

Six-Degree-of-Freedom Accelerations: Linear Arrays Compared with Angular Rate Sensors in Impact Events

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ABSTRACT

SAE J211 provides no definitive specification as to the appropriate procedures for filtering angular rate sensor data prior to differentiation into angular acceleration data, especially for impact data. Accordingly, a 3-2-2 array (nine-accelerometer-package or NAP) of linear accelerometers and a triaxial angular rate sensor were mounted into a Hybrid III 50th-percentile-male ATD headform and compared in a variety of impact events and multibody simulations. Appropriate low-pass digital filter cutoff frequencies for differentiating the angular rate sensor data into angular accelerations were sought via residual analysis in accordance with current SAE J211 guidelines. It was found that there was no specific cutoff frequency that consistently matched the angular accelerations determined by the 3-2-2 array, and the combination of a triaxial linear accelerometer and a triaxial rate sensor could not simultaneously match the 3-2-2 summed square and peak angular accelerations regardless of cutoff frequency. The differentiation step to convert angular velocity data into angular acceleration data was found to insert frequency-dependent noise which low-pass filtering could not adequately eliminate. This may be overcome in the future if the noise floor for angular rate sensors can be constructed to exhibit less noise than contemporary linear accelerometers. While angular rate sensors are essential for long-duration accelerometer recording and are effective for analyses requiring integration, care must be taken when they are used for impact analyses. Currently, for impact analyses, a 3-2-2 array or a 6DOF array paired with an angular rate sensor will generate results with lower noise, and should be used wherever possible.

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INTRODUCTION

Both linear and angular metrics are important to analyses of head injury mechanics. Linear and angular accelerations have been correlated with brain injury, and linear acceleration has been correlated with skull fracture [1, 2, 3, 4, 5, 6]. Many novel head injury metrics, such as BrIC, GAMBIT, HIP, PRHIC, Principle Component Score (PCS), RIC, RVCI, and $\Delta\omega_{\text{peak}}$ require knowledge of angular terms (velocities and accelerations) [7–8]. Angular terms are also influential in determining linear acceleration injury metrics; as six-degree-of-freedom (6DOF) data is required to transfer data collected on the perimeter of an object (usually the head) to center of mass (COM) linear accelerations [9, 10, 11]. 6DOF data is most important when studying brain injury mechanisms and human tolerance to acceleration in situations with complicated kinematics where the activity under study is not amenable to high-speed video analysis [5, 7], such as helmet impacts.

To understand brain injury tolerances, analysis of dynamic loadings in real-world and testing environments must be investigated. Studies performed with human subjects often use angular rate sensors as part of 6DOF systems due to their smaller geometric size requirement,

reduced channel count, and stability in long-duration measurements [12, 13, 14, 15, 16, 17, 18, 19]. It is increasingly common for researchers to perform ATD testing with angular rate sensors mounted in a headform either in replacement of a linear accelerometer array [20–21], or supplementing one [22, 23, 24, 25]. However, while data processing and filtering methods are standardized and straightforward when using a linear accelerometer array [26], how to handle angular rate sensor data is less established, especially for impact or other short-duration events [26–27].

Measuring Angular Velocity and Acceleration

There are a variety of ways to either measure or calculate angular velocity and angular acceleration. Theoretically, only six linear accelerometers are needed to solve the three linear and three angular terms, although in practice nine accelerometers are required because it provides improved error-correction and an algebraic means of calculating angular acceleration [27–28]. Two-dimensional (planar) solutions based on linear accelerometers have existed since the late 1960s and the three-dimensional solution since 1975 (Figure 1), also known as the nine-accelerometer package (NAP) [10].

3-2-2-2 Linear Accelerometer Array (NAP)

Padgaonkar et al. described a nine-accelerometer array of linear accelerometers in a 3-2-2-2 configuration that offered matched pairs of accelerometers to cancel Coriolis components of angular accelerations [28]. This yielded angular acceleration Eqs (1–3) independent of angular velocity terms. In these equations, A represents linear acceleration, ρ represents the offset distance from the center of mass, and ω represents angular velocity. The subscripts describe the axis and sensor position. Angular velocities were calculated as the numerical time integral of the angular accelerometer data. DiMasi provided an exhaustive analysis of the NAP implementation in standard Hybrid-III headforms, including how to handle non-coplanar sensors, alignment errors, and non-centroidal seismic mass locations for “COM” accelerometers [10].

$$\dot{\omega}_x = \frac{(A_{z1} - A_{z0})}{2\rho_{y1}} - \frac{(A_{y3} - A_{y0})}{2\rho_{z3}} \quad (1)$$

$$\dot{\omega}_y = \frac{(A_{x3} - A_{x0})}{2\rho_{z3}} - \frac{(A_{z2} - A_{z0})}{2\rho_{x2}} \quad (2)$$

$$\dot{\omega}_z = \frac{(A_{y3} - A_{y0})}{2\rho_{x2}} - \frac{(A_{x1} - A_{x0})}{2\rho_{y1}} \quad (3)$$

The advantages of the 3-2-2-2 configuration were numerous versus the six-channel configurations, at the cost of increased channel count and size. The NAP configuration, due to the required numerical integration step to calculate angular velocity, is prone to cumulative integration error from bias, drift, misalignment, or vibration. Any measurement error will be propagated through all later times [27, 29].

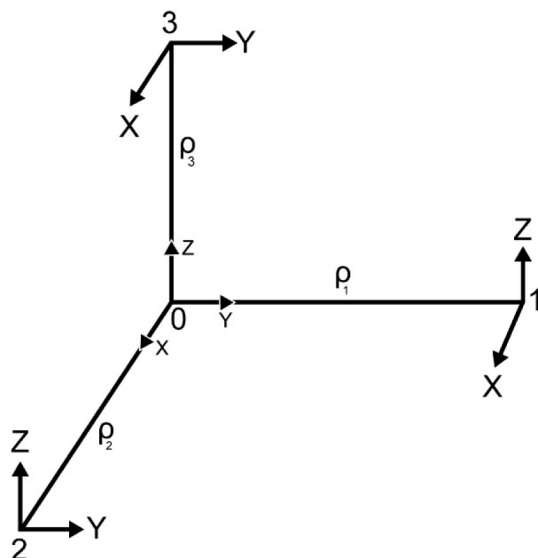


Figure 1. Adapted from Padgaonkar, Krieger et al. 1975

The 3-2-1 or 3-1-1-1 Array

Although theoretically equivalent to the 3-2-1 configuration, in practice the 3-1-1-1 is not favored, as it offers no advantages over the 3-2-1 design in terms of channel count or mathematical rigor, but takes up more space by its three-dimensional nature. The 3-2-1 array is generally presented as a configuration consisting of a triaxial linear acceleration unit at the origin with a pair of y-axis and z-axis oriented accelerometers on an arm along the x-axis with a single z-axis oriented accelerometer located on an arm along the y-axis; the equations for angular acceleration for this configuration consist of Eqs 4–6 [27, 30, 31, 32].

$$\dot{\omega}_x = \frac{(A_{z1} - A_{z0})}{\rho_{y1}} - \omega_y \omega_z \quad (4)$$

$$\dot{\omega}_y = -\frac{(A_{z2} - A_{z0})}{\rho_{x2}} + \omega_x \omega_z \quad (5)$$

$$\dot{\omega}_z = \frac{(A_{y2} - A_{y0})}{\rho_{x2}} - \omega_x \omega_y \quad (6)$$

Eqs 4–6 demonstrate that the angular acceleration components require knowledge of the angular velocity terms. Although these differential equations can be solved numerically, the solution is unstable and because of errors in the experimentally measured accelerations, stepwise integration results in an accumulation of error in the angular accelerations that in turn propagate to the angular velocity terms [31–32]. The result is angular terms that rapidly lose accuracy, regardless of the how the six linear accelerometer channels are analyzed [27, 30, 31, 32].

Mixed Arrays of Linear Accelerometers and Angular Rate Sensors

Studies have also used arrays with more exotic combinations of linear accelerometers [33–34] or 6DOF systems using combinations of triaxial linear accelerometers and triaxial angular rate sensors. Such arrays have been favorably compared to the angular velocity and angular acceleration calculations of linear accelerometer arrays [23, 35, 36, 37]. They offer direct measurement of angular velocity terms and the advantage of reduced size and channel count, which is useful for studies involving sensors worn for more than a brief period of time. The primary difficulty with the use of angular rate sensors is the numerical differentiation required to calculate angular accelerations. Noise components are preferentially increased by numerical differentiation, and over-filtering is usually employed to compensate for this [38]. The error caused by differentiation is transient and applies only to each individual time step, as opposed to the persistent nature of integration errors.

The 3 ω Array

An increasingly common mixed array is a combination of three linear accelerometers and three angular rate sensors, often implemented as two triaxial arrays or one cluster with the three angular rate sensor channels mounted to their corresponding channel of a triaxial linear accelerometer. This array type has been shown to be effective for determining angular kinematics, including during impacts [35, 36, 37, 39]. A six-channel (3 ω) combination of linear accelerometers and angular rate sensors is theoretically advantageous in that no such calculations are needed to directly measure six terms, and angular accelerations can be independently calculated as the time derivatives of the angular velocity terms [24, 27, 36, 40]. Because multiple sensor locations are not required, direct measurement of angular velocities is less susceptible to error from bias, drift, and misalignment.

The primary limitation of 3 ω -based arrays is that a numerical differentiation step is required to calculate the angular accelerations.

Theoretically, differentiation error does not accumulate over time as integration error does, and is less sensitive to drift and bias [27]. However, differentiation preferentially amplifies high-frequency noise and can lead to grossly inaccurate angular acceleration results [24, 27, 38]. In practice, numerical differentiation of inherently noisy experimental data is problematic, and requires care in the selection of filtering methods. Unfortunately, there is little standardization in how data should be filtered prior to differentiation, and methods in the literature vary considerably [13, 14, 19, 21, 36, 40].

The 6 ω Array

One means of avoiding both the numerical integration step required by the 3-2-2-2 linear array and the numerical differentiation step required by the 3 ω array is to construct a 6 ω array, which can take the form of a 3-2-1 array or a 3-1-1-1 array with an additional triaxial array of angular rate sensors. Kang et al. provided an analysis of a 3-1-1-1 array with the angular rate sensors mounted to each non-centroidal arm [25]. They found that the 6 ω array yielded angular acceleration results that were closer to that of the NAP than a 3 ω array, transformed peripheral linear accelerations to the COM better than the 3 ω array and at least as well as the NAP, yielded angular velocity data more accurate than the NAP, and provided angular velocity data which could be integrated into angular displacement data at higher accuracy than the NAP [24–25]. With sufficiently capable angular rate sensors, all channels of the 6 ω array can be filtered to CFC1000, which preserves time-integrity for transformation purposes [24].

The limitation of 6 ω -based arrays is that they are just as channel count and geometrically demanding as the 3-2-2-2 NAP, and the individual sensors are often slightly heavier than the linear accelerometers they replace. This can make the 6 ω array difficult to accommodate in applications with limited space, and it is a more costly solution.

Angular Accelerometers

There is a paucity of data on the use of sensors which directly measure angular acceleration of the head. Angular accelerometers have mostly been used for modal analyses [42], and little is available regarding their efficacy in an automotive or biomechanical context. These sensors tend to be larger and with a lower frequency response than the linear accelerometers and angular rate sensors typically used. A CFC1000-capable angular accelerometer does exist (K-Shear® 8838/8840, Kistler Instrument Corporation, Amherst, NY), however it is approximately four times larger and six times more massive on a per-channel basis than a comparable angular rate sensor. A 3-3 array of linear and angular accelerometers (3 α) may offer no advantages over a 3-3 array of linear accelerometers and angular rate sensors (3 ω), because it merely trades the problems of integration for differentiation, and the time-history for integration accumulates error. A 3-3-3 array of linear accelerometers, angular rate sensors, and angular accelerometers is viable and would directly measure all nine quantities. However, this array offers no means of redundantly calculating linear accelerations, unlike the 3-2-2-2 or 6 ω arrays.

Filtering

For short-duration impact analyses, the numerical integration required to calculate angular velocity by arrays of linear accelerometers is generally preferred to the numerical differentiation required to calculate angular accelerations from angular velocities, as numerical integration acts as a low-pass filter to small errors in angular acceleration calculations. However, these integrations are stable for only a short period of time (on the order of seconds), before cumulative error from sources such as inaccurate zero-bias, sensor drift, and small mismatches between sets of accelerometers overwhelm the signal under analysis [27, 28, 29, 30, 32]. This is an obstacle to impact measurement in scenarios where there may be a series of discrete impacts, where there is significant non-negligible pre-impact motion, or where initial angular velocity is non-zero and of relevance. Examples of these scenarios would be occupant kinematics in rollovers, bicycle-automobile collisions, or forklift tipover or off-dock scenarios.

Short-duration (0-200 ms) Events

For short-duration events, linear arrays of accelerometers are customary and have been in wide use since the mid-1970s. Filtering for linear accelerometers is straightforward and defined by SAE J211 as a CFC1000 (1650 Hz cutoff) low-pass filter for head accelerations [26]. The state of the art is less defined for angular rate sensors. SAE J211 specifies no appropriate channel frequency class for differentiation of angular rate sensor data, and the footnote referenced defines a means of analyzing long-duration events [27].

Many authors have analyzed power spectral densities on a case-by-case basis, or simply resorted to an *ad hoc* cutoff of 300 Hz (sometimes described as CFC180) [13, 14, 19, 21, 24]. Occasionally, higher cutoff frequencies are chosen [36 40–41].

Laughlin compared the angular accelerations from differentiated 1000 Hz magnetohydrodynamic (MHD) angular velocity data to calculated angular accelerations from a 3-2-2-2 linear accelerometer array and found the MHD data to be equal to or superior to the linear array data [40]. Martin et al. investigated MHD angular rate sensors versus a rotary potentiometer and 10000 fps high-speed film, and found 600 Hz to be an appropriate low-pass filter cutoff [36]. Marshall and Guenther compared multiple types of rate sensors to potentiometer data, a 3-2-2-2 linear accelerometer array, and high-speed video data, and found MHDs had acceptable results at CFC1000 for pendulum head-form drops with and without impact [35]. It is noteworthy that the ARS-1 MHD used in that study only had a frequency response up to 1000 Hz; a 1650 Hz low-pass filter would have been of limited use to that rate sensor. Voo et al. similarly compared an ATA DynaCube 3 MHD to high-speed video and a 3-2-2-2 linear accelerometer array, with all electronic data filtered at CFC1000. However, the authors found that the angular velocity data was limited to 100 Hz and the linear acceleration data was limited to 300 Hz, so lower cutoff frequencies could have been used [37]. Kang et al. in an analysis of the 6a ω array, also processed the data as a 3a ω array. They found using low-pass filters of CFC60 (100 Hz) appropriate for low-severity impacts and CFC180 (300 Hz) appropriate for high-severity tests. CFC60 was found to overly reduce peak angular acceleration for high-severity impacts. In general, they found that the 3a ω array was either excessively noisy or excessively reduced in peak magnitude [24]. Funk et al., in a mouthpiece-based study, low-pass filtered all head kinematic data at CFC180. For differentiation into angular acceleration data, they filtered both prior to and after numerical differentiation [14]. Camarillo et al. in a comparison to a mouthpiece accelerometer system, low-pass filtered both angular rate sensor and linear accelerometer data to CFC180 prior to transformation [13]. Suderman et al. analyzed impacts to an ATD wearing a hardhat, and, based on a power spectral density analysis, determined that a 100 Hz low-pass filter was appropriate for all tests in that series [21]. Lloyd and Conidi indicated they filtered COM linear accelerations to CFC1000 and COM angular rate sensor data via a phaseless 8th-order Butterworth low-pass filter with a 500 Hz cutoff. The data were numerically differentiated using a 5-point least-squares quartic equation [41]. For purposes of comparison to the NAP, Siegmund et al., filtered mouthpiece angular rate sensor data to 300 Hz, using a phaseless 4-pole Butterworth filter [19].

Long-Duration (>200 ms) Events

Bussone et al. found per-event appropriate low-pass filter cutoff frequencies for differentiating the angular rate sensor data into angular accelerations by using a residual analysis to find individual cutoff frequencies for the three center of mass (COM) linear accelerometer channels and the three angular rate sensor channels and taking the arithmetic mean of the six cutoffs. This was effective because residual curves were reasonably insensitive to small variations in the cutoff frequency in the band between linear and rotational cutoff frequencies. The angular rate sensors provided more accurate rotational velocities than integrated angular accelerations calculated from a 3-2-2-2 NAP, especially for events lasting longer than 200 ms [27].

Optimal Filtering

Filtering is a process that seeks to balance signal preservation and noise reduction. Any analysis of a real signal requires a compromise between the amount of distortion of true signal accepted and the amount of noise rejected. By necessity, as one reduces noise passed one also increases signal lost. In theory, a given angular velocity signal contains a maximum signal frequency and an optimal cutoff frequency which yields a subjectively noise-free velocity and acceleration which would allow a “true differentiation” of the whole signal [38]. A method used in kinematics analyses and employed in prior analyses of differentiation of angular velocities of the head is to self-analyze a given channel’s frequency content and accept the point at which the lost signal and lost noise are equal, or an analysis of residuals [43]. Winter’s method quantifies the difference between the filtered and unfiltered signals as a function of the filter cutoff frequency across a wide range of possible filter frequencies. The calculation of residuals is shown in Eq 7.

$$R(f_c) = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \hat{X}_i)^2} \quad (7)$$

where X_i is raw data at the i^{th} sample and $\hat{X}_i$ is filtered data at the i^{th} sample.

A theoretical signal containing only random noise would exhibit a residual plot consisting of a straight line decreasing from an intercept at 0 Hz to an intercept on the abscissa at the Nyquist frequency (half the sample rate). The magnitude of the intercept value would represent the root-mean-square (rms) value of the noise. When a signal consists of a true signal combined with noise, the residual will rise above the straight line as the cutoff frequency is reduced. This represents the signal distortion caused as the cutoff is reduced. By setting equal the amount of signal distorted with the amount of noise passed, a cutoff frequency can be chosen. This is found by projecting a horizontal line from the noise intercept until it intercepts the residual line. The index of the intersection is an optimal cutoff frequency. This is depicted in Figure 2. This method assumes noise is randomly distributed and white, and the true signal of interest has a maximum frequency below the Nyquist frequency. This method has been found to be effective for long-duration head accelerations generated by inertial head motions [27] because it tends to arrive at a solution range consistent with least-squares matches to reference solutions. It has not yet been validated for impact-based head kinematics.

A complication of filtering three-dimensional kinematics data is that all three axes channels must be filtered to the same cutoff frequency. Because low-pass filters shift data in the time domain as a function of order and frequency, to avoid inconsistent time-shifting of filtered data, all axes must be filtered to the same cutoff. Accordingly, choosing a universal cutoff requires balancing the filtering needs of each component channel. This is also a complication of integrating angular rate sensor data with linear accelerometer data, especially when the frequencies responses and filtering requirements are not equal.

Winter's Residual Method

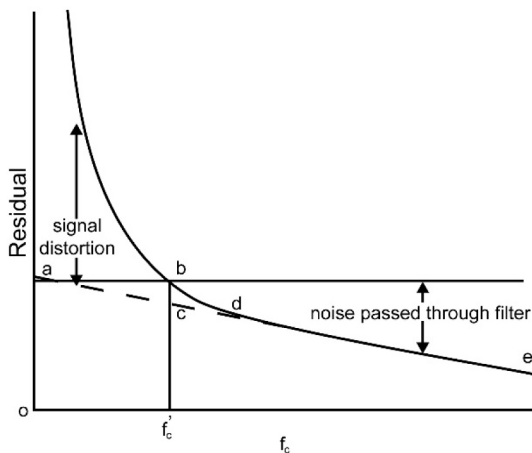


Figure 2. Plot of residuals of filtered and unfiltered signal as a function of filter cutoff frequency. Adapted from Winter, 2004.

Arrays of linear accelerometers, specifically the 3-2-2-2 configuration, are time-proven, standards-compliant, and validated for analysis of impact events. However, it is increasingly common for researchers to use angular rate sensors to capture 6DOF kinematics at reduced cost or with reduced channel counts. Less settled is an equivalent appropriate filtration for angular rate sensor data into angular acceleration data by numerical differentiation for impact events. The purpose of this study was to identify an SAE J211-compliant, *a priori* filtering method for angular velocity data in short-duration (impact) events that most closely matched angular acceleration data produced by linear arrays of accelerometers, and then to generalize that method to determine appropriate filters for angular rate sensor data in impact events. This was performed via a combination of physical testing and multibody simulation. Physical testing provided performance data in real-world environment that allowed the analyses of rate sensor data to be compared to an array of linear accelerometers. The multibody simulation provided data and kinematic results which were objectively correct and not dependent on an intermediate comparison to a reference standard, which while validated and widely-accepted, is not without limitation.

METHODS

ATD Testing

The head of a 50th percentile male Hybrid-III ATD (1846-D head, Denton ATD, Milan, Ohio) was instrumented with nine linear accelerometers (7264-2000, Endevco, San Juan Capistrano, California) in a 3-2-2-2 configuration in accordance with DiMasi [10]. The full scale range of the accelerometers was ± 2000 Gs with a resolution of 0.061 Gs/bit (16-bit system). The ATD head was also instrumented with triaxial angular rate sensors (ARS-1500, Diversified Technical Systems, Seal Beach, California). Channel orientations were in accordance with the SAE J211-2014 standardized dummy head coordinate system: x-axis positive forward, y-axis positive right, z-axis positive down [26]. The full scale range of the angular rate sensors was ± 1500 deg/sec, with a resolution of 0.732 deg/sec/bit (12-bit system). To record the data, the linear accelerometers were connected to a 16-bit data acquisition

system (PicoDAS, EME Corporation, Arnold, Maryland). The rate sensors were connected to a 12-bit data acquisition system (NanoDAS, EME Corporation, Arnold, Maryland), as less resolution was necessary for the rate sensors due to the acceleration events using more of their dynamic range. The data acquisition systems were sampled at frequencies up to 20 kHz per channel.

The ATD was exposed to direct impact head motions with a rigid rubber-covered mallet about each axis for linear and angular motion by the following mechanisms:

1. The ATD was struck in the face at approximately the level of the COM.
2. The ATD was struck in the head from the right side at approximately the level of the COM.
3. The ATD was struck in the head from the right side more forcefully at approximately the level of the COM.
4. The ATD was struck near the vertex of the head from the right side.
5. The ATD was struck upwards at the chin.
6. The ATD was struck in the face from the right side at the level of the nose.

All ATD data processing was completed using GNU Octave (4.0.1). Digital filtering was done using phaseless, four-pole Butterworth low-pass filters, in accordance with SAE J211-2014 [26, 44–45]. Data were untrimmed and feature separate impacts with interval periods without motion. Linear accelerometer data were filtered to CFC600 to match the frequency response characteristics of the ARS-1500 rate sensor, in order to make a like-for-like comparison.

Differentiation of filtered angular velocity data into angular acceleration data was performed using three different numeric differentiation equations:

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad (8)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \quad (9)$$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} \quad (10)$$

In these equations, f represents the value of the angular velocity data at a specific point in time, x represents the specific point in time, and h represents the difference in quantity between two adjacent points in time.

Angular rate sensor data was analyzed by sweeping cutoff frequencies from 1-1000 Hz to determine a single cutoff frequency that best matched the resultant on a sum-of-the-square basis and which best matched the peak value of the resultant. Data were also analyzed to determine cutoff frequencies that yielded least-square or peak value matches within $\pm 5\%$ of the optimum value.