

A Framework for Quantifying Measurement Uncertainties and Uncertainty Propagation in HCCI/LTGC Engine Experiments

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ABSTRACT

In this paper, a framework for estimating experimental measurement uncertainties for a Homogenous Charge Compression Ignition (HCCI)/Low-Temperature Gasoline Combustion (LTGC) engine testing facility is presented. Detailed uncertainty quantification is first carried out for the measurement of the in-cylinder pressure, whose variations during the cycle provide most of the information for performance evaluation. Standard uncertainties of other measured quantities, such as the engine geometry and speed, the air and fuel flow rate and the intake/exhaust dry molar fractions are also estimated. Propagating those uncertainties using a Monte Carlo simulation and Bayesian inference methods then allows for estimation of uncertainties of the mass-average temperature and composition at IVC and throughout the cycle; and also of the engine performances such as gross Integrated Mean Effective Pressure, Heat Release and Ringing Intensity. Throughout the analysis, nominal values for uncertainty inputs were taken from a well-characterized engine test facility. However, the analysis did not take into account the calibration practice of experiments run in that facility and the resulting uncertainty values are therefore not indicative of the expected accuracy of those experimental results. A future study will employ the methodology developed here to explore the effects of different calibration methods on the different uncertainty values in order to evaluate best practices for accurate engine measurements.

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INTRODUCTION

Accurate diagnostics in engine experiments are challenging due to a number of issues including high temperatures and pressures, fast transients (*e.g.* the main combustion event takes place within milliseconds), sensitivity to temperature non-uniformities (in both time and space), limited room for instrument installation, complex geometries of manifolds and combustion chambers that can create acoustic waves, large numbers of chemical species (including short-lived radicals), and, critically, the intrinsically turbulent nature of the combustion process. These issues extend to modeling and simulation efforts that need precise boundary and initial conditions as well as experimental outputs to verify and validate results; and rely on numerical methods and models that are highly non-linear.

Therefore, quantifying measurement uncertainties is extremely important for the engine community at large in order to build more confidence in both experimental and modeling results. This work aims at providing an analysis framework of measurement uncertainties due to both the systematic and random errors in engine measurements and how they propagate when calculating derived quantities, such as the mass-average temperature and mixture composition and engine performance metrics.

Homogenous Charge Compression Ignition (HCCI), and similar Low-Temperature Gasoline Combustion (LTGC) strategies offer promising performance for internal combustion engines, including

higher efficiencies (through lean combustion) and lower emissions (soot, NO_x), discussed in detail in recent reviews [1], [2]. In such engines, the fuel and the oxidizer are well mixed and then compressed until spontaneous ignition. The auto-ignition properties of the mixture as a function of initial and operating conditions need to be understood in order to provide optimized control, *i.e.* ignition and heat release rate at desired timing for best performance. Performance is generally measured using metrics such as gross Indicated Mean Effective Pressure (IMEP_g), Ringing Intensity (RI) and timing of 50% heat release (CA_{50}). Many parametric studies have been carried out over the years, examining the sensitivity of engine performance to intake pressure [3], [4], [5], [6], intake temperature [7], [8], exhaust gas recirculation [9], [10], [11], equivalence ratio [12], [13], swirl [14], [15], compression ratio [16], [17], engine speed [18], piston bowl geometry [19], valve timing [20], stratification [21] [22], fuel cetane number [23], and fuel [24], [25].

The overall approach of this paper is to develop a methodology to estimate the uncertainty of derived quantities (mass average temperature, mixture composition, heat release, ringing, etc.) by determining the uncertainties of the measured experimental quantities (composition at intake, pressure, engine geometry, engine speed) then propagating them using a Monte Carlo approach. First a description of the HCCI/LTGC experimental setup used as baseline for this study is given in [Section 1](#). [Section 2](#) contains a thorough estimation of the measurement uncertainties for the in-cylinder pressure, whose signal

provides much of the information used for analyzing combustion quality, as well as other experimental quantities such as chamber geometry and intake and exhaust molar fractions. The measurement uncertainties are propagated in Section 3, enabling estimation of the uncertainties of many quantities of interest.

1. DESCRIPTION OF THE EXPERIMENTAL BASELINE

In order to provide concrete examples of the uncertainty methodology, experimental parameters were taken from a well characterized HCCI/LTGC engine test facility assembled and extensively used by Dec and co-workers at Sandia National Laboratories [3], [25]. This facility was chosen as a baseline because it represents a very well characterized experimental setup. As the goal of the present work is to lay out a methodology, specific calibration practices and the performance of the actual transducers used by Dec and co-workers are *not* included here. They will be analyzed in a future study to understand the reduction in overall uncertainty that can be obtained by using measured values of key individual uncertainties (such as linearity) for the specific transducers used, and the calibration practices used in the laboratory.

The combustion chamber is derived from a Cummins B-series six-cylinder diesel engine with a displacement of 0.98 liters/cylinder, a flat firedeck, a bore of 102 mm, a stroke of 120 mm and a connecting rod length of 192 mm (see schematic of the engine facility on Fig. 1). The compression ratio (CR) for this configuration is 14:1.

The cylinder pressure is measured using two sensors; a Taber 2911 [26] intake strain gauge pressure transducer with a range from 0 to 200 psi (absolute) and a AVL QC34C [27] piezoelectric in-cylinder pressure transducer with range 0 to 200 bar (relative). The intake pressure transducer is located near the intake manifold, right downstream of the auxiliary heater shown in Fig. 1.a. and is used to accurately measure the pressure during the intake stroke. The piezoelectric in-cylinder pressure transducer is located inside the combustion chamber, mounted nearly flush with the firedeck, approximately 42 mm off center.

The intake pressure transducer uses a bonded foil strain gauge technology, where a metal foil pattern is bonded to a sensing element to which four strain gauges are bonded in an electronic arrangement called a Wheatstone bridge. As pressure changes deform the device, the foil patterns are stretched, changing their electrical resistance in proportion to the deformation. The in-cylinder pressure transducer is of piezoelectric type and converts the mechanical strain from pressure changes into an electric charge, which occurs due to crystal reorientation under stress creating an internal polarization.

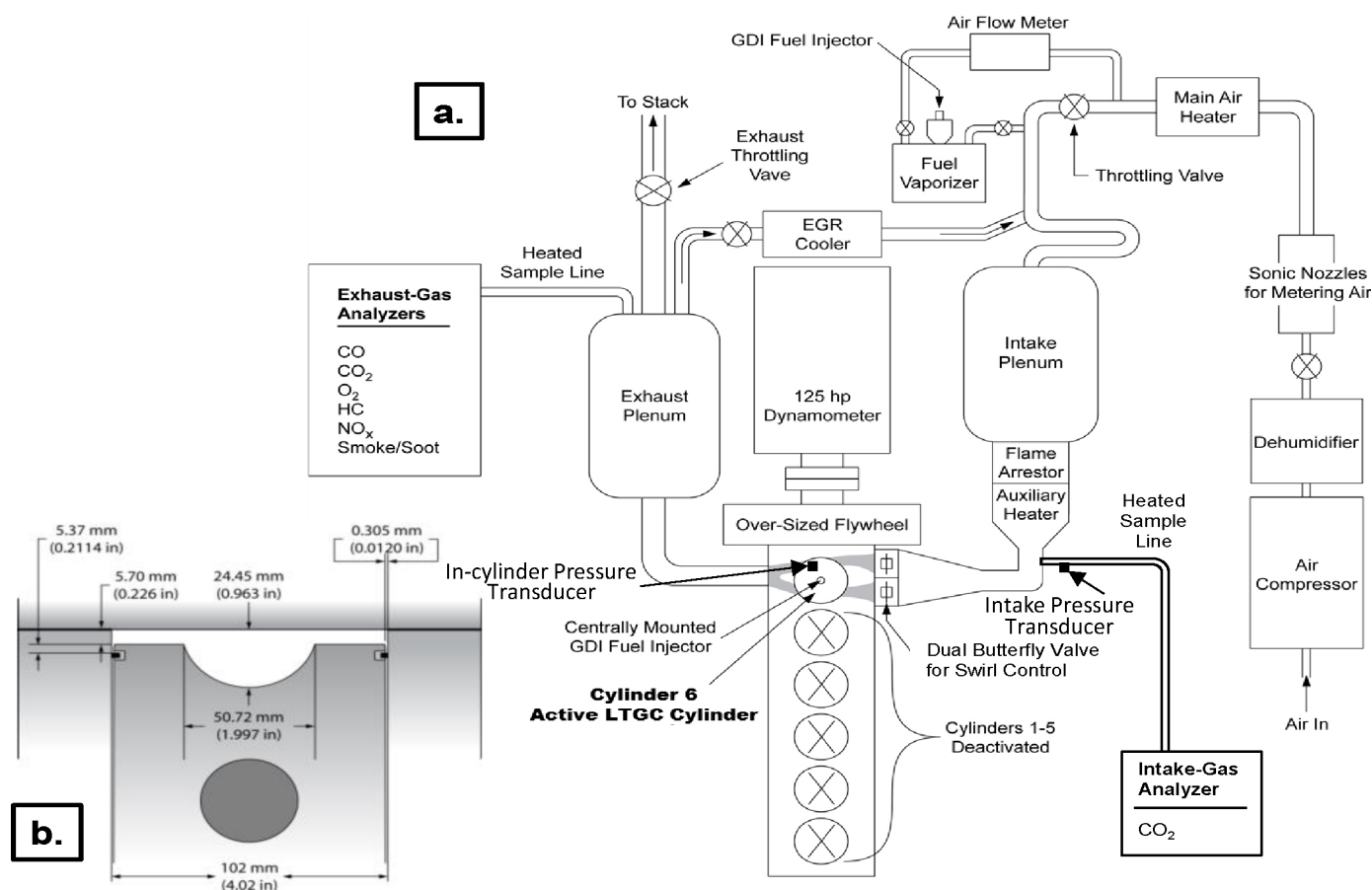


Figure 1. Schematic of the HCCI/LTGC engine facility (a), and combustion chamber geometry of the CR=14:1 piston at Top Dead Center (b). The EGR system is equipped with a gas/liquid separator to remove the condensed water.

Piezoelectric transducers are commonly used due to their high-frequency response capacity, small size, low weight, and low sensitivity to environmental conditions [28]. However, rapid changes in the temperature of the quartz-sensing element change the offset voltage of the transducer output. In addition, piezoelectric transducers respond to pressure differences by outputting a charge referenced to an arbitrary ground, which is converted to a voltage by a charge converter. Therefore, pressures measured by piezoelectric transducers must be referenced to a known (absolute) pressure at some point in an engine cycle, which is done through a process called pegging. Different methods have been used for pegging, such as outlet pressure referencing, two-point referencing, three-point referencing, or least-squares methods. It has been shown that it is preferable to peg when the transducer output is the least affected by the thermally transient nature of the engine cycle, and that referencing the transducer output at inlet bottom dead center to intake manifold pressure provides good accuracy [29]. This latter method is used by Dec on this experimental test bench, by matching the averages of the two transducers around the Bottom Dead Center (located at -180 Crank Angle Degrees, or "CAD") at each cycle. The average Bottom Dead Center (or "BDC") value from the intake transducer is calculated over 5 consecutive measurements while the average from the in-cylinder transducer is calculated over 9 consecutive measurements, during which the pressure is nearly constant. Firing occurs around Top Dead Center (or "TDC") whose timing for this paper is set at 0 CAD. Intake valves are closed around -155 CAD and exhaust valves open around 120 CAD.

The piezoelectric transducer exhibits high output impedance and low signal levels. Therefore, it requires the use of special equipment including low-noise coaxial cables and charge amplifiers in the measurement chain. A Kistler type 5010 charge amplifier [30] is used in the setup described here. Pressure analog signals (from both transducers) are converted to digital signals using a DT3016 data acquisition board with a 12 bit resolution with throughput rates up to 250 kHz [31]. After data acquisition, raw data are first filtered using a low-pass Gaussian progressively cut filter (from 2.5 to 3.9 kHz) then averaged over the 100 cycles. A Gaussian roll-off function is used in order to avoid overshoots in the pressure signal (Gibbs effect [32]). Filtering is a key issue because the heat release calculation relies on the derivative of the in-cylinder pressure signal and differentiation increases signal noise [33].

The engine speed is maintained constant using an electric motor and is measured with a tachometer. The position of the engine crank (or crank angle) is expressed between 0 and 360 degrees and is measured every 0.25 degrees using an infrared encoder. The flow of fuel is measured using a MAX P002 [34] piston flow meter while the flow of air is controlled using a sonic nozzle together with an upstream Omega PX213 [35] pressure transducer. Molar fractions of gaseous species are measured using a Non-Dispersive Infra-Red CAI 600 series sensor [36] for CO₂ (range 0 to 8.5% at intake and 0 to 18% at exhaust) and CO (range 0 to 10,000 ppm, measured at exhaust only), a Rosemount Analytical Inc. Model 755R magnetic detector [37] for O₂ (range 0 to 25%, measured at exhaust only) and a Flame Ionization Detection CAI 600 series analyzer [38] for total hydrocarbons, (THC) (range 0-3,000 ppm CH₄ equivalent, measured at exhaust only).

2. MEASUREMENT UNCERTAINTIES QUANTIFICATION

2.1. Assumptions

In this work, estimations of the uncertainties of measurements were performed in compliance with the EURACHEM/CITAC Guide [39] and the Guide to the Expression of Uncertainty in Measurement [40], that defines the term uncertainty as "A parameter associated with result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. This parameter can be expressed as a standard deviation (or a given multiple of it), or the width of a confidence interval".

The uncertainty of a measurement generally contains many components. The recommended approach in estimating the overall uncertainty (as used in Lee et al. [41] and throughout this paper) is summarized below. First, it is necessary to consider each source of uncertainty and treat it separately to obtain the contribution from that source. Each of the separate contributions to uncertainty is referred to as a standard deviation. Second, the total uncertainty for a measurement result, termed the combined standard uncertainty, $u_c(y)$, is estimated as a standard deviation equal to the positive square root of the total variance obtained by summation of all the single uncertainty components - independence assumption. At last, an expanded uncertainty provides an interval within which the value of the measurand is believed to lie with a certain level of confidence and is generally obtained by multiplying $u_c(y)$ by a coverage factor k . The choice of the factor k is based on the level of confidence desired.

Each identified source of uncertainty must be explicitly quantified. One difficulty in evaluating the uncertainty is determining the distribution of values observed by the sensor manufacturer when measuring the systematic errors, which is rarely reported. Indeed, in order to be combined, all uncertainty contributions must be expressed as standard deviations, which often involves conversion from some other measure of dispersion. When the uncertainty component is evaluated experimentally from the dispersion of repeated measurement, its limits may be written as $\pm a$. Depending on its distribution (rectangular/uniform, triangular or normal with a specified confidence interval), the standard deviation would then be either $a/\sqrt{3}$, $a/\sqrt{6}$ or $a/\sqrt{3.84}$ (assuming a 95% confidence interval), respectively [39]. For this study, we assume that when not available from the manufacturer or other literature, distributions are normal with a 95% confidence interval, except for resolution errors (which are assumed to have an uniform distribution). Random errors (*e.g.* cycle-to-cycle variations) are estimated based on the raw recorded experimental data and measured standard deviations are directly used.

2.2. Uncertainty of the Measurement of In-Cylinder Pressure

In-cylinder pressure is one of the most important quantities to measure accurately on an engine. It represents a relatively homogeneous measurement (as opposed to *e.g.* temperature) and is used to calculate most of the engine's performance metrics. Therefore, recording the undistorted and unbiased cylinder pressure from the transducer is critical.

2.2.1. Identifying Uncertainty Sources

The various sources of uncertainties that affect the measurement of an experimental quantity can be illustrated with a cause-and-effect (or fishbone or Ishikawa [42]) diagram, as shown in Figure 2 for the in-cylinder pressure measurement. As described above in Section 1 regarding pegging, the measurement of in-cylinder pressure depends on both the measurement of pressure at BDC and the cylinder pressure. Two types of errors are considered, *systematic* and *random* errors. Systematic errors affect the accuracy of a measurement, and can be reduced using careful calibration. Random errors, on the other hand, affect the precision of the measurement and can be improved by repeated measurements. The sources of random errors can be difficult to identify, although they can be quantified, due to non-deterministic variations in the measurement process. Also, they are considered “two-sided” errors, because, in the absence of other types of errors, repeated measurements yield results that fluctuate above and below the true or accepted value. Figure 2 shows systematic errors for both transducers above the horizontal line while random errors are shown below that line. The identified systematic errors labeled in Figure 2 are briefly described below. More detailed descriptions and calculations of uncertainties are provided in the subsequent sections.

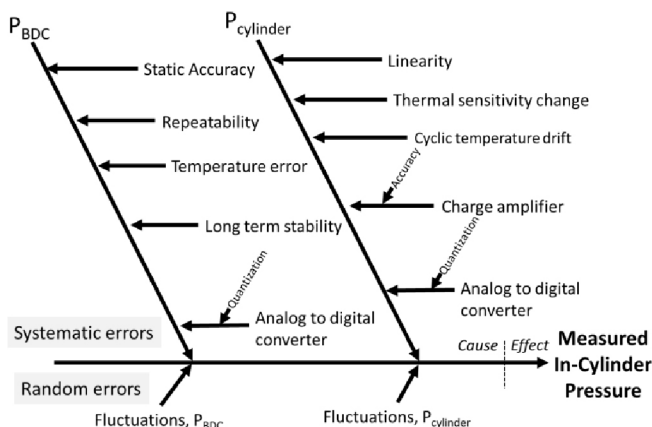


Figure 2. A cause-and-effect diagram for the measurement of in-cylinder pressure in an HCCI/LTGC engine using a “pegging” technique.

The systematic errors for the intake pressure transducer such as static accuracy, repeatability, long-term stability and temperature error band provided by the manufacturer are listed. Repeatability characterizes the extent to which the output signals for successive measurements of the same pressure remains constant; long-term stability refers to a slowly changing output signal over time (in practice, the long-term stability error may be reduced by frequent calibration of the transducer). For the transducer studied here, the static accuracy combines the effects of non-linearity (using a Best Fit Straight Line method), hysteresis and repeatability, and the result is given as a percent of the Full Scale Output. The temperature error band represents the sensitivity of the transducer to variations in temperature. Temperature changes cause the expansion and contraction of the sensor materials, fill fluids, and housing. Lastly, the data acquisition system adds uncertainties when converting the analog signal into a digital one via quantization error.

A similar approach is used for the piezoelectric relative pressure transducer, except no bias error is considered, as the combined uncertainty from the intake absolute pressure transducer is the biased error for the overall pressure measurement. The linearity term refers to deviation that might occur from a constant (ideal) ratio of pressure to electrical charge, due to the design of the housing, elastic deformation, or friction. The thermal sensitivity change term classifies how the average temperature at the mounting position is influencing the sensitivity of the sensor. Other thermal errors result from the contraction and expansion of the piezoelectric pressure transducer diaphragm due to cyclic exposure to large temperature variation during the combustion events, which cause the force on the quartz to be different from that applied by the cylinder pressure alone for part of the cycle. This effect is believed to occur mostly during combustion when the flame is impinging on the diaphragm and though the expansion stroke, underestimating the pressure by up to 0.5 bar for conditions with high-temperature flames, as found in SI engines [43]. This phenomenon is also called cyclic temperature drift and often reported as the maximum misreading within one cycle due to this thermal effect. Data acquisition and signal conditioning errors (charge amplifier, analog to digital converter) are finally considered in the systematic errors category. For this analysis, the quantization (or resolution) error was the most important contributor, with all other data acquisition errors assumed negligible. Specifically, the gain error and thermal noise were addressed by setting the gain to 1 [44] and using shielded twisted pairs, respectively. Spurious tone errors were reduced by double-buffering data and integral and differential non-linearity errors are insignificant for this digital converter (+/- 4 Least Significant Bit). More details on data acquisition and signal conditioning errors are available in the manufacturer’s specifications [31].

Random errors, located below the horizontal line in Figure 2, include all the fluctuations that can affect the measurements even when the engine is operating at steady conditions. The distinction between random errors and repeatability error is that repeatability errors are caused by phenomena contained within the measurement device and random errors are external, i.e. caused by fluctuations in the phenomena being measured. Scattering caused by random fluctuations can have a large number of sources, including, but not limited to, variations in the amount of air and fuel supplied [45], injection timing, exhaust gas recirculation [46], trapped air mass and composition, turbulence in the cylinder; all of which in turn affect the combustion process, amplifying the small variations.

Other sources of uncertainty not considered in the scope of this work include the following: passage restriction and resonance (if the transducer is not exactly flush mounted) [47]; twist in the crankshaft, delay in cylinder volume or crank angle signal [47], combustion chamber resonance [48], and filtering errors.

2.2.2. Quantifying for the Measurement of the Absolute Intake Pressure

In this section, the uncertainty terms related to the absolute BDC pressure (“ P_{BDC} ”, see left side of Figure 2) are described. The standard deviations of each term are then summarized in Table 1.

The static accuracy and the repeatability are given as 0.25% and 0.075% of the Full Scale Output (FSO) [26]. The FSO is defined as the signal difference between the maximum and the minimum stimulus, resulting in this case as FSO=200 psi. Therefore, the static accuracy and repeatability errors are 0.5 and 0.15 psi, respectively.

The temperature error band is given over a compensated temperature range that is the range of temperature within which the transducer output is electronically corrected for errors induced by changes to materials or bridge resistance due to temperature. The temperature error band is referenced to a temperature at which it becomes zero. For this transducer, it is specified to be within 1.5% of the FSO of the compensated range of -20 to 76 °C, and referenced at 28 °C [26]. Assuming the transducer is kept at a constant temperature of 20 °C, the temperature error coefficient is calculated as:

$$\begin{aligned} \text{Temperature error coefficient} &= \pm 1.5 \left(\frac{20 - 28}{76 - 28} \right) \% \\ &= \pm 0.25\% \end{aligned} \quad (1)$$

The error due the temperature is thus 0.25% of the FSO, or 0.5 psi.

The long-term stability is given as 0.1% of the FSO per annum. Assuming a three-year-old transducer (it is considered that such a transducer has a ~5-year typical life), this term becomes 0.3%, i.e. 0.6 psi.

The signal conditioning for the data acquisition of both intake and in-cylinder pressure transducers are discussed in [section 1](#) describing the range of values and units between each step. The main source of error introduced by the analog to digital converter is the quantization error, which is computed as half of the resolution such as:

$$\begin{aligned} \text{Quantization error} &= 0.5 * \text{Resolution} \\ &= 0.5 * \left(\frac{V_D}{V_S \cdot 2^{(n-B)}} E \right) \end{aligned} \quad (2)$$

where V_D is the full-scale input voltage range of the converter, V_S is the full-scale output voltage range of the signal, E is the FSO of the transducer in engineering units, n is the bit resolution of the converted signal and B is the polarity factor (B=1 if the range is bipolar, =0 if unipolar).

The following values are used: $V_D = V_S = 10$ V, $n=12$ bits, $B=1$ and $E = 200$ psi. Therefore, the resolution of the intake transducer is 0.1 psi, hence a quantization error of 0.05 psi, or 0.32 kPa (see also [Figure 3](#) for a comparison of the resolutions between the 2 transducers).

Pegging (described in [Section 1](#)) is carried out at every combustion cycle using an arithmetic average over the five data points closest to BDC timing (i.e. from -180.5 to -179.5 CAD). The uncertainty of the BDC pressure measurement due to random fluctuations can thus be written as the pooled variance [41]:

$$u(\text{fluctuations}) = \sqrt{\frac{\sum_{i=1}^5 s_i^2}{5}} = 0.024 \text{ kPa} \quad (3)$$

where s_i is the measured standard deviation of the i -th data point ($i=1$ corresponds to -180.5 CAD, $i=5$ to -179.5 CAD) and m is the sample size (here 100 engine cycles).

[Table 1](#) lists the errors that need to be taken into account for the measurement of the absolute intake pressure, including the measurements used for the calibration, the intrinsic error of the transducer (from the manufacturer's specifications) and random fluctuations, due, for example to cycle-to-cycle variability. The last column shows standards deviations in the same units (kPa) so that they can be compared, showing that that the long-term stability, the temperature error, and the static accuracy contribute the most to the overall intake pressure uncertainty

Table 1. Uncertainties considered for the intake absolute pressure transducer (Taber 2911 with range 0-200 psi, absolute).

Uncertainty Source	Quoted Error	Assumed Distribution	Standard Deviation [kPa]
Static accuracy	±0.5 psi	Normal (95%)	1.76
Repeatability	±0.15 psi	Normal (95%)	0.52
Temperature error	±0.5 psi at 20 °C	Normal (95%)	1.76
Long term stability	±0.6 psi	Normal (95%)	2.11
Quantization	±0.05 psi	Uniform	0.18
Random fluctuations		Normal (95%)	0.024

The combined overall uncertainty for the measurement of the intake pressure is finally written as:

$$\begin{aligned} u_c^2(P_{\text{intake}}) &= u^2(\text{static accuracy}) \\ &\quad + u^2(\text{repeatability}) \\ &\quad + u^2(\text{temperature error band}) \\ &\quad + u^2(\text{long term stability}) \\ &\quad + u^2(\text{quantization}) \\ &\quad + u^2(\text{fluctuations}) \end{aligned} \quad (4)$$

Therefore:

$$u_c(P_{\text{intake}}) = 33.1 \text{ mBar} = 3.3 \text{ kPa} \quad (5)$$

According to the guidelines for evaluating and expressing the uncertainty of measurement results [40], a coverage factor of $k=2$ for a 95 % confidence level is recommended if the degrees of freedom exceed 10. Since the effective degree of the freedom for the intake pressure measurement is 499 (from random fluctuations), the expanded uncertainty with a 95% confidence interval is given as:

$$u_{\text{expanded,95\% C.I.}}(P_{\text{intake}}) = 2 * u_c(P_{\text{intake}}) = 6.6 \text{ kPa} \quad (6)$$

2.2.3. Quantifying Uncertainties for the Measurement of the In-Cylinder Pressure

The combined overall uncertainty of the measurement of the intake pressure is then used in the estimation of the overall uncertainty of the measurement of the in-cylinder pressure.

Going downstream from the true in-cylinder pressure, P_{cylinder} , on the cause-effect diagram (see Fig. 2), the first term is the linearity of the piezoelectric sensor determined by means of calibration, and thus depends on that method. For this sensor, the linearity error is the maximum deviation of the pressure sensor characteristic from the “best straight line with forced zero point”, *i.e.* the deviation from a constant ratio of pressure to electrical charge. This ideal ratio is the center line which travels through the zero point and lies between two parallel straight lines that enclose the pressure sensors characteristic as closely as possible.

The thermal sensitivity change is given by the manufacturer as a percentage per °C of the nominal sensitivity. The maximum change of sensitivity due to temperature of the sensor is relative to the calibration temperature and is also a function of the pressure. Since the piezoelectric transducer is calibrated in house using the same coolant temperature (~20 °C) as used during the experiment, this term is considered negligible.

The cyclic temperature drift specified by the manufacturer and used in this analysis was measured on a direct injection Diesel engine at 1300 rpm and an Indicated Mean Effective Pressure of 7 bar [27]. Although the manufacturers value is used here for the initial analysis, it should be noted that the actual uncertainty due to this effect in an HCCI engine is likely much smaller because HCCI combustion temperatures at the conditions being investigated are much lower (~1700K) than diesel flame temperatures, which can exceed 2500 K. Cyclic temperature drift can be mitigated by shielding the transducer diaphragm with a thin, insulating layer of room temperature vulcanization glue, a common practice in many engine laboratories including the facility that serves as the baseline for the investigation. Also, the cyclic temperature drift error is applied evenly throughout the cycle: it is considered to be independent of the temperature in the cylinder.

The signal conditioning for the data acquisition of the in-cylinder pressure is discussed in Section 1 with the range of values and units between each step. The piezoelectric sensor has an embedded charge converter that directly outputs data in pico-Coulomb. A charge amplifier then converts the signal to an analog voltage that is converted to a digital signal by an Analog to Digital Converter (ADC) to be stored on the computer. The charge amplifier has accuracy errors (reported to be ±0.5% of the pressure by the manufacturer) while the main source of error introduced by the ADC is the quantization error. Using equation (2) with E equal to 20,000 kPa, the resolution is found to be 9.6 kPa for the in-cylinder transducer. This calculation can be verified by observing the measured pressure from

both transducers, as can be seen on Figure 3 where a comparison between single cycle data acquired by the two transducers near BDC is shown.

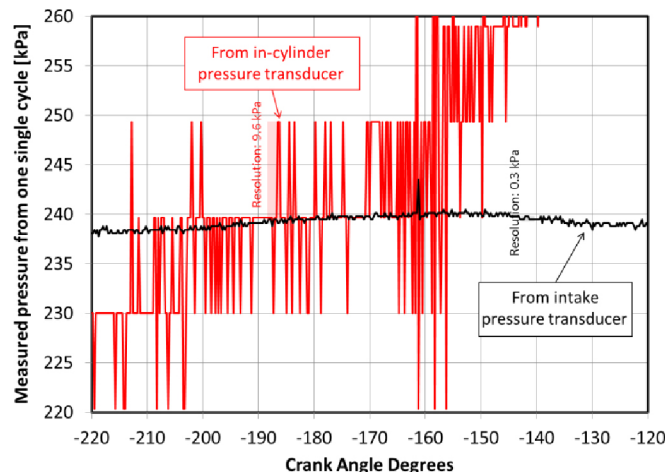


Figure 3. Comparison of the resolutions between the measured pressure from the in-cylinder and intake transducers from a single cycle around BDC.

Table 2. Uncertainties for the systematic errors of the in-cylinder pressure transducer (piezoelectric, AVL QC34C with range 0-200 bar, relative).

Uncertainty Source	Quoted Error	Assumed Distribution	Standard Deviation [kPa]
BDC Pressure	See Section 2.2.1	Normal (95%)	3.3 (from Eq. (5))
Linearity	±0.5 bar	Normal (95%)	25
Thermal sensitivity change	negligible		negligible
Cyclic temperature drift	±0.3 bar	Normal (95%)	17.3
Charge amplifier accuracy	±0.5% of Pressure	Normal (95%)	0.5 to 30
ADC resolution	±0.5*9.6 kPa	Uniform	2.8

The random component of the uncertainty for the measurement of the in-cylinder pressure is estimated by calculating the standard deviation among 100 consecutive cycles at a stable experimental operating point. Figure 4 shows the measured standard deviation of both motored (with two separate sets of measurements taken on consecutive days) and fired cycles in order to understand better the signal fluctuations with and without combustion. The observed random fluctuations for the motored data can be attributed mainly to variations in air mass flow and valve timing, while for the fired case they can additionally result from variations in fuel mass flow, intake temperature and composition, and combustion phasing and duration. We can see that the measured standard deviations are very similar between the motored and fired cases until ignition (crank angle less than 0 degree), including during the intake around -155 crank angle degrees. The standard deviation of the fired case then takes off at ignition (with a maximum at 800 kPa which is not shown for clarity), but quickly drops to 50 kPa at 20 crank angle degrees and becomes only 20% higher than the motored case (6.5 vs. 5.5 kPa) after the exhaust (~120 crank angle degrees). The slightly larger fluctuations during the last third of the fired cycle are attributed to combustion-related effects (likely, temperature) on the piezoelectric transducer.

pressure transducer. The slightly increased fluctuations are cancelled out during the next cycle due to pegging (described in [Section 1](#)) at Bottom Dead Center (-180 crank angle degrees).

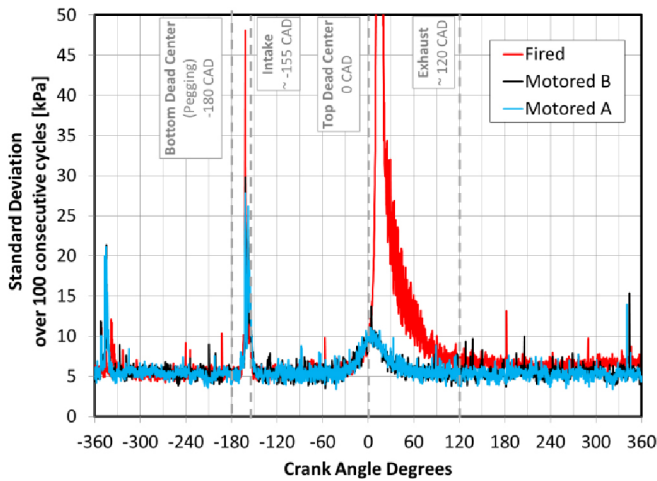


Figure 4. Comparison between the raw measured standard deviation over 100 consecutive cycles in 2 motored conditions ("Motored A" and "Motored B", measured one day apart) and in fired conditions ("Fired"). The standard deviation of the "Fired" case peaks at ~800 kPa, but is not shown here for clarity.

Using the pooled-standard deviation equation [39], the uncertainty due to the fluctuations is estimated to be around 0.5 kPa ($=5 \text{ kPa}/\sqrt{100}$) for most of the cycle with peaks near ignition and intake valve closure of approximately 80 kPa and 5 kPa, respectively.

Comparing fluctuations with the systematic uncertainties listed in [Table 2](#), it appears that the main sources of uncertainties for the measurement of the in-cylinder pressure are the linearity and the cyclic temperature drift (systematic errors) during most of the cycle, except near the point of maximum pressure where the random fluctuations (random error) become the largest source of uncertainty.

Following the cause-and-effect diagram on [Figure 2](#), we can derive the combined overall uncertainty of the in-cylinder pressure as:

$$\begin{aligned}
 u_c^2(P_{\text{cylinder}}) &= u^2(P_{\text{BDC}}) + u^2(\text{linearity}) \\
 &+ u^2(\text{cyclic temperature drift}) \\
 &+ u^2(\text{charge amplifier accuracy}) \\
 &+ u^2(\text{quantization}) + u^2(\text{random fluctuations})
 \end{aligned}
 \quad (7)$$

[Figure 5](#) shows the contribution of each source of uncertainty in absolute terms to the combined overall uncertainty for the measurement of cylinder pressure (bottom) and the corresponding relative combined overall uncertainty (top). The absolute combined overall uncertainty varies from 30 kPa (until -45 CAD) to 86 kPa at the maximum pressure. The contribution of each uncertainty source is represented as a percentage to the total combined absolute uncertainty value (bottom figure). As shown, the linearity and temperature drift uncertainties account for more than 90% of the overall uncertainty until except in the -45 to +45 CAD range, when the charge amplifier accuracy (white area) then the experimental fluctuations (black area)

becomes increasingly predominant near the top dead center and the combustion event. Quantization and intake pressure uncertainties play a negligible role throughout the cycle.

This analysis should be balanced with consideration of the relative combined overall uncertainty (top of [Figure 5](#)) that shows that the measurement uncertainty has the largest impact near the intake stroke (-155 CAD, ~13%), where the measured pressure is rather low (~240 kPa), as opposed to the top dead center (<0.5%), where the pressure is much larger (~11,000 kPa).

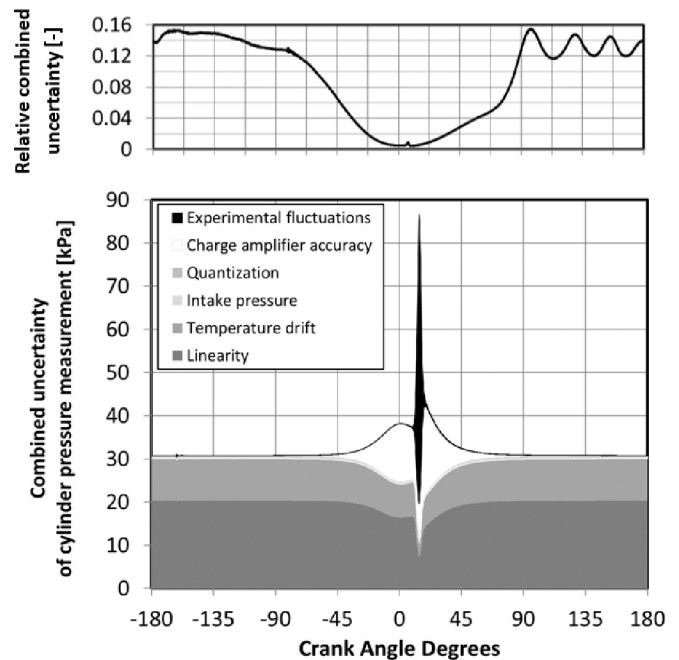


Figure 5. Combined uncertainties for the measurement of in-cylinder pressure during the closed cycle: relative (top) and absolute (bottom) contributions. The contributions from each of the 6 sources of uncertainty (5 systematic errors - see [Table 2](#), and 1 experimental fluctuation error) are represented on a percentage basis using different colors.

The effective degree of freedom for the measurement of in-cylinder pressure is 99 (100 measurements are available every 0.25 crank angle), therefore the expanded uncertainty at a 95% confidence level is computed by multiplying the overall combined uncertainty by a factor of 2 [40]. [Figure 6](#) shows expanded uncertainties with a 95% confidence interval for both the intake (green) and in-cylinder (red) pressure measurements around the pegging and intake valve closure events, where the uncertainty is largest relative to the measured quantities. Dotted lines represent 95% Confidence Intervals.

As can be seen in [Figure 6](#), pegging is applied at -180 CAD to provide the absolute basis for the in-cylinder transducer from the intake transducer. It is tempting to apply a similar logic to the transducer uncertainties and thus assert that the in-cylinder pressure confidence interval should be smaller, at least near -180 CAD. Theoretical support for this argument is difficult to come by and would, at minimum, require a more sophisticated physical uncertainty model and hence is left for future research. In the present work, no relationship between the uncertainties from the two pressure transducers was assumed.