming process, the mechanical behavior of advanced high strength steel (AHSS) can be dramatically influenced by non-proportional strain histories, based on the reports from [1, 2, 3, 4, 5]. For example, the loading condition in a deep drawing could alter into biaxial tension from pure shear when the material flows into the die from the flange area, resulting in a non-proportional deformation [6]. In addition to reverse loading, the strain path in real applications can be very complicated. One type of test commonly used in the lab is the uniaxial tension test with pre-strain along different orientations with regard to the rolling direction. The pre-strain could be uniaxial tension, equi-biaxial tension or shear. It is reported that the flow stress for the second step may exceed the monotonic strain-stress curve with a few stagnation, dependent on the angle change of orientation [7, 8, 9, 10, 11]. This phenomenon is called cross-loading where inactive slip systems in the first loading step were activated in the second step. Microscopic-based hardening rules and crystal plasticity models were developed for the cross-loading phenomenon. Dislocation structure based state variables, expressed as a fourth order tensor, were applied in a continuum plasticity theory by [12]. This model can also provide good results for reversal loading as well as any possible strain path changes. A special case of the Teodosiu model has been evaluated by [13], neglecting distortional hardening. Another similar modification of Teodosiu model was done by [14-15], capturing higher curvature of the yield

surface in loading direction and the respective flattening in the orthogonal direction. A homogeneous yield function based anisotropic hardening (HAH) model [16], was extended to the cross-loading case with latent hardening effect for steel sheets proposed by [10-11]. This approach could also capture the Bauschinger effect without using the concept of traditional back stresses. A modularized framework was introduced by [17] to predict isotropic, kinematic, and cross hardening behaviors, with using a "dummy" back stress to identify the loading conditions.

The objectives of this paper are to, a) develop a simpler constitutive model for AHSSs, covering both monotonic and non-proportional loading conditions, b) correlate the modeled results with a sufficient set of experiments under non-linear (reversal) strain path. Its implementation into finite element analysis, together with cross-hardening case will be provided in a subsequent manuscript, in order to predict more comprehensive non-linear historical plasticity behavior.

CONSTITUTIVE MODELING

"Dummy" Back Stress

A unity "dummy" back stress α is defined in order to predict the kinematic hardening effect. The concept of "dummy" back stress tensor, introduced in [17], can be interpreted as a set of status

variables to identify the loading history, without explicitly incorporated into the yield criterion. α is evolving towards the direction of the Cauchy stress tensor σ by rotating only, in general stress spaces (6-dimensional for 3D element and 3-dimensional for shell element). A 3-dimensional stress space ($\sigma_{11}, \sigma_{22}, \sigma_{12}$) is applied for shell element throughout this manuscript since all thickness components are considered as zero. The angle $\psi \in [0, \pi]$ between σ and α is then defined as

$$\cos \psi = \frac{\sigma : \alpha}{\|\sigma\| \|\alpha\|} \quad (1)$$

This follows the definition introduced in [17] and similar to the strain history variable in [8] and stress history variable in [16]. It can be also normalized as

$$\bar{\psi} = \frac{1 - \cos \psi}{2} \quad (2)$$

Therefore $\bar{\psi}$ has a range of $[0, 1]$, corresponding to angle ψ whose range is $[0, \pi]$. The spherical coordinate system is applied for both stress tensors to concisely depict the rotational mechanism, in which

$$\sigma = [r_\sigma, \vartheta_\sigma, \varphi_\sigma]^T \quad (3)$$

$$\alpha = [1, \vartheta_\alpha, \varphi_\alpha]^T \quad (4)$$

where r , ϑ , and φ indicates the length, polar angle, and azimuthal angle, respectively. The range of $[0, 2\pi)$ is assigned to both angles. The Cauchy stress tensor angular components can then be expressed as follows

$$\vartheta_\sigma = \text{atan2}\left(-\sigma_{12}, -\sqrt{\sigma_{11}^2 + \sigma_{22}^2}\right) + \pi \quad (5)$$

$$\varphi_\sigma = \text{atan2}(-\sigma_{22}, -\sigma_{11}) + \pi \quad (6)$$

where atan2 is the two-parameter arctangent function. With the given definition, the polar angle ϑ is measured referring to the combined vector direction of both σ_{11} and σ_{22} , while the azimuthal angle φ is inclined to the positive direction of σ_{11} . Every single point on the yield surface, represented as a 3D surface (elliptical sphere if von-Mises yield criterion is assumed), can be uniquely distinguished for the purpose of loading condition generality. Therefore, any possible loading condition transitions (such as reversal tension/compression, reversal shear, or tension-shear, etc.) can be unambiguously described, which is also equivalent to the traditional kinematic modeling methods.

The “dummy” back stress α is redesigned to follow the Cauchy stress by rotation only. Therefore, a possible rotational rate function ω is assumed to be proportional to the normalized angle difference $\bar{\psi}$ between Cauchy stress and back stress tensors, reads

$$\omega = A_1 \bar{\psi} \exp(A_0 \bar{\epsilon}^p) \quad (7)$$

where $A_1 > 0$ is a material coefficient controlling the shape of transitional area in stress-strain curves under abrupt loading condition changes (e.g. reversal loading). A larger value of A_1 yields a shorter and sharper transitional area. $A_0 \geq 0$ is an optional parameter controls the gradually decreasing of the rotational rate over the growth of the equivalent plastic strain. The rotating direction functions are respectively defined as $l(\vartheta)$ and $l(\varphi)$, reads

$$l(\vartheta) = \text{sign}\{\Delta\vartheta - \pi[1 + \text{sign}(\Delta\vartheta - \pi)]\text{sign}(\Delta\vartheta)\} \quad (8)$$

$$l(\varphi) = \text{sign}\{\Delta\varphi - \pi[1 + \text{sign}(\Delta\varphi - \pi)]\text{sign}(\Delta\varphi)\} \quad (9)$$

where $\Delta\vartheta = \vartheta_\sigma - \vartheta_\alpha$ and $\Delta\varphi = \varphi_\sigma - \varphi_\alpha$. Equations (8) and (9) yield either positive or negative signs for the purpose of ensuring the subsequence towards ϑ_σ and φ_σ along their shortest paths, respectively. Therefore, the evolution of unity tensor α is then simply following the sphere coordinate system, reads

$$d\alpha = dr\mathbf{r} + r d\vartheta\boldsymbol{\theta} + r \sin \vartheta d\varphi\boldsymbol{\phi} = d\vartheta\boldsymbol{\theta} + \sin \vartheta d\varphi\boldsymbol{\phi} \quad (10)$$

where the same rotational rate ω is applied for both angles while rotation directions are following $l(\vartheta)$ and $l(\varphi)$, as follows

$$d\vartheta = l(\vartheta)\omega d\bar{\epsilon}^p \quad (11)$$

$$d\varphi = l(\varphi)\omega d\bar{\epsilon}^p \quad (12)$$

where $\bar{\epsilon}^p$ is the increment of equivalent plastic strain. The evolution of “dummy” back stress α can be illustrated by Figure 1 below.

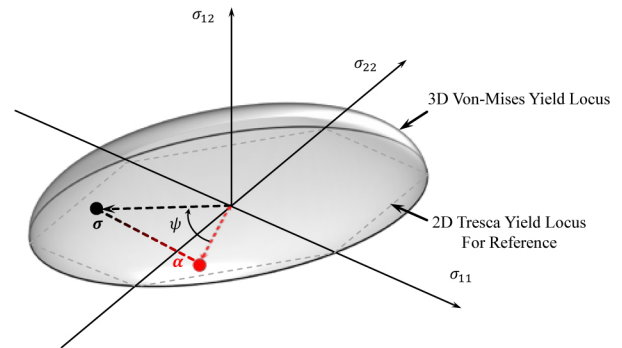


Figure 1. Schematic graph of “dummy” back stress evolution. The black dash line indicates the current Cauchy stress tensor, while the red dash line indicates an elongated “dummy” back stress tensor (its actual magnitude is unity). The latter is to rotate towards the former in the sub-plane, represented by the triangular area. Both of them are located at a von-Mises yield locus in a stress space of ($\sigma_{11}, \sigma_{22}, \sigma_{12}$).

The traditional isotropic hardening model $\bar{\sigma}_{iso}$ is compatible within this framework and should act independently of the hardening effect. The permanent softening behavior in reversal/cyclic loading

can be captured by adding a new item χ , for recovering the magnitude of equivalent stress. It is defined by a sigmoidal differential equation associated with rotational rate between both tensors and the equivalent stress from isotropic hardening, reads

$$d\chi(q, \bar{\psi}) = A_{-1} q \bar{\psi} (2 - \chi) (q - \chi) d\bar{\epsilon}^p \quad (13)$$

where $A_{-1} \geq 0$ is an optional material coefficient controlling the rate of the permanent softening. The equivalent stress recovery will be more apparent with a larger value of A_{-1} . It can be set as zero if the isotropic hardening model has an early saturated value (e.g. Voce model). q is the ratio between $\bar{\sigma}_{Iso}(0)$ and $\bar{\sigma}_{Iso}(\bar{\epsilon}^p)$ indicates the proportionality between the stress recovery and strength of isotropic hardening comparing to the initial yield stress, reads,

$$q = \frac{\bar{\sigma}_{Iso}(0)}{\bar{\sigma}_{Iso}(\bar{\epsilon}^p)} \quad (14)$$

It is noted that χ is originally set as unity and it only evolves when α is not aligned with σ , where the kinematic hardening effect occurs. $\bar{\sigma}_{Iso}(\bar{\epsilon}^p)$ can be any of the traditional isotropic hardening model, such as Swift, Voce, or experimental data column, etc..

Stress Triaxiality and Lode Angle Dependency

The dependency of stress triaxiality and Lode angle was also introduced through a correction item based on [4,18-19].

$$f_{ext}(\eta, \bar{\theta}) = [1 - c_\eta (\eta - \eta_0)^2] \left[c_\theta^s - \frac{m+1}{m} (c_\theta^{ax} - c_\theta^s) \left(\gamma - \frac{\gamma^{m+1}}{m+1} \right) \right] \quad (15)$$

where

$$c_\theta^{ax} = \begin{cases} c_\theta^t & \bar{\theta} \geq 0 \\ c_\theta^c & \bar{\theta} < 0 \end{cases} \quad (16)$$

Here c_η , η_0 , c_θ^s , c_θ^t , c_θ^c , and m are material coefficients. γ is defined as

$$\gamma = \frac{\sqrt{3}}{2 - \sqrt{3}} \left[\sec\left(\frac{\bar{\theta}\pi}{6}\right) - 1 \right] \quad (17)$$

where $\bar{\theta}$ is the normalized Lode angle parameter. It is expressed, under plane stress condition, as follows,

$$\bar{\theta} = 1 - \frac{2}{\pi} \arccos \left[-\frac{27}{2} \eta \left(\eta^2 - \frac{1}{3} \right) \right] \quad (18)$$

The material parameters were optimized to simultaneously correlate with the tested equivalent stress-strain curves under different monotonic loading conditions, if available. In the current framework,

only the coefficient c_θ^c is required since the tension/compression difference is specifically considered, while the rest of them can be set as, $c_\eta = 0$ (η_0 is thus nullified), $c_\theta^t = c_\theta^s = 1$, and $m = 0.25$.

New Model Derivation

The material governing equation can then be obtained, with a) the angle difference between Cauchy stress tensor and the “dummy” back stress tensor, b) the permanent softening item χ , and c) the stress triaxiality dependency,

$$f(\sigma) - \chi \bar{\sigma}_{Iso}(\bar{\epsilon}^p) (1 - A_3 \bar{\psi}^{A_2}) f_{ext}(\eta, \bar{\theta}) = 0 \quad (19)$$

where $f(\sigma)$ is the yield criterion, χ is the permanent softening item, and $A_2 > 0$ is the second material coefficient to control both the magnitude and the shape of the transitional area, which is generated by the normalized angle difference $\bar{\psi}$. $A_3 \in [0, 0.95]$ is the third parameter to control the initial yield stress under reversal loading condition. When $A_3 = 0$ the model degrades into a pure isotropic hardening framework. It is noted that a semi-associative plastic flow rule is applied for this model, which indicates the plastic flow is dependent on the yield criterion $f(\sigma)$ only. This model is also fully compatible with anisotropic yield criteria and fracture models. The involvement of which would be independent on calibrated material coefficients in the presented model. If the anisotropic yield criterion was applied, the isotropic-kinematic hardening behavior is not coupled with $f(\sigma)$, and thus the calibrated parameters will remain.

Model Calibration

The presented model has three essential material coefficients (A_1 , A_2 , and A_3) to be determined, after the calibration of the isotropic hardening rule. Typically this model can be calibrated using a true stress-true strain curve generated by uniaxial load with a full tension-compression-tension (TCT) or compression-tension-compression (CTC) cycle. Several optional parameters are also available to further improve the correlation results if more experimental results are provided. Specifically, uniaxial tests with multiple cycles can be applied to verify the part of the permanent softening, with the material parameters A_0 or, A_{-1} . The determination of the material parameters are given as follows:

1. A_1 can be equivalent to the derivatives of the isotropic hardening item with regards to the equivalent plastic strain, because A_1 controls the rate of the back stress evolution, and therefore the stress-strain relationship under reversal loading condition. A good estimation of A_1 can be the derivative above when the equivalent plastic strain equals to 0.2%, reads

$$A_1 = \left. \frac{d\bar{\sigma}_{Iso}}{d\bar{\epsilon}^p} \right|_{\bar{\epsilon}^p=0.2\%} \quad (20)$$

It is noted that the value of 0.2% is not the actual equivalent plastic strain but a constant value. It can be varied based on the users' calibration for the purpose of improving the correlation.