

Improved Modeling of Near-Wall Heat Transport for Cooling of Electric and Hybrid Powertrain Components by High Prandtl Number Flow

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ABSTRACT

Reynolds-averaged Navier-Stokes (RANS) computations of heat transfer involving wall bounded flows at elevated Prandtl numbers typically suffer from a lack of accuracy and/or increased mesh dependency. This can be often attributed to an improper near-wall turbulence modeling and the deficiency of the wall heat transfer models (based on the so called P-functions) that do not properly account for the variation of the turbulent Prandtl number in the wall proximity ($y^+ < 5$). As the conductive sub-layer gets significantly thinner than the viscous velocity sub-layer (for $Pr > 1$), treatment of the thermal buffer layer gains importance as well. Various hybrid strategies utilize blending functions dependent on the molecular Prandtl number, which do not necessarily provide a smooth transition from the viscous/conductive sub-layer to the logarithmic region. This work relies on the $k\text{-}\zeta\text{-}f$ turbulence model and the underlying hybrid wall treatment, which is capable of predicting the near-wall momentum and heat transfer with more fidelity, compared to the standard or low-Re variants of the $k\text{-}z\text{-}\epsilon$ turbulence model. Based on a new DNS database for turbulent flow and heat transfer in a heated pipe ($Re_\tau = 360$, $Pr = 1, 10$, and 20), a two-layer wall heat transfer model has been formulated. A priori analysis and RANS predictions of the reference heated pipe flow are encouraging, showing improvements of the near-wall heat transfer predictions with respect to accuracy and mesh independence. The potential of the proposed model in real engineering applications is demonstrated in the cooling of electric/hybrid powertrain components, by simulating heat and fluid flow in the e-motor water jacket model.

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INTRODUCTION

Numerical simulations are widely used in the design of vehicle thermal management systems. Nowadays, there exists a particular interest in development of various electric/hybrid powertrain components. Their proper cooling is of decisive importance for reliable performance under various operating conditions. This kind of industrial applications is challenging for computational fluid dynamics (CFD) as coolant flow properties give rise to a wide range of molecular Prandtl numbers, at the same time being associated with relatively low Reynolds numbers. Reynolds-averaged Navier-Stokes (RANS) computations of heat transfer involving wall bounded flows at elevated Prandtl numbers suffer from a lack of accuracy and/or increased mesh dependency. This can be often attributed to an improper near-wall turbulence modeling and the deficiency of the wall heat transfer models (based on the so called P-functions) that do not properly account for the variation of the turbulent Prandtl number in the wall proximity, [see e.g. Kays [1], Irrenfried and Steiner [2] and Antoranz et al. [3]]. As the conductive sub-layer gets significantly thinner than the viscous velocity sub-layer (for $Pr > 1$), treatment of the thermal buffer layer gains importance as well. Various hybrid strategies utilize blending functions dependent on the molecular

Prandtl number, which do not necessarily provide an adequate behavior in the buffer layer, thus leading to underestimated velocity and temperature profiles [4].

Recently, Irrenfried and Steiner [2] reported on the extensive study of a uniformly heated pipe flow featuring high Prandtl numbers by means of direct numerical simulations (DNS). They proposed a significantly improved P-function modeling concept that is applicable irrespective of the non-dimensional wall distance (y^+). Utilizing this new DNS database, the objective of the present work is to develop a comparably accurate, but simpler wall heat transfer model suitable for implementation in the RANS framework.

PRESENT MODEL

The RANS turbulence model employed in the present work is the $k\text{-}\zeta\text{-}f$ model (Hanjalić et al. [5]) which is based on the elliptic relaxation concept. The variable ζ represents the ratio v^2 / k (v^2 is a scalar property in the Durbin's $v^2\text{-}f$ model [6]), which reduces to the wall-normal stress in the near-wall region, thus providing more convenient formulation of the equation for ζ and especially of the wall boundary conditions for the elliptic function f . Compared to the

simpler two-equation eddy viscosity models, it is more robust and capable of capturing turbulent stress anisotropy near wall and predicting heat transfer with more fidelity.

The set of equations constituting the k - ζ - f model reads:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P - \varepsilon + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \quad (1)$$

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{C_{\varepsilon 1} P - C_{\varepsilon 2} \varepsilon}{T} + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] \quad (2)$$

$$\frac{\partial \zeta}{\partial t} + U_j \frac{\partial \zeta}{\partial x_j} = f - \frac{\zeta}{k} P + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\zeta} \right) \frac{\partial \zeta}{\partial x_j} \right] \quad (3)$$

$$L^2 \nabla^2 f - f = \frac{1}{T} \left(C_1 + C_2 \frac{P}{\varepsilon} \right) \left(\zeta - \frac{2}{3} \right) \quad (4)$$

with the wall boundary condition for f : $f_{\text{wall}} = \lim_{y \rightarrow 0} (-2\nu\zeta / y^2)$

Here, T represents a switch between the turbulent time scale $\tau = k / \varepsilon$ and the Kolmogorov time scale $\tau_K = (\nu / \varepsilon)^{1/2}$:

$$T = \max \left[\frac{k}{\varepsilon}, C_\tau \left(\frac{\nu}{\varepsilon} \right)^{1/2} \right] \quad (5)$$

The corresponding length scale L is obtained as a switch between the turbulent and Kolmogorov length scales:

$$L = C_L \max \left[\frac{k^{3/2}}{\varepsilon}, C_\eta \left(\frac{\nu^3}{\varepsilon} \right)^{1/4} \right] \quad (6)$$

Values of the coefficients appearing in the model equations are outlined in [Table 1](#).

Table 1. Model coefficients

C_1	C_2	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ	
0.4	0.65	$1.4(1+0.045/\zeta^{0.5})$	1.9	0.22	
σ_k	σ_ε	σ_ζ	C_τ	C_L	C_η
1.0	1.3	1.2	6.0	0.36	85

Popovac and Hanjalić [7] proposed the so-called compound wall treatment with a blending formula following the work of Kader [11], for the flow properties for which boundary conditions are required at the first near-wall grid node P:

$$\phi_P = \phi_\nu e^{-\Gamma} + \phi_t e^{-1/\Gamma} \quad (7)$$

where ' ν ' denotes the viscous and ' t ' the fully turbulent value. The variable ϕ represents the dimensionless velocity, wall shear stress, production and dissipation of the turbulent kinetic energy with the blending coefficient being dependent only on the normalized distance to the wall:

$$\Gamma = 0.01 \frac{y^{+4}}{1+5y^{+}} \quad (8)$$

The hybrid wall treatment employed here represents a somewhat simplified approach. Whereas the original compound wall treatment of Popovac and Hanjalić [7] includes the tangential pressure gradient and convection terms, the standard wall functions are used as the upper bound. Hence, the method blends the integration up to the wall (exact boundary conditions) with the high-Reynolds number wall functions, enabling well-defined boundary conditions irrespective of the position of the wall-closest computational node. This approach ensures numerical robustness which is required in industrial computations. More specific details about the model developments can be found in the reference publications [7,8,9].

The temperature wall function used in most of the CFD codes is defined assuming a constant turbulent Prandtl number ($Pr_t=0.9$) as follows:

$$T^+ = Pr_t \left(\frac{1}{\kappa} \ln Ey^+ + P \right) \quad (9)$$

with the so-called P-function proposed by Jayatilke [10]. It relies on numerous experimental data, being sensitive to variations of the molecular Prandtl number. The present hybrid wall heat transfer model employs the originally proposed blending function of Kader [11], which is also a function of the Pr number:

$$\Gamma = 0.01 \frac{(Pr y^+)^4}{1+5Pr^3 y^+} \quad (10)$$

The performance of the k - ζ - f turbulence model is illustrated by [Figures 1](#) and [2](#) which compare the predicted dimensionless velocity and temperature profiles against the recently published DNS results of Irrenfried and Steiner [2]. A turbulent, uniformly heated pipe flow ($Re=5300$) at various Prandtl numbers is computed by means of RANS using simple, practically axisymmetric configurations with different mesh resolutions considered throughout this investigation. Constant fluid properties and boundary conditions are specified according to the reference DNS.

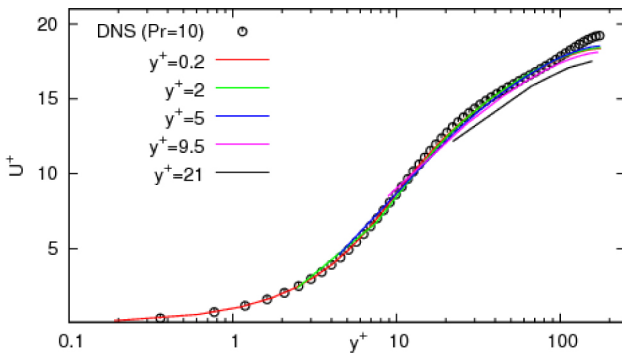


Figure 1. Comparison of the predicted non-dimensional velocity profiles for various mesh resolutions against the DNS data [2]

At this point, it is important to emphasize that the standard coefficients in the velocity log-law in [equation \(9\)](#) have been intentionally used ($\kappa=0.41$ and $E=9.0$) without modifications for low Reynolds number flows [2], so as to retain its applicability to flows at moderate and higher Reynolds numbers. Consequently, the U^+ profile for the mesh with $y^+=21$ is slightly under-predicted, while all other mesh resolutions appear to be sufficient to properly reproduce the reference DNS results. Provided that a fine mesh is used, (e.g. $y^+=0.2$), the model is evidently capable of capturing the temperature distribution for all Prandtl numbers, as depicted in [Figure 2](#).

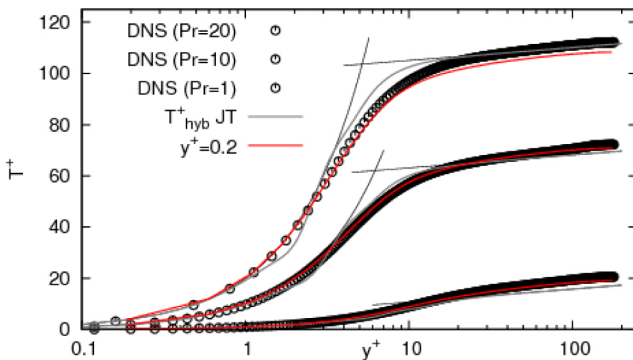


Figure 2. Predicted T^+ evolutions compared to the DNS data for $Pr=1, 10$ and 20 [2]

For the sake of an a priori analysis, the hybrid Jayatilleke model is plotted as a reference, revealing its advantages and potential issues. The P-function of Jayatilleke is expected to be a suitable model for higher Prandtl number flows (10 and 20), as long as a proper wall y^+ (within the logarithmic layer) is ensured. However, by careful inspection of [Figure 2](#), deficiency of the model is observed for $Pr=1$, representing also an important outcome, as far as such a low Reynolds number is concerned. For $Pr \gg 1$, the conductive sub-layer gets substantially thinner than the viscous velocity sub-layer, necessitating a more sophisticated treatment of the thermal buffer layer. This is particularly due to the fact that hybrid strategies utilize the Pr-dependent blending functions, which do not necessarily provide a smooth transition from the viscous/conductive sub-layer to the logarithmic region. With increase in Prandtl number, performance of the hybrid model in the thermal buffer region is expected to deteriorate, in particular if the first numerical grid point falls around $y^+=2$ and $y^+=8$. This near-wall region is of main interest in the present

work that aims at improvement of heat transport modeling of flows featuring high Prandtl numbers, keeping the original hybrid wall treatment for turbulence [12].

TWO-LAYER WALL HEAT TRANSFER MODEL

The P-function based modeling concept has been recently revisited by Irrenfried and Steiner [2]. The authors addressed deficiencies of the P-function of Spalding [13] and proposed an improved model applicable for all y^+ values down to the wall. This model, derived using comprehensive DNS data for a turbulent heated pipe flow at elevated Prandtl numbers, is briefly outlined by the following set of equations:

$$T^+ = Pr_{t,\infty} (U^+ + P) \quad (11)$$

$$U^+ = \frac{1}{\kappa} \ln y^+ + \beta \quad (12)$$

$$P = \int_0^{U^+} \left(\frac{1 + \frac{\nu_t}{\nu}}{\frac{Pr_{t,\infty}}{Pr} + \frac{Pr_{t,\infty}}{Pr} \frac{\nu_t}{\nu}} - 1 \right) dU^+ \quad (13)$$

$$\frac{dU^+}{dy^+} = \frac{2}{1 + \sqrt{1 + 4l_m^{+2}}} \quad (14)$$

with $Pr_{t,\infty}=0.9$, $\kappa=0.34$, $\beta=4.5$ and l_m^+ representing van Driest's mixing length as described in [2]. Using a more advanced formulation for turbulent viscosity ratio and the DNS-based, enhanced model for turbulent Prandtl number (Irrenfried and Steiner [2]), [equation \(13\)](#) can be numerically integrated to obtain the distribution of $P(U^+)$. In order to cover the entire y^+ range, it is essential that the velocity is computed from the expression (14) instead of the log-law given by [equation \(12\)](#). In terms of implementation and robustness, a model that requires numerical integration is not appealing for the majority of the computational fluid dynamics (CFD) codes. In what follows, utilizing the same DNS database, a simpler two-layer model will be presented, whereas the numerically integrated T^+ profile is going to serve as an additional reference to ensure comparable accuracy.

According to the foregoing discussion, it is crucial to provide a suitable model of T^+ in the vicinity of the wall (accounting for the Prt variation shown in [Figure 3](#)) by means of analytical integration of the following equation:

$$dT^+ = \frac{1}{\left(\frac{1}{Pr} + \frac{1}{Pr_t} \frac{\nu_t}{\nu}\right)} dy^+ \quad (15)$$