

A Geometrically Exact Isogeometric Blended Shell: Formulation, Benchmarking, and Automotive Application

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ABSTRACT

We present a geometrically exact isogeometric blended shell formulation. In other words, all geometric quantities appearing in the blended theory are evaluated exactly and no approximations are employed. The blended approach allows higher-order shell theories, like Kirchhoff-Love, to be combined with Reissner-Mindlin shell formulations, which have rotational degrees of freedom. In this way, rotations can be employed only where needed to simplify shell modeling such as at kinks and intersections. Away from these regions shear locking free formulations can be employed to improve robustness, accuracy, and efficiency. We compare our approach to standard shell elements commonly used in industry on several benchmarks to explore the behavior of the element. We then model an inner car hood to demonstrate our ability to handle complex CAD geometry in a simple manner without geometry cleanup and mesh generation steps. This shell element forms the basis of the newly developed Isogeomtrx structural solver.

Keywords: shells, isogeometric analysis, Reissner-Mindlin, Kirchhoff-Love, automotive applications

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INTRODUCTION

We present a geometrically exact isogeometric blended shell formulation. Our formulation is inspired by the blended shell formulation presented in [1]. However, we avoid geometric approximations leading to far more accurate solutions in the presence of curved geometry. We restrict our blending to include both five and six degree-of-freedom shear deformable Reissner-Mindlin shell kinematics as well as a three degree-of-freedom Kirchhoff-Love shell kinematic. However, our framework is general, and additional shell kinematics can be included (for example the shear deformable five degree-of-freedom rotation-free formulations in [2, 3]) with minimal effort. The blending simplifies structural modeling with smooth, higher-order shell elements in that kinks, complex shell intersections, and boundary conditions can be handled in a simple manner. Additionally, proper blending can dramatically reduce the number of degrees-of-freedom required to solve a problem. For example, five or six degree-of-freedom Reissner-Mindlin control points can be used to capture kinks and intersections while three degree-of-freedom control points can be used elsewhere since the basis is smooth enough to handle a higher-order Kirchhoff-Love kinematic description. This is critical to employing isogeometric shell elements for industrial sized problems. Eliminating rotational degrees-of-freedom, by increasing the smoothness and order of the basis, also leads to elements that are

shear locking free, an important consequence of employing the isogeometric paradigm. We benchmark the blended shell against the most common standard commercial shell finite elements. We make these comparisons on several carefully selected benchmark problems in the linear elastic regime.

Kirchhoff-Love (KL) shell theory assumes zero transverse shear strains and is appropriate for thin shells. A finite element implementation of KL theory requires higher-order C^1 -continuous basis functions. Traditionally, building finite element meshes for C^1 -continuous basis functions was prohibitively difficult. This resulted in the widespread adoption of thick-shell or shear deformable theories like the Reissner-Mindlin (RM) shell theory as the basis for shell finite elements. RM theory only require C^0 basis functions. However, RM shells introduce additional rotational degrees-of-freedom which complicate the implementation. With the advent of isogeometric analysis (IGA) and the adoption of smooth Computer-Aided Design (CAD) basis functions in engineering analysis the use of KL shell theory in the finite element method has greatly increased [4, 5, 6, 7] and resulted in new possibilities for the modeling of complex shell configurations.

B-Spline and Nurbs Fundamentals

A univariate B-spline basis is defined by a knot vector $\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+p+1}\}$, which consists of a non-decreasing sequence of real numbers, $\xi_i \leq \xi_{i+1}$, $i = 1, \dots, n + p + 1$ where p is the degree of the B-spline basis functions and n is the number of basis functions. The A^{th} B-spline basis function of degree p , denoted by $N_A^p(\xi)$, can be recursively defined as

$$N_A^0(\xi) = \begin{cases} 1, & \text{if } \xi_A \leq \xi < \xi_{A+1} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$N_A^p(\xi) = \frac{\xi - \xi_A}{\xi_{A+p} - \xi_A} N_A^{p-1}(\xi) + \frac{\xi_{A+p+1} - \xi}{\xi_{A+p+1} - \xi_{A+1}} N_{A+1}^{p-1}(\xi). \quad (2)$$

A B-spline curve can be written as

$$C(\xi) = \sum_A \mathbf{P}_A N_A^p(\xi) \quad (3)$$

where $\mathbf{P}_A$ is called a control point. To exactly capture conic sections a rational variant of the B-spline basis function is often used. A Non-Uniform Rational B-spline (NURBS) curve can be written as

$$C(\xi) = \sum_A \mathbf{P}_A R_A^p(\xi) \quad (4)$$

where

$$R_A^p(\xi) = \frac{w_A N_A^p(\xi)}{\sum_{A=1}^n w_A N_A^p(\xi)} \quad (5)$$

is a rational basis function and w_A is the weight corresponding to the A^{th} basis function. For notational clarity we often suppress the degree superscript on the basis functions and use N_A and R_A interchangeably.

THE PRINCIPLE OF VIRTUAL WORK

In this paper the geometrically exact blended shell formulation is restricted to linear elastic problems. The starting point is the principle of virtual work in the spatial configuration,

$$\delta \Pi(\mathbf{u}) = \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) : \delta \boldsymbol{\epsilon} d\Omega - \int_{\Gamma} \mathbf{t} \cdot \delta \mathbf{u} d\Gamma - \int_{\Omega} \mathbf{b} \cdot \delta \mathbf{u} d\Omega = 0 \quad (6)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress, $\boldsymbol{\epsilon}$ is the small strain tensor, $\mathbf{t}$ is the traction, and $\mathbf{b}$ is the body force.

SHELL GEOMETRY AND KINEMATICS

The initial shell body Ω_0 is described by the reference surface $\bar{\mathbf{X}}$ and an inextensible director vector field $|\mathbf{D}| = 1$ which defines the shell thickness. $\bar{\mathbf{X}}$ is parameterized by the convective coordinates s_1 and s_2 while the thickness of the shell is parameterized by s_3 , which ranges from -1 to 1. The reference configuration $\mathbf{X}$ in the global coordinates can be written as

$$\mathbf{X}(s_i) = \bar{\mathbf{X}}(s_\alpha) + s_3 \frac{h}{2} \mathbf{D}(s_\alpha) \quad (7)$$

where h is the shell thickness and the established convention for Latin and Greek indices (i.e., $i = 1, 2, 3$ and $\alpha = 1, 2$) is adopted. The reference director vector $\mathbf{D}(s_\alpha)$ is chosen to be the unit normal to the reference surface

$$\mathbf{D}(s_\alpha) = \frac{\bar{\mathbf{X}}_{,1} \times \bar{\mathbf{X}}_{,2}}{|\bar{\mathbf{X}}_{,1} \times \bar{\mathbf{X}}_{,2}|} \quad (8)$$

where $(\cdot)_{,\alpha}$ denotes $\partial(\cdot)/\partial s_\alpha$. Similarly, the current configuration $\mathbf{x}$ can be written as

$$\mathbf{x}(s_i) = \bar{\mathbf{x}}(s_\alpha) + s_3 \frac{h}{2} \mathbf{d}(s_\alpha) \quad (9)$$

where $\bar{\mathbf{x}}(s_\alpha)$ is the reference surface in the current configuration and $\mathbf{d}$ is the current director vector. Note that for the KL shell theory the current director vector $\mathbf{d}$ is set to the unit normal vector $\mathbf{n}$ with respect to the current reference surface, i.e.,

$$\mathbf{d} = \mathbf{n} = \frac{\mathbf{p}}{||\mathbf{p}||} \quad (10)$$

where

$$\mathbf{p} = \mathbf{x}_{,1} \times \mathbf{x}_{,2}. \quad (11)$$

The RM shell theory takes the shear deformation into account so that $\mathbf{d} \neq \mathbf{n}$.

The displacement of the shell is written as

$$\mathbf{u}(s_i) = \mathbf{x}(s_i) - \mathbf{X}(s_i) \quad (12)$$

$$= \bar{\mathbf{u}}(s_\alpha) + s_3 \frac{h}{2} [\mathbf{d}(s_\alpha) - \mathbf{D}(s_\alpha)] \quad (13)$$

$$= \bar{\mathbf{u}}(s_\alpha) + s_3 \Delta \mathbf{d}(s_\alpha) \quad (14)$$

where $\bar{\mathbf{u}}$ is the displacement of the reference surface and $\Delta \mathbf{d}$ is the increment of the director.

For a blended shell the increment of the current director, $\Delta \mathbf{d}$, is split into contributions from several different kinematic descriptions. The parts of the domain affected by each kinematic description are determined by placing surface control points into sets. The set containing all of the control points which define the surface is denoted by S . We denote the set of control points corresponding to KL shell theory by $KL3$, the set corresponding a standard RM 5-parameter shell theory by $RM5$, and the set corresponding to a standard RM 6-parameter shell theory by $RM6$. Note that

$$S = KL3 \cup RM5 \cup RM6 \quad (15)$$

and

$$KL3 \cap RM5 \cap RM6 = \emptyset. \quad (16)$$

Then the displacement $\mathbf{u}$ can be further specialized to

$$\mathbf{u} = \bar{\mathbf{u}} + s_3 (\Delta \mathbf{d}^{KL3} + \Delta \mathbf{d}^{RM5} + \Delta \mathbf{d}^{RM6}). \quad (17)$$

Note that control points in $KL3$ are assumed to be associated with basis functions defined over smooth portions of the domain. The control points in $RM5$ and $RM6$ are associated with basis functions which are smooth or non-smooth.

THE GRADIENT EXTRACTION OPERATOR

The evaluation of the current director, $\Delta \mathbf{d}$, for different shell formulations requires that certain quantities be lifted to control points. The lifting is achieved through a simple application of Bézier projection [8]. We compute a gradient extraction operator $G = [G_{BA}]$ such that

$$N_{B,\alpha} = \sum_A G_{BA}^\alpha N_A. \quad (18)$$

Using G the gradient of any spline field, ψ , with respect to the convective coordinates can be computed as

$$\psi_{,\alpha} = \sum_B N_{B,\alpha} \psi_B \quad (19)$$

$$= \sum_B \sum_A G_{BA}^\alpha N_A \psi_B \quad (20)$$

$$= \sum_A N_A \psi_A^\alpha \quad (21)$$

where

$$\psi_A^\alpha = \sum_B G_{BA}^\alpha \psi_B. \quad (22)$$

For blended shells the lifting operation is restricted to element domains.

THE KL SHELL DIRECTOR, $\Delta \mathbf{d}^{KL3}$

The KL shell director presented in this paper makes no geometric approximations. This leads to very accurate shell formulations for highly curved geometry regardless of mesh density. We have that

$$\mathbf{p} = \mathbf{x}_{,1} \times \mathbf{x}_{,2} \quad (23)$$

$$\mathbf{p}_{,\alpha} = \mathbf{x}_{,1\alpha} \times \mathbf{x}_{,2} + \mathbf{x}_{,1} \times \mathbf{x}_{,2\alpha} \quad (24)$$

$$\|\mathbf{p}\|_{,\alpha} = \frac{\mathbf{p}_{,\alpha} \cdot \mathbf{p}}{\|\mathbf{p}\|} \quad (25)$$

and

$$\mathbf{d} = \frac{\mathbf{p}}{\|\mathbf{p}\|} \quad (26)$$

$$\mathbf{d}_{,\alpha} = \left(\frac{(\mathbf{p}_{,\alpha} \otimes \mathbf{p}) - (\mathbf{p} \otimes \mathbf{p}_{,\alpha})}{\mathbf{p} \cdot \mathbf{p}} \right) \mathbf{d}. \quad (27)$$

The linearized increment $\Delta \mathbf{d}^{KL3}$ can be written as

$$\Delta \mathbf{d}^{KL3} = \frac{h}{2} \frac{1}{\|\mathbf{p}\|} (\mathbf{I} - \mathbf{d} \otimes \mathbf{d}) (\bar{\mathbf{u}}_{,1} \times \mathbf{x}_{,2} + \mathbf{x}_{,1} \times \bar{\mathbf{u}}_{,2}) \quad (28)$$

$$= \frac{h}{2} \frac{1}{\|\mathbf{p}\|} (\mathbf{I} - \mathbf{d} \otimes \mathbf{d}) \left(\sum_{A \in KL3} N_A \left(\sum_{B \in S} G_{BA}^1 \bar{\mathbf{u}}_B \right) \times \mathbf{x}_{,2} + \mathbf{x}_{,1} \times \sum_{A \in KL3} N_A \left(\sum_{B \in S} G_{BA}^2 \bar{\mathbf{u}}_B \right) \right) \quad (29)$$

$$= \frac{h}{2} \frac{1}{\|\mathbf{p}\|} (\mathbf{I} - \mathbf{d} \otimes \mathbf{d}) \sum_{A \in KL3} N_A \sum_{B \in S} (G_{BA}^2 \text{skew}(\mathbf{x}_{,1}) - G_{BA}^1 \text{skew}(\mathbf{x}_{,2})) \bar{\mathbf{u}}_B \quad (30)$$

$$= \mathbf{P} \sum_{A \in KL3} N_A \sum_{B \in S} \mathbf{C}_{BA} \bar{\mathbf{u}}_B \quad (31)$$