

Bond Graph Based Approach for Modeling of Automatic Transmission Dynamics

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ABSTRACT

This paper describes a method for modeling the dynamics of planetary automatic transmissions by using bond graphs. Bond graph models for the most relevant planetary gear sets (single, double and stepped pinion and Ravigneaux) are summarized and used as building blocks to construct the "device-mapping" bond graphs of complete planetary transmissions. Examples used include the legendary Ford Model T transmission and state-of-the-art six- and ten-speed transmissions. A systematic procedure of deriving mathematical models based on the constructed bond graph models is described which can be applied to any clutch application combination. It is illustrated how device-mapping bond graph models can be converted into conceptual torque-path bond graphs for better insight on the analysis of transmission shift events. Lastly, the effect of system rotational inertias on shift dynamics is discussed for practical concerns.

CITATION: Ivanovic, V. and Tseng, H., "Bond Graph Based Approach for Modeling of Automatic Transmission Dynamics," *SAE Int. J. Engines* 10(4):2017, doi:10.4271/2017-01-1143.

INTRODUCTION

Modern step gear planetary automatic transmissions are complex dynamic systems that can be configured differently through various clutch application combinations. Appropriate mathematical models are required to enhance the understanding of the drivetrain dynamics during transmission shifts, reduce the effort of the hardware design optimization, and improve the efficiency of software development and calibration.

Modeling and analysis of planetary automatic transmission has been subject of a number of prior publications. For example, reference [1] proposes the lever analogy, a kinematics-based method that has been very popular in the transmission engineering community for analyzing and optimizing kinematic arrangements of planetary transmissions. The pioneering work on modeling the transmission dynamics by using bond graphs is given in [2,3]. A control oriented model for a four-speed automatic transmission is developed by using bond graph modeling method and utilized for simulation and analysis of garage shifts in [4,5]. Bond graph method is used in [6] for analysis of shifts and opportunity to utilize an extra clutch to improve the shift performance on an example of a ten-speed transmission. A six speed transmission model is developed in [7] by using Newton's method for the purpose of developing a method for torque phase control based on clutch torque estimation. Another example of modeling the transmission dynamics for simulation purposes by using Newton's method for an eight speed transmission is given in [8]. Two degree of freedom transmission model with lumped inertias and effect of rotational inertias on shift quality are analyzed in [9,10]. The system inertias and inertia coupling are obtained based on power flow

equations in [9] and from high order simulation model in [10]. Both publications demonstrate that power flows of new transmissions with high number of speeds and compact design with packaging similar to conventional six-speed transmissions are characterized by a significant impact of inertia effects on the coordination of torque actuators as result of inertial torques due to high internal speed multiplication of rotational components. Reference [11] describes a method for systematic derivation of the system Lagrange equation based on Newton's method applied to the free-body diagram on examples of a four- and six-speed transmission. The Lagrange equation is an efficient way to describe dynamic models of planetary transmissions and suitable for simulation purposes. The equation has a high order where the size of the system is equal to the number of free bodies plus internal torques. Because of that the Lagrange equation can be effectively used for simulation purposes but as such it is not suitable for equation based system dynamics studies unless its order is reduced. It is demonstrated in [11] how the Lagrange equation can be reduced to a corresponding minimum (reduced) order model by eliminating internal variables (internal torques) and applying the speed constraints. This process can be challenging for complex kinematic arrangements with a large number of components. Reference [12] proposes and describes in detail the Lagrange method as a systematic procedure for deriving the equations of planetary gear set configurations and applies it to a four-speed gearbox consisting of two simple planetary gear sets. Although this energy method represents a more convenient method compared to Newton's method when dealing with complex planetary gear set configurations, deriving equations based on this method can still be demanding for very complex configurations. The Lagrange method shows clear

benefits if effects related to the change of potential energy of pinion gears are considered, but these effects are negligible as also stated in [12].

Based on authors' experience, the bond graph method [13,14,15] provides advantage over the Newton's and Lagrange methods when dealing with complex kinematic arrangements. Bond graph models provide direct representation of a complex transmission/driveline mechanical system with a graph that can be converted into static or dynamic model equations in a straightforward way based on application of straightforward causality rules that eliminate dependent states, large number of which often appears in planetary gearboxes.

The objective of this paper is to provide a detailed, generic description on using bond graph method for modeling dynamics of step gear planetary automatic transmissions and systematic derivation of system equations, illustrate the method on examples of the historic Ford Model T transmission and state-of-the-art six- and ten-speed transmissions, and discuss the effect of system rotational inertias on shift dynamics based on derived generic mathematical model.

BOND GRAPH MODELS OF SUBSYSTEMS FOR MODELING TRANSMISSIONS

Bond graphs can be used to model any physical system. They are convenient for modeling systems of any level of complexity or interaction between different energy domains. From authors' experience bond graphs are particularly convenient for modeling mechanical dynamic systems with a large number or dependent states and planetary transmissions are certainly one of the best examples of such systems. The former is because a bond graph of a mechanical system directly reflects the structure of the system which simplifies their derivation. The latter is result of straightforward causality rules that "eliminate" dependent variables. Readers new to the bond graphs are referred to [13,14] or [Appendix A](#) which gives a brief overview of the method, basic elements relevant for transmission dynamics, and concept of causality.

Modeling planetary transmission dynamics by using bond graphs is particularly convenient and efficient when using existing, previously developed building blocks, i.e. bond graph models of characteristic subsystems. They can be easily linked by using junction elements and adding elements such as inertias, compliances, resistors and transformers to describe the power flow through the system. In case of planetary transmissions these building blocks represent planetary gear sets that can be found in different configurations.

Mostly used planetary gear sets (PGS), their bond graph models, and corresponding torque and speed equations are outlined in [Appendix B](#). Bond graph models of two complexities are given for each PGS. The full model accounts for all rotational elements. The simplified model neglects the pinion gear torque which is valid in case when effects related to the pinion gear inertial torques and friction are negligible. This assumption is a valid assumption for automotive planetary gearboxes. The PGS bond graph models from [Appendix B](#) are kinematic models derived assuming torque and speed sign

convention as illustrated in the corresponding schematics. Because of that these models do not include energy storage elements associated with element inertias, which are added when bond graphs of complete transmissions are constructed. The causality strokes are also not included as their assignment is relevant only for complete transmission models.

The bond graph models from [Appendix B](#) have been derived by graphically representing the torque and speed equations by using the "Willis formula" [18]. Additional information about derivation of the bond graph models for the single pinion and double pinion PGS can be found in [3]. A great advantage of using bond graph method is that the torque relations are obtained as results of describing the speed equations which can be observed from illustrations in [Appendix B](#). It should be noted that the torque and speed sign convention determines the direction of power flow, i.e. the orientation of arrows. Therefore, the arrows orientation of given PGS models cannot be changed independently and a care should be taken if the change of power flow is done in order not to introduce an error in the speed and torque relations.

The PGS bond graph models are characterized by triangular shapes consisting of **0**- and **1**-junctions and transformers with modules reflecting the speed and torque relations between the PGS components. Model of the simplified single pinion PGS for example consists of two **0**-junction and one **1** junction and one transformer with module reflecting the ratio between the ring and sun gear teeth counts ($\beta = z_r/z_s$). The **1**-junction represents equilibrium of torques across the PGS. The **0**-junctions describe relations of speeds between the PGS components and torques of the sun (lower **0**-junction) and ring gear (upper **0**-junction).

BOND GRAPH MODELS FOR COMPLETE PLANETARY TRANSMISSIONS

This section demonstrates how to construct bond graph models for complete transmissions on examples of the historic Ford Model T transmission and state-of-the-art six- and ten-speed transmissions for RWD applications. We like to call such bond graph models "device mapping bond graphs" because they directly reflect the mechanical structure of observed systems.

[Fig. 1](#) shows the stick diagram for the Ford Model T transmission. The Model T transmission is a planetary transmission essentially very similar to modern automatic transmissions. It consists of one planetary gear set with one stepped pinion and three sun gears. Two band brake and a direct clutch are used to provide two forward and reverse speeds. The third band brake is used as the vehicle brake acting directly on the transmission output. The clutches and brakes are mechanically operated by three pedals and a hand lever. The left pedal applies the low speed band and releases the direct when forward, allows neutral when centered, and engages the direct clutch when released (i.e. low-neutral-direct). The center pedal controls the reverse band. The right pedal controls the foot brake. The hand lever allows engagement of the direct clutch when forward and emergency brakes when backwards.

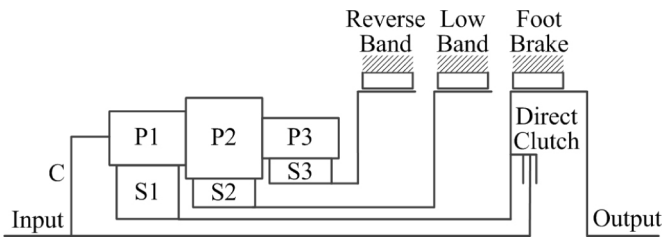


Fig. 1. Stick diagram for Ford Model-T transmission.

Fig. 3 shows the final bond graph model for the Model T transmission. First step in constructing a bond graph model is to observe the mechanical structure of the system, select appropriate models and elements and place them such as they appear in the real system. In this case we need models for a PGS with three sun gears and a long pinion, **Se** elements for interaction with the input and output, **I** elements for rotating inertias, **R** elements for clutches, **Sf** elements for defining grounding speed constraints on band brakes, **1**-junctions for single speed points which are physically located at locations of individual rigid rotational transmission components, and **0**-junctions for single torque points at locations of physical contact between components (gear mesh) or at locations where exists a relative motion between components such as in clutches and brakes.

The PGS with three sun gears and one stepped pinion shares similarity with the stepped pinion PGS from Appendix B, but they are not the same and therefore a new bond graph model needs to be constructed. As mentioned before, this model can be derived based on the "Willis formula" [18] applied to the three sun gears connected to the common pinion gear via external gear mesh. Assuming that all components except the pinion rotate in the same direction the following is valid

$$\frac{\omega_{Si} - \omega_C}{-\omega_P - \omega_C} = -\frac{z_{Pi}}{z_{Si}} = -\beta_i \Rightarrow \omega_{Si} = \omega_C + \beta_i(\omega_P + \omega_C) . \quad (1)$$

where $i=1 \dots 3$, ω_P is the pinion speed, ω_C is the carrier speed, and ω_{Si} is the i^{th} sun gear speed, and z_{Pi} and z_{Si} are teeth counts for the i^{th} pinion and sun gear. By taking that torques on all components act in the same direction except the carrier torque the torque equation reads

$$\tau_C = \tau_P + \tau_{S1} + \tau_{S2} + \tau_{S3} . \quad (2)$$

Bond graph model of Eqs. (1) and (2) is shown in Fig. 2. It consists of four **0**-junctions representing single torque points representing torques of sun gears and the pinion gear and speed relations from Eq. (1). Four **1**-junctions correspond to single speed junctions related to component speeds, upper one for the pinion gear and lower for the sun gear. The top left **1**- junction models the torque equation (2) and represents the single speed point for the carrier. A **1**- junction in the middle represents the single speed point ($\omega_P + \omega_C$) which is needed to implement Eq. (1) but at the same time defines the torque relations between the sun gears and the pinion along with the three transformers which represent gear ratios (β ratios).

When all building blocks are in place (see Fig. 3), the next step is to connect them by bonds in the same way as they appear in the stick diagram. Finally, arrows are added to describe the power flow through the system and torque and speed relations between physical components. In order to capture the transmission efficiency effects the model can be readily extended by adding power dissipation **R** elements at corresponding locations related to the friction in bearings, sealings, and gear mesh, oil churning, or transmission fluid pump driving torque. The friction effects are relevant for fuel economy studies and generally negligible for shift control studies with an exception on garage shifts where the gearbox input and output friction have significant effects on the accurate vehicle disturbance prediction [5].

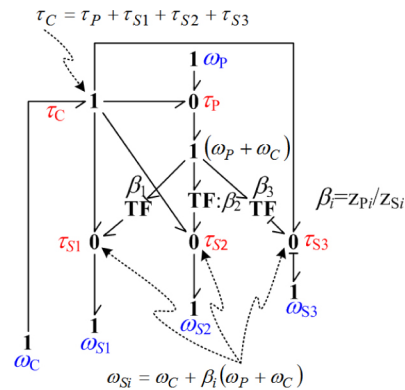


Fig. 2. Bond graph model of Ford Model T transmission planetary gear set with three sun gears and one stepped pinion.

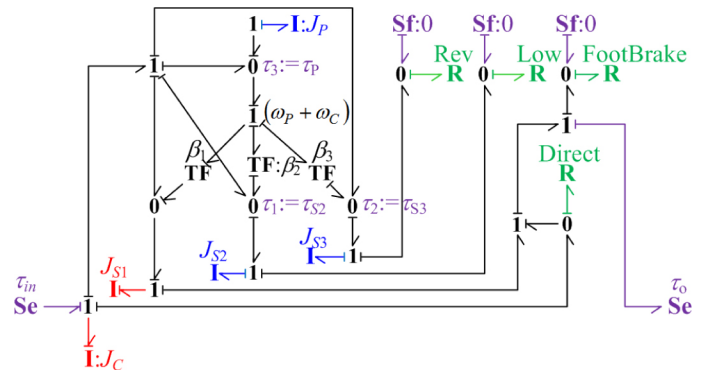
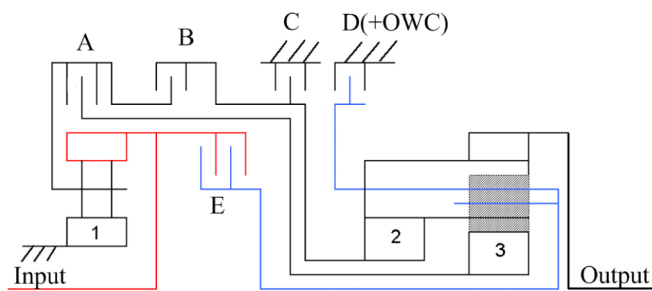


Fig. 3. Bond graph model for Ford Model T transmission.

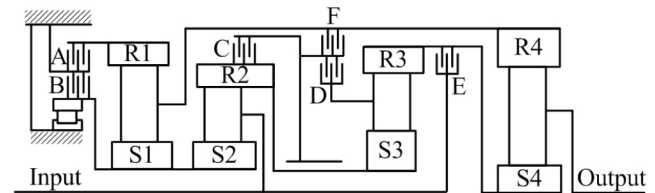
It should be mentioned that the final bond graph model for Ford Model T transmission which is given in Fig. 3 includes three internal torques variables (τ_1 , τ_2 , and τ_3). They are defined for the purpose of derivation of the system equations which is discussed in next section.

Fig. 4 shows the stick diagram and clutch application chart for the six-speed transmission. This transmission consists of one simple PGS with grounded sun gear, one Ravigneaux PGS, 5 friction clutches and one One-Way-Clutch (OWC) in parallel to the D clutch. Two clutches are closed in each gear. Fig. 5 shows the corresponding transmission bond graph model. The model is obtained in a similar way as explained for the Model T transmissions and by using PGS bond graph models for the simple and Ravigneaux PGS from Appendix B.



Gear	Clutches					Ratios	Steps
	A	B	C	D	E		
1 st	X	X				4.171	
2 nd	X		X			2.340	1.783
3 rd	X			X		1.521	1.538
4 th	X				X	1.143	1.331
5 th				X	X	0.867	1.318
6 th			X		X	0.691	1.255
R		X		X		-3.403	Total 6.035

Fig. 4. Stick diagram and clutch application chart for 6-speed transmission.



Gear	Clutches						Ratios	Steps
	A	B	C	D	E	F		
1 st	X	X		(O)	X		4.696	
2 nd	X	X	X	X			2.985	1.5732
3 rd	X		X	X	X		2.156	1.3845
4 th	X		X	X		X	1.779	1.2219
5 th	X		X		X	X	1.526	1.1658
6 th	X			X	X	X	1.278	1.1941
7 th			X	X	X	X	1	1.2780
8 th		X		X	X	X	0.854	1.1710
9 th		X	X		X	X	0.689	1.2395
10 th		X	X	X		X	0.636	1.0833
R	X	X		X		X	-4.866	Total 7.384

Fig. 6. Stick diagram and clutch application chart for 10-speed transmission [17].

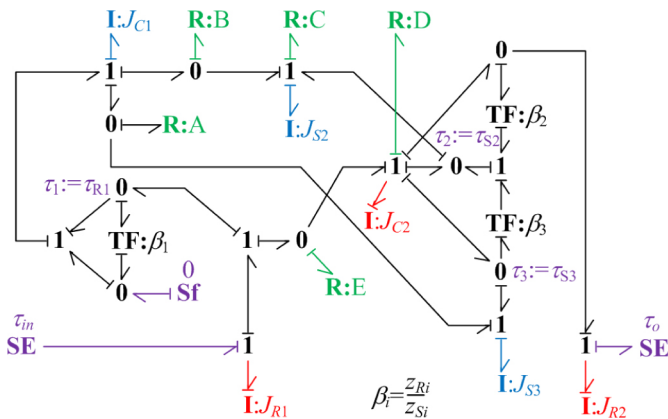


Fig. 5. Bond graph model for 6-speed transmission.

Fig. 6 shows the stick diagram and clutch application chart for the ten-speed transmission [17]. This transmission consists of 6 clutches with an OWC in parallel to the B clutch, four simple PGSSs, and the so-called node elements with three clutches (C, F, and D). As such any combination of two of these clutches can be represented as one clutch. Clutch D in the first gear is optional and applied as preparation for second, third, or reverse gear. Four clutches closed in each gear with only two open clutches which positively impacts the transmission losses. Following the previously described procedure the corresponding bond graph can be constructed as shown in Fig. 7.

Before deriving system equations, the causality analysis needs to be conducted in order to identify independent and dependent states (see Appendix A). The causality analysis provides at-the-glance information about the order of the system, i.e. the number of state, which in this case are speeds of transmission elements associated with inertia elements with an integral causality. The following guidelines can be applied for the process of assigning the causalities. The speed input causality is taken for all R elements. The integral causality is taken for I elements related to the input and output elements and elements that do not support any other causality type (e.g. node element $I:J_0$ in ten-speed transmission). For the remaining I elements there exist different possible combinations of causality

assignments. It is reasonable to assign an integral causality on I-elements associated with elements which speed measurement is practically feasible.

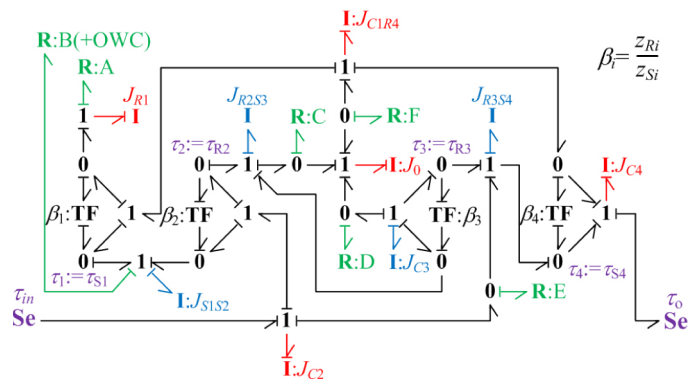


Fig. 7. Bond graph model for 10-speed transmission.

Color coding is used in bond graph models in Figs. 3-7 to easier distinguish between the independent and dependent speed variables. The independent states (I elements with integral causality) are denoted with red and dependent ones (I-elements with derivative causality) with blue color. Note that the number of independent states corresponds to the number of speed sensors required to provide a full visibility of clutch relative speeds during the transmission operation. During a typical shift there are only two independent states (input and output speed) implying that only two sensors are needed. However, for the purpose of diagnostics it might be important to provide a full visibility in any possible transmission operation mode including the full neutral state and therefore to provide speed sensors on all independent speed states if possible in terms of packaging and acceptable in terms of cost. The causal bond graph models from Figs. 3-7 show that the Model T transmission has two independent and three dependent states, the six-speed transmission has three independent and three dependent states, while the ten-speed transmission has five independent and four dependent states.

The constructed bond graph models can be directly used for simulation studies in dedicated packages such as 20-Sim™. For vehicle system analysis, the transmission bond graphs described above can be integrated into the complete drivetrain bond graph as demonstrated in Fig 8 [3] where the transmission is abbreviated/represented by a **RI** field, i.e. a collection of resistive and inertia elements. The derived bond graph models can also be used for a systematic derivation of minimum order system equations which facilitate effective control analysis and synthesis.

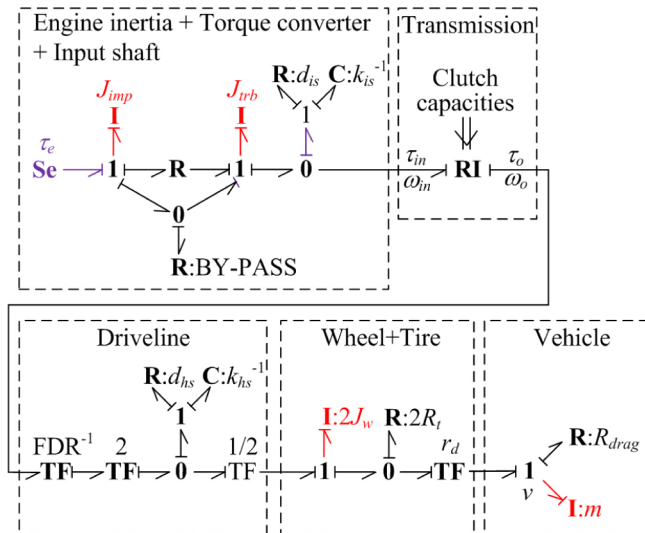


Fig. 8. Bond graph model of powertrain (nomenclature: *imp*, *trb*=torque converter impeller and turbine, *is*=input shaft, FDR=Final Drive Ratio, *hs*=half shaft, *t*=tire, *w*=wheel, *rd*=tire dynamic rolling radius, *m*=vehicle mass, *drag*=roll+aero+grade resistance, **RI**= resistance-inertia field).

DERIVATION OF DYNAMIC EQUATIONS

Depending on the transmission clutch application arrangement the system of dynamic equations is considered either as of full or reduced order. The full order system corresponds to the arrangement with all slipping clutches where all clutches have the control authority. Full order system is generally used for simulation purposes. The reduced order corresponds to clutch arrangements of specific speeds, transmission shifts where clutches can be either in open, slipping or locked states. The number of possible reduced order systems is 2^r where r is the number of clutches.

Full Order System

As explained above, the causality analysis provides information on the order of the system and classification of state variables associated with **I**-elements into independent and dependent. Transmission dynamics equations are derived by solving the system of equations comprising equations of equilibrium of torques on **1**-junctions related to n independent and m independent speed states and m equations of speed constraints in terms of time derivatives of momentums related to independent states.

Matrix equation of equilibrium of torques on **1**-junctions related to the independent states can be written as

$$\dot{\mathbf{p}}_{id} + \mathbf{\Gamma}_1 \boldsymbol{\tau} + \mathbf{\Gamma}_2 \boldsymbol{\tau}_{int} = \mathbf{0} \quad (3)$$

where $\dot{\mathbf{p}}_{id} \in \mathbb{R}^{n \times 1}$ is the vector of momentum time derivatives associated with independent states; $\boldsymbol{\tau} = [\tau_{in}, \tau_o, \boldsymbol{\tau}_c]^T$ is the vector of inputs where τ_{in} is the input shaft torque, τ_o is the output shaft torque, and $\boldsymbol{\tau}_c \in \mathbb{R}^{r \times 1}$ is the vector of clutch torques with r equal to the number of clutches; $\boldsymbol{\tau}_{int} \in \mathbb{R}^{m \times 1}$ is the vector of internal torques; $\mathbf{\Gamma}_1 \in \mathbb{R}^{n \times r+2}$; and $\mathbf{\Gamma}_2 \in \mathbb{R}^{n \times m}$. The internal torques are used to simplify the process of derivation of equations. The number of internal torques is equal to the number of independent states m . A general recommendation is to assign the internal torques to **0**-junctions that are adjacent or as close as possible to the dependent states as demonstrated on examples in Figs 3, 5, and 7.

Similarly, matrix equation of dynamics equilibrium of torques on **1**-junctions related to the dependent states is

$$\dot{\mathbf{p}}_d + \mathbf{\Gamma}_3 \boldsymbol{\tau} + \mathbf{\Gamma}_4 \boldsymbol{\tau}_{int} = \mathbf{0} \quad (4)$$

where $\dot{\mathbf{p}}_d \in \mathbb{R}^{m \times 1}$ is the vector of momentum time derivatives associated with dependent states, $\mathbf{\Gamma}_3 \in \mathbb{R}^{m \times r+2}$, and $\mathbf{\Gamma}_4 \in \mathbb{R}^{m \times m}$.

Matrix equation of speed constraints includes equations of element speeds on **0**-junctions related to the internal torques:

$$\mathbf{\Lambda}_1 \boldsymbol{\omega}_{id} + \mathbf{\Lambda}_2 \boldsymbol{\omega}_d = \mathbf{0} \Rightarrow \boldsymbol{\omega}_d = \underbrace{-\mathbf{\Lambda}_2^{-1} \mathbf{\Lambda}_1}_{\mathbf{N}} \boldsymbol{\omega}_{id} \quad (5)$$

where $\mathbf{\Lambda}_1 \in \mathbb{R}^{m \times n}$, $\mathbf{\Lambda}_2 \in \mathbb{R}^{m \times m}$, and $\mathbf{N} \in \mathbb{R}^{m \times n}$ is the matrix that relates independent and dependent states.

Combining Eqs. (4) and (5), taking $\mathbf{p}_{id} = \mathbf{I}_{id} \boldsymbol{\omega}_{id}$ and solving for $\boldsymbol{\tau}_{int}$ gives

$$\boldsymbol{\tau}_{int} = \mathbf{\Gamma}_4^{-1} (\mathbf{\Lambda} \dot{\mathbf{p}}_{id} - \mathbf{\Gamma}_3 \boldsymbol{\tau}) \quad (6)$$

Inserting Eq. (6) into Eq. (3) and taking $\mathbf{p}_d = \mathbf{I}_d \boldsymbol{\omega}_d$ yields the final system dynamic equation

$$\underbrace{(\mathbf{I}_{id} + \mathbf{\Gamma}_2 \mathbf{\Gamma}_4^{-1} \mathbf{\Lambda}_d \mathbf{\Lambda}_1^{-1} \mathbf{\Lambda}_1)}_{\mathbf{M}} \cdot \dot{\boldsymbol{\omega}}_{id} = \underbrace{(\mathbf{\Gamma}_2 \mathbf{\Gamma}_4^{-1} \mathbf{\Gamma}_3 - \mathbf{\Gamma}_1)}_{\mathbf{T}} \cdot \boldsymbol{\tau} \quad (7)$$

where $\mathbf{I}_{id} = \text{diag}(J_{d,1}, \dots, J_{d,m})$ and $\mathbf{I}_d = \text{diag}(J_{id,1}, \dots, J_{id,n})$ are diagonal inertia matrices of independent and dependent states, $\mathbf{M}$ is the inertia matrix and $\mathbf{T}$ is the torque matrix. Eq. (7) can be also written as