

Optimization of Contact Profiles using Super-Ellipse

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ABSTRACT

In many rolling or sliding contact mechanisms, like roller bearings, journal bearings, gearings or any contact between two semi-infinite bodies, concentrated stress occurs at the edges of the contact. This stress, due to what is called edge effect, is known to be very fatigue damaging. To avoid this concentrated stress in common roller bearings, the rolling surface of the roller and/or of the track is crowned. The crowning shapes usually include a straight line and one or several circular arcs. Even if such crowning profiles are efficient in avoiding edge effects, they are far from being optimized in terms of contact pressure distribution. In recent years, crowning shapes based on a logarithmic function developed by Lundberg have been improved and optimized for the purpose of industrialization. However, the resulting profiles are fairly complex and have only been developed for roller bearings and not for other types of rolling or sliding contacts. This paper presents a new profile type based on a super-ellipse equation (ellipse generalized to the order n). With this profile, it is quite straightforward to set parameters according to the intended application. It can be simply adapted to all kinds of contact by modifying the order of the super-elliptic profile. The benefits of the super-elliptic profile are an even contact pressure distribution and no edge effect, while it remains easy to manufacture. Several practical applications are presented, all of them related to the VCRI variable compression ratio technology developed by MCE-5 DEVELOPMENT. This technology features a wide range of contact mechanisms of various shapes, either sliding (side-pusher contact, journal bearings), rolling (synchronized roller contact), or both rolling and sliding (gear mesh contact).

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INTRODUCTION

The VCRI Variable Compression Ratio technology, developed by MCE-5 DEVELOPMENT, enables significant reduction in fuel consumption and pollutant emissions of a vehicle. This technology is based on a double gear and rack mechanism: a gear wheel meshes with the combustion rack (linked to the piston) and the control rack (enabling compression ratio variation). The combustion rack is guided by mean of a synchronized roller. These four innovative components induce new interactions between the different parts of the engine (see [Figure 1](#)).

The synchronized roller is rolling without sliding between a synchronizing plate, fitted to the crankcase, and the moving combustion rack. These interfaces can be considered as cylinder-to-cylinder contacts with one of the body having an infinite radius. To ensure a constant centre distance between the wheel and the two racks, the three components are fitted with tracks, one on the pitch circle of the gear wheel, the others on the pitch line of each rack. The two contacts between those tracks can also be considered as rolling cylinder-to-cylinder contacts, the length of the cylinders being far smaller than their radius. The contact between the gear wheel teeth and the racks teeth is a conventional spur gear contact, both rolling and sliding. Finally, the control rack is linked to the crankcase by mean of a side pusher. The back side of the control rack is slightly

arched to allow a small degree of freedom in rotation. Therefore, the contact between the side pusher and the control rack can be considered as a sliding cylinder-to-cylinder contact.

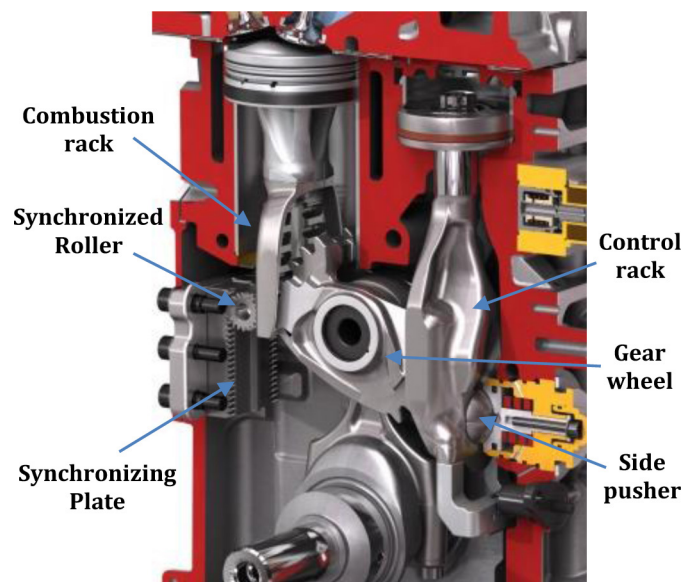


Figure 1. VCRI technology innovative components

All these interactions can be calculated with conventional formulas of the theory of elastic contact. For instance, the contact pressure p_0 between 2 cylinders of length L and radius R_1 and R_2 , with parallel axes can be calculated as:

$$p_0 = \left(\frac{E^* F}{\pi L R} \right)^{1/2} \quad (1)$$

Where F is the applied load and R and E^* the equivalent radius and Young modulus expressed as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

However this kind of formula considers a homogeneous distribution of the contact pressure along the length of the cylinder. In reality, when a finite plane surface comes into contact with another surface, a concentrated contact pressure appears at the edge of the contact. **Figure 2** shows the difference between the basic theoretical pressure, as calculated by the theory of elastic contact, and a more realistic contact pressure distribution obtained by Finite Element Analysis (FEA).

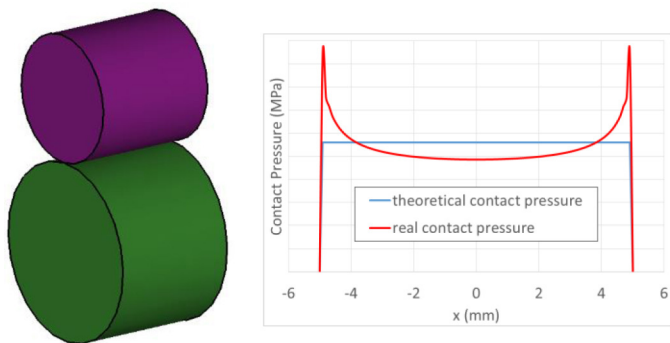


Figure 2. Difference between theoretical contact pressure and real contact pressure in the contact between 2 cylinders with parallel axes

With this FEA model, the edge contact pressure is nearly twice the nominal contact pressure, although in theory, with perfectly sharp edges, the edge contact pressure should be infinite. This concentrated stress is known to generate the maximum contact fatigue damage, as described in Glaeser and Shaffer (1).

Each of the interactions in a VCRi engine is subjected to edge effects. The purpose of this paper is to define, for each of these interactions, a profile correction which is tolerant to misalignments, easy to manufacture and suppressing any contact pressure concentrations.

CORRECTION PROFILES

Many profiles have been developed over the years to suppress the edge effects in such contacts. This chapter gives a quick overview of these profiles. To illustrate the effect of each profile on the contact pressure distribution, a simple FEA model is used. This model consists in a rectangular block pressed against a semi-infinite solid. The rectangular block is supposed to have an infinite length.

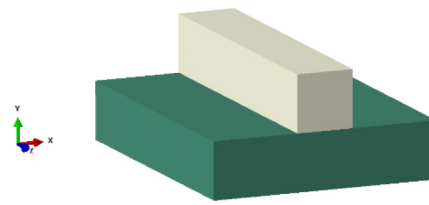


Figure 3. Theoretical model of a rectangular block

The different types of correction profile include the circular profile, which is the most commonly used, and the combination of a straight line and one or several circular arcs. A logarithmic profile has also been developed for roller bearings in order to optimize the contact pressure and the stress distribution.

Circular Profile

The circular profile is the simplest profile. It can be found for example on piston skirts or spur gears. It consists in a continuous circle, defined by its radius or by the maximum depth of the profile at the edge of the contact surface. **Figure 4** shows an example of contact pressure distribution with a circular crowning profile for a section of the simplified model of rectangular block.

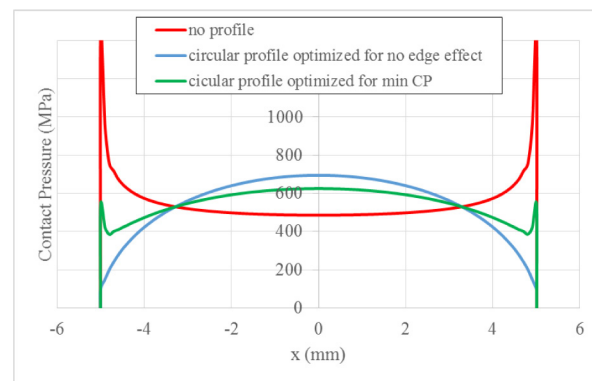


Figure 4. Contact pressure distribution for a circular profile

In this example, the mean contact pressure, which would be the ideal contact pressure if the distribution were perfectly homogeneous, is 534 MPa. The circular profile is very efficient to completely suppress the edge effect (blue line) but it has the disadvantage of creating a higher contact pressure at the centre of the contact (695 MPa, +30% compared with the ideal contact pressure distribution). One can parametrize this shape to allow a minor edge effect, keeping it smaller than the contact pressure in the centre of the contact (green line). In that case, the maximum contact pressure is 625 MPa (+17% compared with the ideal contact pressure distribution). However, this type of optimized profile is very sensitive to small misalignments between the 2 bodies in contact, resulting in an increased edge effect on one side, eventually causing damages to the bodies.

Trumpet Shape Profile

The trumpet shape profile is the combination of a straight line and a circular arc or a parabolic curve. It is to be noticed that with the small trumpet dimensions usually applied, the circular arc and the parabolic curve can be considered as coincident. This profile can be defined by

the length L_1 of the straight line or the length L_a of the circular arc, and the maximum depth D of the profile or the radius R of the circular arc. Figure 5 illustrates the 2 possible parameter configurations for trumpet shape profiles.

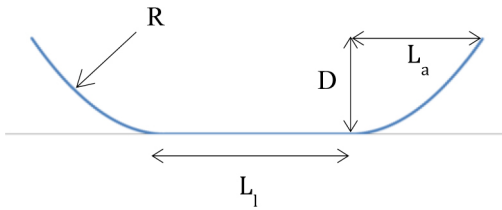


Figure 5. Parameters for the trumpet shape profile

Figure 6 shows the contact pressure distribution on the rectangular block model for a trumpet shape profile with a straight line of half the contact length. The contact pressure at the centre of the surface is lower than with a circular profile, and the maximum contact pressure is located at the transition between the straight line and the circular arc. The maximum contact pressure is 662 MPa for the profile optimized for no edge effect (24% higher than the ideal contact pressure distribution) and 613 MPa for the profile with limited edge effect (+15%).

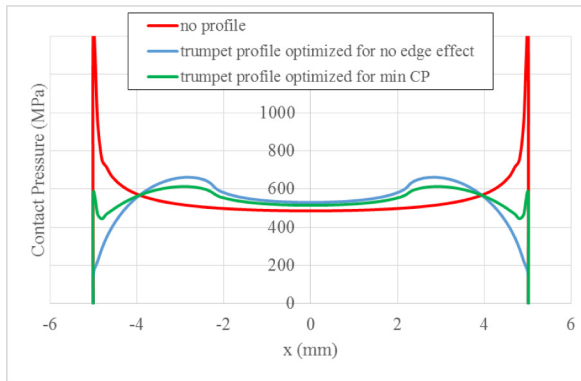


Figure 6. Contact pressure distribution for a trumpet shape profile

This type of profile is commonly used in piston pin bosses and in connecting rod small end bushes, in the latter case because, aside from being easier to machine, it has one main advantage over a circular profile in lubricated contact, i.e. the lower contact pressure in the centre of the surface tends to trap the oil between the two peaks of pressure, where the friction heat can be difficult to dissipate.

Yet, none of the 4 interactions in a VCRi engine, as described in the introduction, are in perfect lubricated condition, and contact pressure distributions with trumpet profiles are far from being perfectly homogeneous.

Logarithmic Profile

The logarithmic profile has been studied for decades. It was first developed for roller bearings by Lundberg (2), who was the first to establish crowning shape based on a logarithmic function:

$$y = A \ln \frac{1}{1 - K \left(\frac{x}{l} \right)^2} \quad (2)$$

Where l is half the surface length. The scale factor A works as a multiplier for the whole function and the Lundberg's profile factor K prevents that infinite dimensioning be given at profile ends.

Figure 7 shows the effect of a logarithmic profile on the theoretical model of rectangular block. The results show that the logarithmic profile gives a noticeably lower contact pressure and a very homogeneous distribution, compared with the circular profile modification. The maximum contact pressure for a profile optimized for no edge effect is 573 MPa, which is only 7% above the ideal contact pressure distribution. With limited edge effect, the value is even lower, with 562 MPa (+5%).

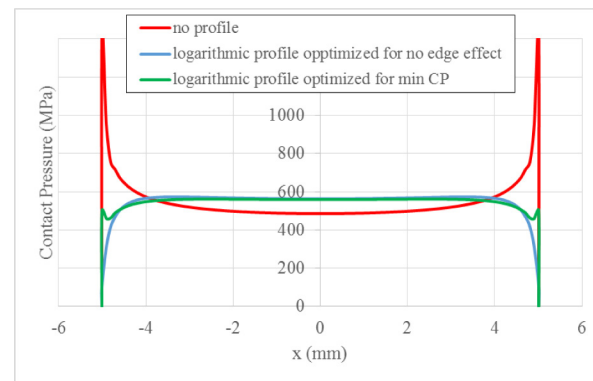


Figure 7. Contact pressure distribution for a logarithmic profile

A comparison was performed by Bergseth and Björklund (3) between a logarithmic profile and a circular profile on spur gears. It showed that the maximum contact pressure was indeed smaller with a logarithmic profile than with a circular profile. However, with important misalignments, this result could be reversed, with edge effects occurring even with the logarithmic profile. Johns (2) has developed a derivation of the Lundberg's profile factor K . Based on this derivation, Fujiwara and Kawase (4) introduced three new parameters to make it possible to design a logarithmically crowned shape that would not produce edge stress in a roller bearing even if the roller is tilted. Here is the resulting equation:

$$y = \frac{2K_1 F}{\pi l_e E^*} * \ln \left(\frac{1}{1 - (1 - 0.3033 K_m \frac{t}{s}) \left\{ \frac{x - (s - x_m)}{\frac{l_e}{2}} \right\}^2} \right) \quad (3)$$

Where x_m denotes the length of the straight section and is given by

$$x_m = \frac{1}{2} \sqrt{\frac{1 - \exp(-y_m \frac{\pi l_e E^*}{2 K_1 F})}{1 - 0.3033 K_m \frac{t}{s}}} \quad (4)$$

K_1 , K_m and y_m are design parameters, F is the applied load, E^* is the equivalent Young modulus, l_c is the effective contact length, s is half of the effective contact length, and t is half of the contact width.

However, this equation is quite complicated, with many parameters, and requires an optimization algorithm to be correctly customized for various applications. Other shapes are sometimes used such as the combination of multiple arcs or the combination of a logarithmic curve and a circular arc, but all these shapes are also very complex to set-up and optimize. The purpose of this research is to find an alternate profile that would be very adaptable and easy to customize.

SUPER-ELLIPTIC PROFILE

Definition

Super-ellipses are also known as Lamé's curves, and have been developed by Gabriel Lamé as a generalization of the ellipse. In the Cartesian coordinate system, the equation of a super-ellipse can be written as:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1 \quad (5)$$

Where a and b are the minor and major semi-axes of the super-ellipse, and n its order. If n is smaller than 1, the super-ellipse has the shape of a four-armed star with concave sides. For $n = 1$, the shape is a rhombus. When n is between 1 and 2, the curve looks like a rhombus with convex sides. For $n = 2$, the curve is an ordinary ellipse. When n is higher than 2, the super-ellipses are curves that lie between ellipse and rectangle (or circle and square if both semi-axes are equal). As an example, Figure 8 presents super-elliptic curves for $a = 2$ and $b = 1$ and for various values of the order. Only values of n equal or above 2 are of any interest in terms of optimized profiles for contact pressure, because below 2, tangent discontinuity at the centre of the profile would create a contact pressure concentration.

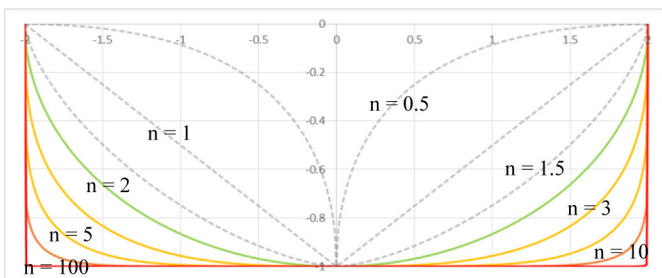


Figure 8. Shapes of the super-ellipse, depending on the order

It is important to note that even if super-ellipses look as though they have straight sides joined by curves, they are actually curved all along. Even where a segment looks straight it is actually slightly curved, with a calculable radius and centre of curvature. All those curves (with n equal or above 2) offer a wide range of shapes that are very easy to customize, depending on the application.

Application on the Rectangular Block

For primary test, the super-elliptic profile is applied to the simple FEA model of rectangular block. A parameter study is performed to determine the influence of the order of the super-ellipse and the depth of the profile on the block contact pressure distribution. First, the influence of the depth, the value of which is the minor semi-axis a of the super-ellipse, is studied for a simple ellipse (super-ellipse of order 2). Figure 9 shows the result of this parametric study. As one might have surmised, edge effects still appear on the limits of the surface if the depth of the profile is not sufficient ($6 \mu\text{m}$). If the depth of the profile is well adjusted ($12 \mu\text{m}$), the contact pressure distribution is very homogeneous. It is important to notice that when the profile depth is larger than the optimum depth (for example $18 \mu\text{m}$, i.e. 50% larger), the contact pressure distribution is still satisfyingly homogeneous, with a maximum value (593 MPa) still lower than the maximum value for an optimized circular shape (625 MPa).

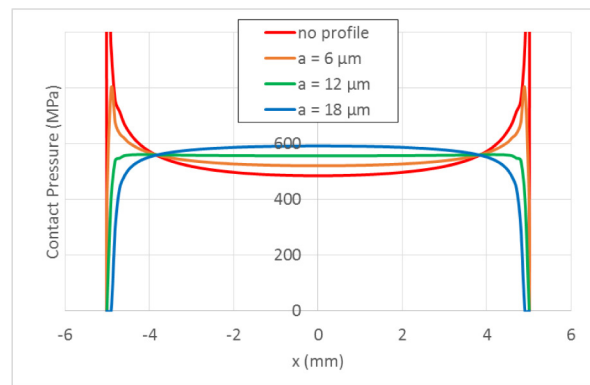


Figure 9. Influence of the depth of the super-elliptic profile on the contact pressure distribution for a rectangular block

The influence of the order of the super-elliptic profile is calculated for the adjusted profile depth of $12 \mu\text{m}$. The results are shown in Figure 10. When the order of the super-ellipse is increased, the contact pressure at the centre decreases, while the contact pressure on the edges increases. The optimum depth has to be adapted to the order of the super-ellipse to minimize the contact pressure. But it is worth noticing that, with the same $12 \mu\text{m}$ depth, the more important pressure on the edges does not look like ordinary edge effects, with very high and abrupt peaks.

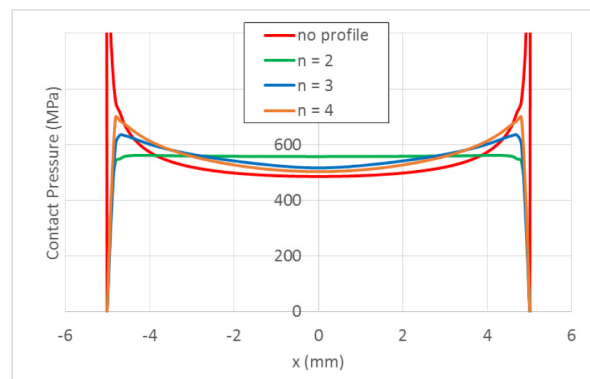


Figure 10. Influence of the order of the super-elliptic profile on the contact pressure distribution for a rectangular block