

Longitudinal Vibration Analysis of Electric Wheel System in Starting Condition

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ABSTRACT

Due to coupling of in-wheel motor and wheel/tire, the electric wheel system of in-wheel motor driven vehicle is different from tire suspension system of internal combustion engine vehicle both in the excitation source and structural dynamics. Therefore emerging dynamic issues of electric wheel arouse attention. Longitudinal vibration problem of electric wheel system in starting condition is studied in this paper. Vector control system of permanent magnet synchronous hub motor considering dead-time effect of the inverter is primarily built. Then coupled longitudinal-torsional vibration model of electric wheel system is established based on rigid ring model and dynamic tire/road interface. Inherent characteristics of this model are further analyzed. The vibration responses of electric wheel system are simulated by combining electromagnetic torque and the vibration model. The results indicate that abrupt changes of driving torque will cause transient vibration of electric wheel system. To be specific, vehicle body mainly undergoes longitudinal shake in 4Hz and unsprung mass suffers from longitudinal shock in 38Hz; the transient vibration of electric wheel system is most remarkable at zero velocity and decreases with vehicle speed. In addition, torque ripple will arouse $6p^{\text{th}}$ order vibration, which is centralized in 40Hz and 100Hz and becomes noticeable in certain speed range due to resonance. The longitudinal vibration of electric wheel system in starting condition is highlighted and deserves attention. The detailed features and characteristics of this vibration are studied in this paper, which is favorable for design and analysis of in-wheel motor driven vehicle.

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INTRODUCTION

The rapid growth of car ownership has brought serious problems such as energy shortages, environmental pollution, and so on. Developing eco-friendly electric vehicles has become an effective way to mitigate these problems. Among numerous structural forms of electric vehicles, in-wheel motor driven type has gained great attention due to its obvious advantages in structural layout, integrated chassis control and manipulation. As a critical part of in-wheel motor driven vehicles, electric wheel system becomes the focus of research [1, 2, 3].

The electric wheel assembly is a multi-physical and multi-body system, which includes tire/wheel, in-wheel motor and controller. Interaction between electromagnetic excitation of in-wheel motor and structural dynamics of tire results in the emergence of dynamic problems in electric wheel system. From the perspective of electromagnetic excitation, due to non-ideal factors such as non-sinusoidal magnetic field distribution, cogging torque and current harmonics, the electromagnetic torque of motor will inevitably fluctuate [4]. In addition, the driving torque of in-wheel motor changes rapidly in unsteady conditions such as starting and acceleration [5]. The driving torque will act as main source exciting vibration of electric wheel system. From the perspective of structural dynamic features, direct connection between in-wheel motor and wheel without buffer in the driveline makes the electric wheel system a weak damping system [6]. As a result, abrupt change of driving torque in combination with torque ripple will cause transient

vibration problem of electric wheel system in starting condition. As the tire is coupled in longitudinal and circumferential directions through tire/road interface, driving torque will easily cause longitudinal vibration of electric wheel. Therefore, unlike the extensively studied vertical vibration of traditional suspension tire system caused by road roughness, longitudinal vibration in electric wheel system is worthy of attention.

Scholars have made some researches with respect to longitudinal dynamic problems of electric wheel system. It is demonstrated through bench test that longitudinal vibration of electric wheel system is noticeable. Subsequent analysis indicates that this longitudinal vibration is caused by torque ripple of in-wheel motor [7]. Furthermore, the influence mechanism of electromechanical coupling on torque ripple and longitudinal vibration performance are studied in [8]. In order to attenuate longitudinal vibration of electric wheel system, flexible connection between rim and in-wheel motor is adopted in [9] to modify the inherent characteristics of electric wheel and decrease longitudinal vibration in particular frequency bands. The above studies analyze electromechanical coupling of electric wheel system in steady conditions and neglect transient behavior of electric wheel system in unsteady conditions. In [10] and [11] the acceleration condition of in-wheel motor driven vehicle is simulated and transient dynamics of electric wheel system are analyzed. Slip ratio of tire fluctuates violently because of instantaneous change of driving torque. As a result, longitudinal dynamic characteristics of

vehicle are affected. These papers study dynamic behavior of electric wheel system in unsteady condition, whereas the influence of torque ripple is not considered.

In light of the above, longitudinal vibration of electric wheel system caused by abrupt change of driving torque in combination with torque ripple in the starting condition is studied. This paper is organized as follows. Vector control system of permanent magnet synchronous hub motor considering dead-time effect of the inverter is primarily built. Torque ripple which results from dead-time is derived theoretically. Based on rigid ring model and dynamic tire/road interface, coupled longitudinal-torsional vibration model of electric wheel system is then established and inherent characteristics of this model are analyzed. The longitudinal vibration responses of electric wheel system are simulated by combining electromagnetic torque and dynamic model. The detailed features and characteristics of the longitudinal vibration caused by driving torque in starting condition are analyzed.

MODELING OF ELECTROMAGNETIC TORQUE OF IN-WHEEL PMSM

Permanent magnet synchronous motor (PMSM) is considered to be an optimal choice for in-wheel motor as its considerable advantages in power density and efficiency. Fractional-slot concentrated-winding is widely employed in PMSM because of high slot fill factor, short end turns and so on. Therefore, PMSM with concentrated fractional-slot windings is adopted in this paper. The parameters of this motor are listed in [Table 1](#).

Table 1. The parameters of in-wheel PMSM

Number of pole pairs	12
d, q -axis stator inductances	700 μ H
Stator resistance	0.08462 Ω
Flux linkage established by magnets	0.085 Vs
Switching frequency	10kHz
Inverter DC bus voltage	120V
Dead time	2 μ s

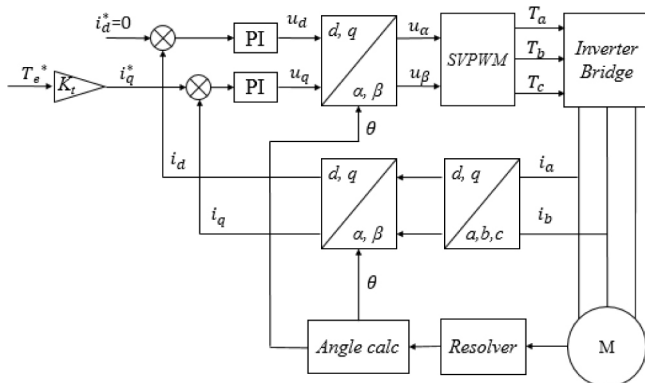


Figure 1. Control diagram of FOC for PMSM

Field-oriented control (FOC) is a classical control project for alternating current (AC) drive system. Based on the FOC system for PMSM, the dynamic behavior of in-wheel motor in starting condition is simulated. The control diagram of FOC for PMSM is shown in [Fig.](#)

1. The control diagram with the strategy of $i_d=0$ includes space vector PWM algorithm, PI control module, inverter, motor module and sensors for collecting phase currents and rotor position. The detailed introductions of the related modules are referred in [12]. Inverter dead time is considered in the simulation model because dead time has influence on low-order current harmonics and torque ripple.

As for star connection type of stator winding, dead time will induce $6k \pm 1$ th order harmonics in phase current, among which 5th and 7th harmonics are dominant[13]. Thus the phase current can be expressed as follows:

$$\begin{cases} i_a = i_1 \sin(p\omega t + \varphi_1) \\ \quad + \sum_k i_{6k \pm 1} \sin[(6k \pm 1)p\omega t + \varphi_{(6k \pm 1)}] \\ i_b = i_1 \sin(p\omega t - \frac{2}{3}\pi + \varphi_1) \\ \quad + \sum_k i_{6k \pm 1} \sin[(6k \pm 1)(p\omega t - \frac{2}{3}\pi) + \varphi_{(6k \pm 1)}] \\ i_c = i_1 \sin(p\omega t + \frac{2}{3}\pi + \varphi_1) \\ \quad + \sum_k i_{6k \pm 1} \sin[(6k \pm 1)(p\omega t + \frac{2}{3}\pi) + \varphi_{(6k \pm 1)}] \end{cases} \quad (1)$$

Where: i_a, i_b and i_c are A, B, C phase current respectively; p is number of pole pairs; ω is the rotation speed of motor; i_1 is magnitude of fundamental current; $i_{6k \pm 1}$ is magnitude of $6k \pm 1$ th harmonic current; φ_1 is phase of fundamental current; $\varphi_{6k \pm 1}$ is phase of $6k \pm 1$ th harmonic current; k is integer.

The torque ripple of PMSM is mainly caused by non-sinusoidal distribution of magnetic field of the permanent magnet, cogging torque and current harmonics. Due to structural optimization such as skewed slot, the contribution of non-sinusoidal distribution of magnetic field and cogging torque to torque ripple is minor. For this reason, the non-ideal structural factors are neglected and the mathematical model of PMSM reads:

$$\begin{cases} i_d = \frac{2}{3} \left[i_a \cos p\omega t + i_b \cos \left(p\omega t - \frac{2}{3}\pi \right) + i_c \cos \left(p\omega t + \frac{2}{3}\pi \right) \right] \\ i_q = -\frac{2}{3} \left[i_a \sin p\omega t + i_b \sin \left(p\omega t - \frac{2}{3}\pi \right) + i_c \sin \left(p\omega t + \frac{2}{3}\pi \right) \right] \\ T_e = 1.5p \left[\varphi_f i_q + (L_d - L_q) i_d i_q \right] \end{cases} \quad (2)$$

Where: $i_d (i_q)$, $L_d (L_q)$ are $d(q)$ axis current and inductance, φ_f is the flux linkage established by magnets, T_e is electromagnetic torque. Combining [Eq. \(1\)](#) and [Eq. \(2\)](#) the electromagnetic torque can be expressed as

$$T_e = 1.5 p \varphi_f \left\{ \begin{aligned} & i_1 \cos \varphi_1 + \sum_k i_{6k-1} \cos[6kp\omega t + \varphi_{(6k-1)}] \\ & - \sum_k i_{6k+1} \cos[6kp\omega t + \varphi_{(6k+1)}] \end{aligned} \right\} \quad (3)$$

It can be seen from the Eq. (3) that the $6k \pm 1$ th current harmonics give rise to $6pk$ th order torque ripple. The dominant orders of current harmonics are 5th, 7th and the main order of torque ripple is $6p$ th.

MODELING AND DYNAMIC ANALYSIS OF ELECTRIC WHEEL SYSTEM

Dynamic Model of Electric Wheel System

As driving torque will easily arouse longitudinal vibration of electric wheel through tire/road interface, coupled longitudinal-torsional vibration model of electric wheel system is established according to preliminary studies of the authors[7,8,9]. This model illustrated in Fig. 2 consists of rigid ring tire model, tire contact model and quarter vehicle suspension model.

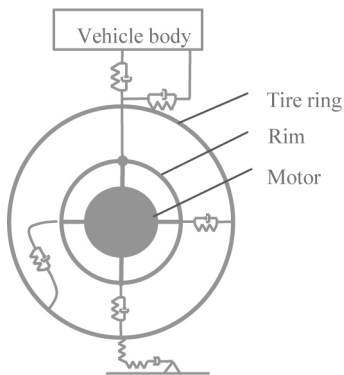


Figure 2. Electric wheel system model

In the rigid ring tire model, tire carcass is simplified as a rigid ring considering longitudinal translation and circumferential rotation. The rim and motor are rigidly connected and modeled as a rigid body. The tire ring and the rim/motor are connected through the sidewalls with pressurized air, which are modeled as lumped longitudinal and rotational spring and damper.

The coupling of electric wheel system in longitudinal and circumferential direction is represented by tire contact model, which contains the brush model and tire relaxation characteristics approximated by first order filter. The electric wheel system and vehicle body are integrated together through quarter vehicle suspension, which is equivalent as a linear stiffness and damper in parallel. The equations of motion of electric wheel dynamic model are shown in Eq. 4. The parameters of this model are identified experimentally and the validity of this model has been proved in [7].

$$\left\{ \begin{aligned} & I_{dy} \ddot{\theta}_a + c_{b\theta} (\dot{\theta}_a - \dot{\theta}_b) + k_{b\theta} (\theta_a - \theta_b) = T_e \\ & I_{by} \ddot{\theta}_b + c_{b\theta} (\dot{\theta}_b - \dot{\theta}_a) + k_{b\theta} (\theta_b - \theta_a) = -R_e F_{cx} \\ & m_b \ddot{x}_b + c_b (\dot{x}_b - \dot{x}_a) + k_b (x_b - x_a) = F_{cx} \\ & m_d \ddot{x}_a + c_b (\dot{x}_a - \dot{x}_b) + k_b (x_a - x_b) \\ & + c_{ux} (\dot{x}_a - \dot{x}_u) + k_{ux} (x_a - x_u) = 0 \\ & m_u \ddot{x}_u + c_{ux} (\dot{x}_u - \dot{x}_a) + k_{ux} (x_u - x_a) = 0 \\ & \dot{F}_{cx} \frac{1}{2c_{px}\sigma} + F_{cx} \frac{V_x}{c_{px}\sigma^2} = \dot{\theta}_b R_e - \dot{x}_b \end{aligned} \right. \quad (4)$$

Where: m_b , m_d , m_u are mass of tire ring, rim/motor and quarter vehicle body, respectively; I_{by} , I_{dy} are moment of inertia of tire ring and rim/motor; k_b , c_b are longitudinal stiffness and damping coefficient of tire; $k_{b\theta}$, $c_{b\theta}$ are circumference stiffness and damping coefficient of tire; k_{ux} , c_{ux} are longitudinal stiffness and damping coefficient of suspension; c_{px} is stiffness of tire tread element; σ is the relaxation length of tire. σ equals half contact length of tire a at zero slip ratio. V_x is longitudinal velocity of vehicle; x_a , θ_a are longitudinal displacement and circumferential rotational angle of rim/motor, x_b , θ_b are longitudinal displacement and circumferential rotational angle of tire. F_{cx} is longitudinal contact force of tire, T_e is driving torque on rim/motor, R_e is tire radius. The parameters of electric wheel system are listed in Table 2.

Table 2. The parameters of electric wheel system

Mass of tire	6 kg
Mass of rim/motor	30 kg
Mass of quarter vehicle body	300kg
Moment of inertia of tire	0.7 kg m ²
Moment of inertia of rim/motor	0.4 kg m ²
Longitudinal stiffness of tire	2588400 N/m
Longitudinal damping coefficient of tire	800Ns/m
Circumference stiffness of tire	82688Nm/rad
Circumference damping coefficient of tire	17 Nms/rad
Stiffness of tire tread element	10000000N/m ²
Half contact length of tire	0.0355m
Longitudinal stiffness of suspension	26232 N/m
Longitudinal damping coefficient of suspension	1273.9Ns/m

Inherent Characteristics of Electric Wheel System

For simplicity, the nonlinear factors such as variation of relaxation length and fluctuation of vehicle velocity are neglected and thus the electric wheel system represented by dynamic equations in Eq. (4) is simplified to a linear time invariant system. Based on theory of linear system, inherent characteristics of electric wheel system are analyzed. The poles distribution of this linear system is shown in Fig. 3. Without poles in right-half plane, this system is stable. The modal frequencies and damping ratios of electric wheel system can be obtained according to coordinates of conjugate complex poles in each pair. The determining rules are as follows.