

A Theoretical Investigation of the Influence of Powertrain Mounts on Transmission Torsional Dynamics

Enrico Galvagno, Pablo Gutierrez, Mauro Velardocchia, and Alessandro Vigliani
Politecnico di Torino

ABSTRACT

This paper investigates the effect of the powertrain mounting system on the linear and nonlinear torsional dynamical behaviour of a transmission system. To this aim, two dynamic models, one with rigid mounts and the other with flexible mounts, are presented and compared: the first model considers only the torsional dynamics of transmission and driveline, while the second model includes also a 3 degrees-of-freedom powertrain block. The mechanical coupling and interaction between the powertrain block and transmission system is discussed and formulated. These models are then analyzed in terms of vibrational mode shapes, natural frequencies and Frequency Response Functions (FRFs); a sensitivity analysis of the main transmission parameters, e.g. the gear ratio, is also presented. From the comparison of the two simulated configurations (with and without powertrain mounts) both in time and frequency domain, a significant interaction between the two subsystems is noticed, particularly in the vehicle acceleration signal. Neglecting the powertrain mounts may lead to underestimating the real vibration level. A nonlinear model of a dual clutch transmission, including the main backlashes present along the transmission path (clutch spline, gear meshes, synchronizers), is coupled to the powertrain mounting system model. Finally, the effect of mount stiffness on transmission NVH during critical manoeuvres is shown.

CITATION: Galvagno, E., Gutierrez, P., Velardocchia, M., and Vigliani, A., "A Theoretical Investigation of the Influence of Powertrain Mounts on Transmission Torsional Dynamics," *SAE Int. J. Veh. Dyn., Stab., and NVH* 1(2):2017, doi:10.4271/2017-01-1124.

INTRODUCTION

There is limited literature available on the interaction between the torsional dynamic behavior of automotive transmissions and the powertrain mounting system. A very common assumption in transmission modelling is that the powertrain mounts are highly damped and therefore they do not interact with the torsional dynamical behavior of the transmission ([1]-[6]).

The powertrain mounting system, see e.g. Figure 1, is designed to optimize NVH performance in terms of filtering the engine harmonics and more generally to reduce the transmissibility of vibrations from engine and transmission to the vehicle body/chassis. A review of automotive vehicle engine mounting systems can be found in [7].

In this paper a 3 DOF model of a powertrain is introduced and used to study the rigid body motion of the powertrain unit attached to the ground by means of its mounts. A modal analysis is carried out (natural frequencies and mode shapes) and the instantaneous center of rotation of each mode shape is calculated, also considering variations of mount parameter (i.e., the differential mount stiffness).

Then a 3 DOF purely torsional transmission model is presented: this model is named rigid mount model, because the presence of powertrain mounts is neglected, as if mounts were infinitely stiff. The main reasons why powertrain suspensions influence the transmission

and longitudinal vehicle dynamics are discussed; afterwards a complete model (transmission and powertrain) is formulated. The dynamic behavior of the complete system is analyzed both in frequency and in time domain by means of numerical simulations. Finally, to test the mutual interactions of transmission and powertrain motion even in the nonlinear field, a nonlinear torsional model of the transmission is coupled with the linear powertrain model: transient simulations are used to excite the system nonlinearities. All the models are populated with realistic numerical data, typical of compact cars.

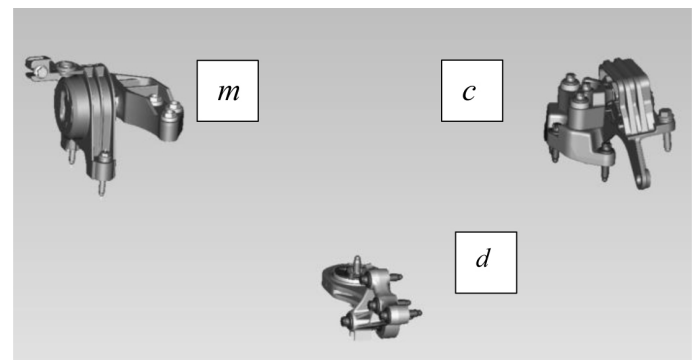


Figure 1. The powertrain mounting system: m – engine mount, c – gearbox mount, d – differential mount.

Powertrain Stand-Alone Model

A 3 DOF linear model, as depicted in Figure 2, is used to study the dynamic behavior of the powertrain unit alone, without considering any couplings and/or interactions with the vehicle and transmission systems. The lateral displacements are locked and the roll axis of the powertrain is supposed horizontal (perpendicular to the plane of the drawing) since we are interested solely in longitudinal vehicle dynamics and torsional powertrain dynamics.

A rigid body, i.e. the powertrain block, is suspended by means of three mounts having lumped elastic and damping properties. The two upper mounts, called engine mount (subscript *m*) and gearbox mount (subscript *c*) react in both longitudinal X and vertical Z directions, while the lower one, called differential mount (subscript *D*), is composed of a rod with two bushing at the ends and reacts only along X axis. Considering the static and low frequency tasks of this mounting system, the first two mounts must support the powertrain weight while the third one must oppose the powertrain rotations induced by the reaction torque exchanged with the engine and transmission. The typical nonlinear force vs displacement characteristics of these elements are linearized in the neighborhood of the mean actual operating conditions: the static force for the engine and gearbox mounts and zero force for the differential mount.

Note that, for the sake of clarity, in the drawing reported in Figure 2, lengths *l_{cx}* and *l_{mx}* have been increased respect to their real values.

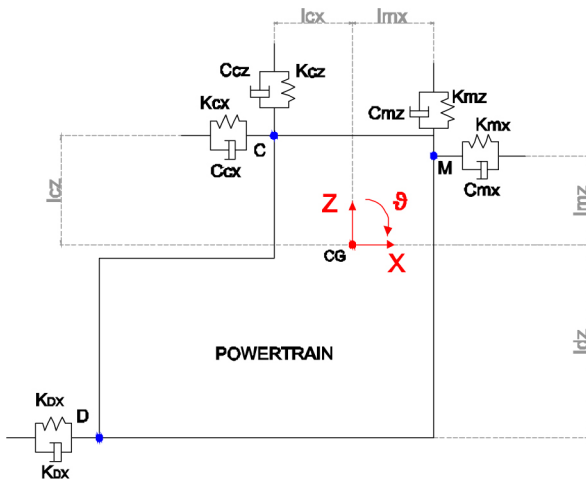


Figure 2. Dynamic model for the study of rigid powertrain modes.

The equations of motion in matrix form are:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = 0 \quad (1)$$

where the generalized coordinate vector is: $\{q\} = \{X \ Z \ \vartheta_p\}^T$, *X* and *Z* are the longitudinal and vertical displacements of the powertrain center of gravity and ϑ_p is the angular displacement about the powertrain roll axis, with respect to the static equilibrium condition. Matrices $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices:

$$[M] = \begin{bmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & I_{CG,YY} \end{bmatrix}$$

$$[C] = \begin{bmatrix} C_{1,1} & 0 & C_{1,3} \\ 0 & C_{2,2} & C_{2,3} \\ C_{3,1} & C_{3,2} & C_{3,3} \end{bmatrix}$$

where

$$C_{1,1} = C_{mx} + C_{cx} + C_{dx}; C_{1,3} = C_{3,1} = C_{mx}l_{mz} + C_{cx}l_{cz} - C_{dx}l_{dz}$$

$$C_{2,2} = C_{mz} + C_{cz}; C_{2,3} = C_{3,2} = -C_{mz}l_{mx} + C_{cz}l_{cx}$$

$$C_{3,3} = C_{mx}l_{mz}^2 + C_{cx}l_{cz}^2 + C_{dx}l_{dz}^2 + C_{mz}l_{mx}^2 + C_{cz}l_{cx}^2$$

$$[K] = \begin{bmatrix} K_{1,1} & 0 & K_{1,3} \\ 0 & K_{2,2} & K_{2,3} \\ K_{3,1} & K_{3,2} & K_{3,3} \end{bmatrix}$$

where

$$K_{1,1} = K_{mx} + K_{cx} + K_{dx}; K_{1,3} = K_{3,1} = K_{mx}l_{mz} + K_{cx}l_{cz} - K_{dx}l_{dz}$$

$$K_{2,2} = K_{mz} + K_{cz}; K_{2,3} = K_{3,2} = -K_{mz}l_{mx} + K_{cz}l_{cx}$$

$$K_{3,3} = K_{mx}l_{mz}^2 + K_{cx}l_{cz}^2 + K_{dx}l_{dz}^2 + K_{mz}l_{mx}^2 + K_{cz}l_{cx}^2$$

By solving the characteristic eigenvalue problem:

$$([K] - \omega^2[M])v = 0, \quad (2)$$

the natural frequencies f_i of the undamped system and the corresponding mode shapes, described by eigenvectors ψ_i , have been computed, considering the parameter values in Table 1 of a compact passenger car.

Table 1. Parameters of the stand-alone model

Powertrain mass (m_p)	230 kg
Moment of inertia yy direction (I_{CG,yy})	8 kg·m ²
X distance from CG to engine mount (l_{mx})	5 mm
Z distance from CG to engine mount (l_{mz})	160 mm
X distance from CG to gearbox mount (l_{cx})	4 mm
Z distance from CG to gearbox mount (l_{cz})	190 mm
X distance from CG to differential mount (l_{dx})	360 mm
Engine mount stiffness in X direction (K_{mx})	180 kN/m
Engine mount stiffness in Z direction (K_{mz})	180 kN/m
Gearbox mount stiffness in X direction (K_{cx})	180 kN/m
Gearbox mount stiffness in Z direction (K_{cz})	140 kN/m
Differential mount stiffness in X direction (K_{dx})	200 kN/m

$$f_1 = 6.3 \text{ Hz}; \quad f_2 = 7.5 \text{ Hz}; \quad f_3 = 8.7 \text{ Hz}$$

$$\psi_1 = \begin{bmatrix} 2 \cdot 10^{-4} \text{ m} \\ 1 \text{ m} \\ 1.3 \cdot 10^{-2} \text{ rad} \end{bmatrix}; \quad \psi_2 = \begin{bmatrix} 1 \\ -3 \cdot 10^{-4} \\ 0.32 \end{bmatrix}; \quad \psi_3 = \begin{bmatrix} -1.1 \cdot 10^{-2} \\ -4 \cdot 10^{-4} \\ 1 \end{bmatrix}$$

The first mode is a translational mode in the vertical direction, the second mode is mainly a translational mode in the longitudinal direction with a certain degree of rotation and the third mode is a rotational mode.

In general, the design of the mounting system and parameter selection is performed with the aim of producing a very low coupling between the modes. In order to investigate this aspect, the study of the position of the instantaneous center of zero velocity is here proposed.

Instantaneous Center of Velocity (IC)

Under the assumption made in this work, the movement of the powertrain is planar. It is therefore interesting to study the position of the IC of velocity when the system vibrates in each mode. Since the first mode, associated to the vertical displacement, is strongly decoupled from longitudinal and rotational motions, this DOF will be considered locked for the rest of this section, thus simplifying the following calculations.

Let consider a generic motion, identified by position $\{X, \vartheta\}^T$ and velocity $\{\dot{X}, \dot{\vartheta}\}^T$, as visible in Figure 3.

The IC position can be found as the intersection between the normal directions to the velocity vectors of two distinct points of the rigid body. We selected the center of gravity G and a generic point A at a distance l to the right of G.

The vertical position of the IC is expressed as the sum of two contributions:

$$Z_{IC} = Z_1 + Z_2 = -l \sin \vartheta - l \cos \vartheta \cotan \alpha \quad (3)$$

Introducing the auxiliary variables a , b and h , angle ϑ is eliminated from the former equation:

$$Z_{IC} = -l \frac{a}{l\dot{\vartheta}} - l \cotan \alpha \frac{h}{l\dot{\vartheta}} \quad (4)$$

$$Z_{IC} = -\frac{a}{\dot{\vartheta}} - \frac{b}{\dot{\vartheta}} = -\frac{\dot{X}}{\dot{\vartheta}} \quad (5)$$

The distance between IC and G is therefore simply the ratio of the translational speed $\dot{X}$ and the rotational speed $\dot{\vartheta}$. The larger the translational speed is respect to the rotational speed, the larger the distance between points IC and G.

The IC has been calculated for each mode and a sensitivity analysis considering the effects of different differential mount stiffness has been performed: the results are summarized in Figure 4 and Figure 5.

In Figure 4, the position of the IC versus the differential mount stiffness is plotted for the two modes (eq.(5) is applied to the eigenvectors computed for each differential mount stiffness value). The two dashed horizontal lines correspond to the differential mount position (the lower one) and to the mean position of engine and gearbox mounts (the upper one). The vertical dot-dashed lines represent the analyzed stiffness values, i.e. 10^3 , 2×10^5 and 10^8 N/m,

correspondent to the mode shapes shown. It is of interest observing that there is a vertical asymptote at a certain value K_{dx}^* of the differential stiffness. This value guarantees the best decoupling of the translational and rotational motion of the powertrain, because the IC for one mode is the center of gravity G, while its position tends to infinity for the other mode. Any other combination would give a stronger coupling between vertical and rotational motion.

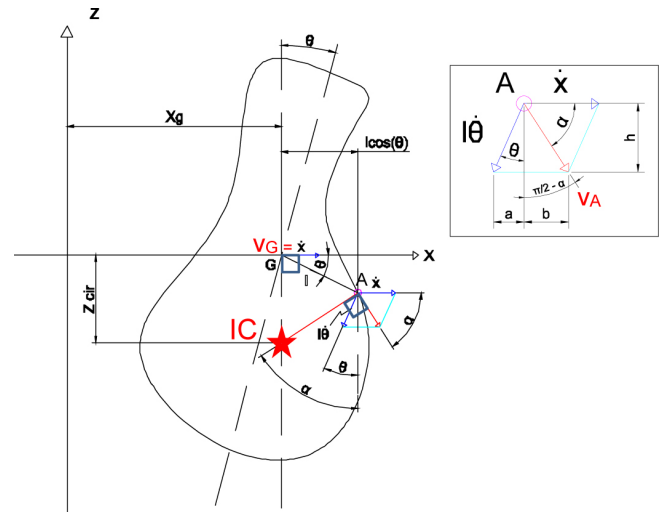


Figure 3. Sketch for the determination of the instantaneous center of velocity.

The differential stiffness modification evidently produces also a change in the natural frequencies, as illustrated in Figure 5. Three zones can be highlighted in this chart:

- a first zone characterized by low stiffness, where the frequency of the lowest mode increases progressively with K_{dx} , while the frequency of the other mode is almost constant;
- a central zone (best decoupling) where the two frequencies are very similar; it is worth noting that also their rate of change with respect to the stiffness, i.e. df/dK , is similar;
- a third region in which the lowest natural frequency is insensitive to any further increase of stiffness, while the other frequency increases rapidly with stiffness, thus deteriorating the isolation capacity of the suspension system.

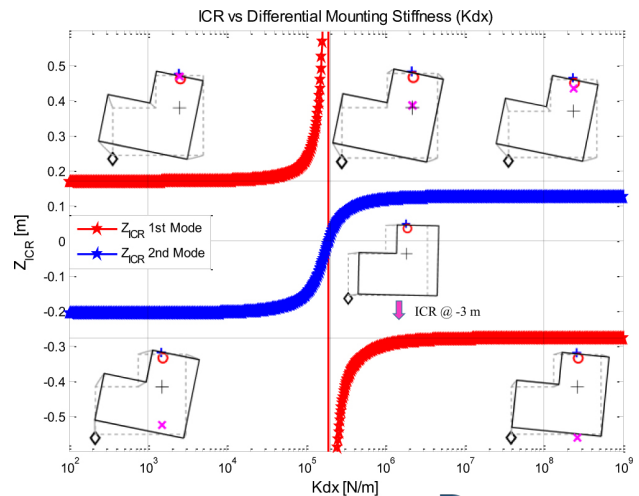


Figure 4. Sensitivity analysis: effect of differential mount stiffness in terms of vertical position of the IC. Where $\diamond$: engine mount, $\square$: gearbox mount, $\diamond$: differential mount, $\times$: IC, $+$: the center of gravity.

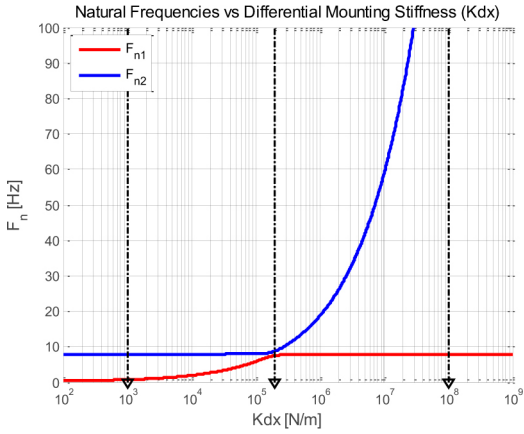


Figure 5. Sensitivity analysis: effect of differential mount stiffness in terms of natural frequencies of the two modes.

Complete Linear Model

In order to study the effect of the powertrain mounting system on transmission torsional dynamics and more generally the mutual interaction between transmission and powertrain systems, the equations of motion (EoM) of the powertrain stand-alone model are coupled with those describing the torsional behavior of the transmission system.

Transmission Dynamics: The 3DOF Rigid Mounts Model

In particular, a 6-speed Dual Clutch Transmission model has been developed, considering the torsional compliances of the torsional damper (a dual mass flywheel in this case), and of the half-shafts. See e.g. [2] for details on DCT working principles and its kinematic and dynamic behavior.

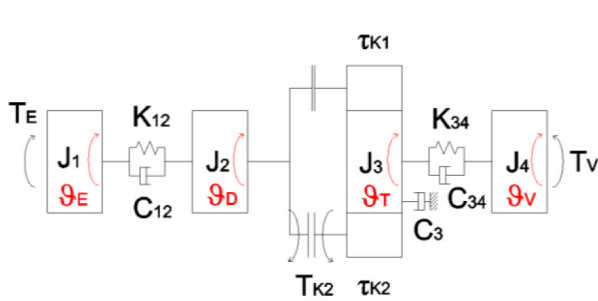


Figure 6. The Dual Clutch Transmission torsional model.

Depending on the specific operating condition, especially correlated with clutches (slipping or engaged) and synchronizers status, the transmission system may possess a variable number of degrees of freedom. In the present paper, only one of the two clutches is engaged (the other one is in slipping condition or completely disengaged) and the synchronizers sleeves are located either in neutral position or at an endstop, where the gear is surely engaged.

Under these assumptions, the transmission system has three torsional DOFs. The coordinates reported in Figure 6 represent the angular positions of: θ_E engine, θ_D clutches drum, θ_T transmission output shaft and θ_V vehicle. If one clutch is closed, the second and third

coordinate are linearly related through the actual gear ratio, i.e. $\dot{\theta}_D = \dot{\theta}_T \tau_{K1}$. The equations of motion of the DCT alone considering clutch K1 engaged are:

$$J_E \ddot{\theta}_E + C_D (\dot{\theta}_E - \dot{\theta}_D) + K_D (\theta_E - \theta_D) = T_E \quad (6)$$

$$\left(J_D + \frac{J_T}{\tau_1^2} \right) \ddot{\theta}_D + \frac{C_D}{\tau_1} \left(\frac{\dot{\theta}_D}{\tau_1} \right) + \frac{C_T}{\tau_1} \left(\frac{\dot{\theta}_D}{\tau_1} - \dot{\theta}_V \right) + \frac{K_T}{\tau_1} \left(\frac{\theta_D}{\tau_1} - \theta_V \right) - C_D (\dot{\theta}_E - \dot{\theta}_D) - K_D (\theta_E - \theta_D) = T_{K2} \left(\frac{\tau_2}{\tau_1} - 1 \right) \quad (7)$$

$$J_V \ddot{\theta}_V - C_T \left(\frac{\dot{\theta}_D}{\tau_1} - \dot{\theta}_V \right) - K_T \left(\frac{\theta_D}{\tau_1} - \theta_V \right) = -T_V \quad (8)$$

This 3-DOF model is used to describe the torsional behavior of the transmission and the longitudinal vehicle dynamics when the powertrain mounts flexibility is completely ignored. It will be used as the benchmark for the comparison with the flexible mounts model (6 DOF) that will be introduced in the next sections.

Powertrain-Chassis Interaction

There are three main reasons why the powertrain suspension could have an influence on transmission and longitudinal vehicle dynamics:

1. The real constraint between powertrain and vehicle body is a flexible suspension, which allows relative motion between the two parts and transfers the dynamic forces through the bushings. With reference to Figure 7, the dynamic equation of vehicle longitudinal behavior changes from the rigid case

$$F_x - F_{res} = (m_v + m_p) a_x \quad (9)$$

to the new model

$$F_{t,x} - F_{res} - (F_{mx} + F_{cx} + F_{dx}) = m_v a_{x,v} \quad (10)$$

where F_{mx} , F_{cx} and F_{dx} are the longitudinal components of the forces in each mount, F_{res} is the vehicle total drag force due to aerodynamic and rolling resistances, $F_{t,x}$ is the total longitudinal force exerted by the driving axle tires.

Considering linear elastic and viscous damping characteristics, the transmitted force for a generic i -th mount is

$$F_{ix} = K_{ix} \Delta X_i + C_{ix} \Delta \dot{X}_i, \quad (i = m, c, d) \quad (11)$$

where $\Delta X_i = X_v - (X \pm l_{ix} \theta_p)$, $\Delta \dot{X}_i = \dot{X}_v - (\dot{X} \pm l_{ix} \dot{\theta}_p)$, + for the upper mounts, - for the lower one.

Hence, the powertrain dynamics in terms of X , $\dot{X}$, θ_p , $\dot{\theta}_p$, enters explicitly into the computation of the vehicle acceleration, potentially affecting the vehicle drivability.

The other remaining actions exchanged with the vehicle through the engine mount system, i.e. the resultant torque T_{My} and the total vertical force F_{Mz} , are perfectly balanced by the vehicle, which is assumed rigid in these directions. The roll and vertical