

Optimal Tire Force Control & Allocation for Longitudinal and Yaw Moment Control of HEV with eAWD Capabilities

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ABSTRACT

Hybrid Electric Vehicles (HEV) offer improved fuel efficiency compared to their conventional counterparts at the expense of adding complexity and at times, reduced total power. As a result, HEV generally lack the dynamic performance that customers enjoy. To address this issue, the paper presents a HEV with eAWD capabilities via the use of a torque vectoring electric rear axle drive (TVeRAD) unit to power the rear axle. The addition of TVeRAD to a front wheel drive HEV improves the total power output. To further improve the handling characteristics of the vehicle, the TVeRAD unit allows for wheel torque vectoring at the rear axle. A bond graph model of the proposed drivetrain model is developed and used in co-simulation with CarSim. The paper proposes a control system which utilizes tire force optimization to allocate control to each tire. The optimization algorithm is used to obtain optimal tire force targets to at each tire such that the targets avoid tire saturation. The Youla parameterization technique is used to develop robust tracking controllers for each axle. The proposed control system is ultimately tested on the drivetrain model with a high fidelity CarSim vehicle model for validation. Simulation results show that the control system is able to maximize vehicle longitudinal performance while avoiding tire saturation on a low μ surface. More importantly, the control system is able to track the desired yaw moment request on a high speed double lane change maneuver through the use of the TVeRAD to improve the handling characteristic of the vehicle.

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INTRODUCTION

In an effort to reduce emissions and improve fuel economy, many automotive manufacturers have developed several successful Hybrid Electric Vehicle (HEV) designs. Fuel consumption of HEVs are in the range of 20-30% lower than their conventional counterparts[1]. A significant factor in the reduction of fuel consumption is the use of smaller internal combustion engines and, in some cases, an overall reduction in output power available from the hybrid powertrain. As a result, HEVs lack the dynamic driving characteristics found in similar sized conventional vehicles. In an effort to maintain high fuel economy and improve driving performance, HEVs with electric All Wheel Drive (eAWD) capabilities have been proposed. Such a vehicle utilizes a traditional hybrid drivetrain at the front axle and electrifies the rear axle with electric rear axle drive (eRAD). eAWD HEV using mechanical couplings between the front and rear axles are a popular form of implementation. It has been shown that these type of systems can improve performance and vehicle stability via proper coordination of rear axle torque control [2] and by controlling the mechanical coupling itself [3].

The eAWD drivetrain architecture presented in this paper uses "through the road" eAWD functionality – the front and rear axles are completely mechanically independent from one another. This type of architecture requires a much more sophisticated control system to actively manage wheel torque to the front and rear axles as compared to mechanically coupled eAWD systems. In an effort to further improve the performance of the base vehicle, the drivetrain architecture uses a torque vectoring eRAD (TVeRAD) which can actively distribute wheel torque from one side of the axle to the other.

This paper proposes a control system for vehicle longitudinal and yaw control utilizing longitudinal tire force optimization and allocation. The proposed control system is an extension of the slip ratio control and allocation strategy proposed in[4]. The proposed control system utilizes optimization as a means of longitudinal tire force target generation. The optimization allocates tire force targets to the front and rear axles such that the driver demands are met and tire force saturation is avoided. Low level controllers use the appropriate motor torque and torque vectoring inputs to track the optimal tire force targets.

The paper is broken up as follows:

- A model of the eAWD HEV drivetrain with the TVeRAD is developed using bond graphs. A linear system analysis of the TVeRAD is presented in order to obtain crucial information on the system for control development
- The tire force optimization and allocation strategy is developed which uses driver demands and feedback signals from the vehicle to dynamically generate optimal tire force targets for the front and rear axle.
- The development of low level tracking controllers is presented using the Youla parametrization technique.
- Finally, simulation results are presented to showcase the performance of the proposed control system under a low μ launch and a high speed double lane change maneuver.

SYSTEM MODELING

Models of sufficient complexity are required in order to represent the dynamics of the system. Bond graphs allow easy and intuitive modeling of multi energy domain systems; therefore, all systems modeled in this paper are modeled using Bond Graphs[5]. In this section a drivetrain model of a HEV with eAWD capabilities is introduced. The torque vectoring eRAD (TVeRAD) hardware is introduced and a simple model is developed.

eAWD HEV Drivetrain Model

The general drivetrain architecture used in the eAWD HEV proposed in this paper is given in Figure 1. As can be seen from the figure, the front axle is powered by the traditional Power Split (PS) series-parallel hybrid transaxle with an open differential. The rear axle is powered by an electric rear axle drive (eRAD) unit. The particular eRAD proposed uses a main traction motor to provide the traction force to the wheels and a small torque vectoring motor that can actively distribute wheel torque from one wheel to the other. Thus the eAWD HEV utilizes a torque vectoring eRAD (TVeRAD) to power the rear wheels.

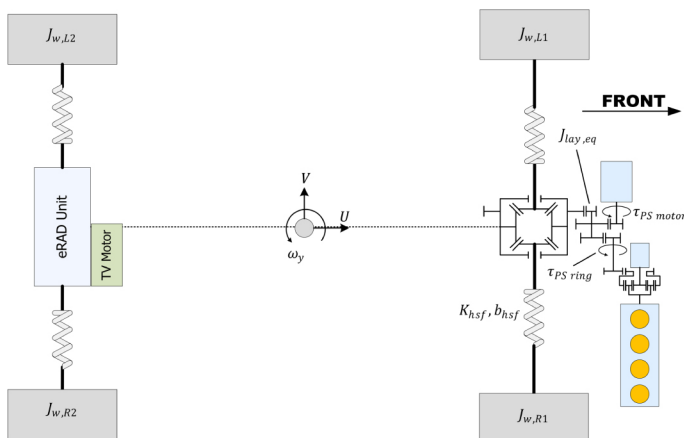


Figure 1. Drivetrain architecture of proposed eAWD HEV

To give a better representation of the system, Figure 2 presents a much more detailed schematic of the TVeRAD. The main traction motor is placed between two planetary gearsets and powers the sun gear in each of the gear sets. The carrier of each planetary gear is connected to the halfshaft of each wheel. The ring gears are connected together via a balance shaft and an idler gear. Note that the idler gear does not introduce a gear ratio, but simply changes the direction of rotation to the R2 wheel. Finally, the small torque vectoring motor is geared directly to the balance shaft[6].

During straight line driving, the ring gear and balance shaft remain stationary and thus the two wheels move forward at the same speed. During cornering, the balance shaft rotates to balance the speeds of the ring gears and thus allows for differential action i.e. the two wheels can have independent speeds. Note that the torque vectoring (TV) motor is geared directly to the balance shaft. Therefore, applying a torque on the TV motor will directly change the speed of the ring gears. Thus, the TV motor can distribute wheel torque from one wheel to the other, in equal and opposite quantities, based on the direction of torque application.

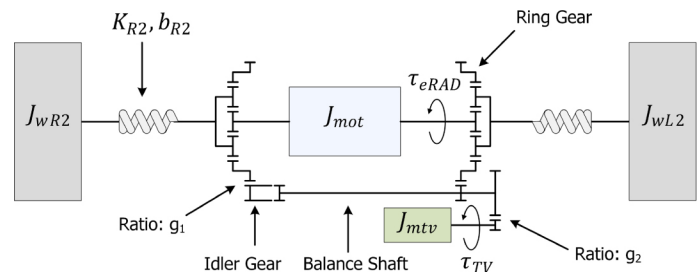


Figure 2. Detailed schematic of the torque vectoring eRAD (TVeRAD)

To develop a dynamic mathematical model of the system described above, several simplifying assumptions are used for the front axle:

- Under most operating conditions, the generator in the Power Split (PS) front axle is used to control the engine speed, therefore, the engine and generator torque are reduced to a ring gear torque, τ_{ring}
- Since the front axle uses an open differential, the two halfshafts are lumped into an equivalent halfshaft. It has been shown that the proper way of lumping halfshafts together is as two springs in series rather than in parallel[7].
- The total inertia of the PS front axle is lumped into the PS traction motor

Similarly, the following simplifying assumptions are used for the rear axle:

- It is assumed that the masses of the carrier, ring and balance shaft are negligible
- Gear lash in the system is negligible
- The only inertias in the system are those of the main traction motor, the TV motor and the wheels
- The only compliance in the system are that of the two rear halfshafts.

Given the simplifying assumptions above and the kinematic constraint of a planetary gear set, the bond graph models of the front and rear axles are given in Figure 3 and Figure 4 respectively.

$$\omega_{carrier} = \frac{\rho}{1+\rho} \omega_{sun} + \frac{1}{1+\rho} \omega_{ring} \quad \text{where } \rho = \frac{r_{sun}}{r_{ring}} \quad (1)$$

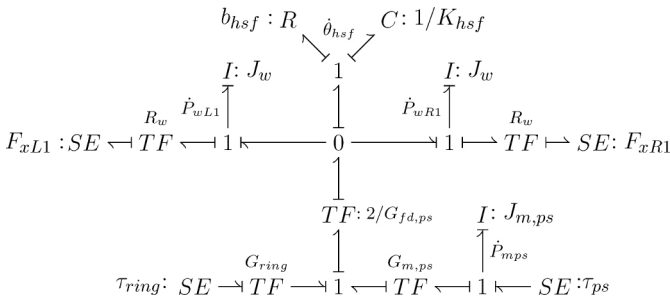


Figure 3. Bond graph model of eAWD HEV front axle

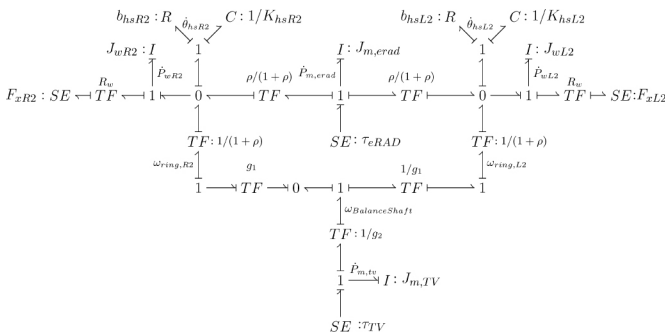


Figure 4. Bond graph model of eAWD HEV TVeRAD

In Figure 3, $G_{m,ps}$ represents the PS motor gear down ratio to the layshaft, G_{ring} is the gear ratio from the ring to the layshaft and $G_{fd,ps}$ is the PS final drive ratio. In Figure 4, g_1 represents the gear ratio from the balance shaft to the ring gears whereas g_2 is the TV gear down ratio to the balance shaft. Note that the zero junction to the left of the $\omega_{BalanceShaft}$ one junction represents the idler gear – the sign of the speed changes while maintaining the same torque.

The resulting bond graph models output ten state equation describing the dynamics of the system. The states, control inputs and dynamic equations are given as

Table 1. eAWD HEV model states and inputs

State	Description	State	Description
$\omega_{m,ps}$	Power Split traction motor speed, [rad/s]	θ_{hsL2}	Rear left (L2) halfshaft displacement, [rad]
θ_{hsf}	Front axle lumped halfshaft displacement, [rad]	θ_{hsR2}	Rear right (R2) halfshaft displacement, [rad]
$\omega_{m,erad}$	TVeRAD traction motor speed, [rad/s]	$\omega_{w,ij}$	Wheel speeds, [rad/s]
$\omega_{m,tv}$	TVeRAD torque vectoring motor speed, [rad/s]		
Control Input		Description	
τ_{ps}		τ_{ps}	Power Split traction motor torque, [Nm]
τ_{erad}		τ_{erad}	TVeRAD traction motor torque, [Nm]
τ_{TV}		τ_{TV}	TVeRAD torque vectoring motor torque, [Nm]

$$\dot{\omega}_{m,ps} = \frac{1}{J_{m,ps}} \left(\tau_{ps} - G_{m,ps} \left(\frac{2}{G_{fd,ps}} \tau_{hsf} - \frac{1}{G_{ring}} \tau_{ring} \right) \right) \quad (2)$$

$$\dot{\theta}_{hsf} = \frac{2G_{m,ps}}{G_{fd,ps}} \omega_{m,ps} - \omega_{w,L1} - \omega_{w,R1} \quad (3)$$

$$\dot{\omega}_{m,erad} = \frac{1}{J_{m,erad}} \left(\tau_{erad} - \frac{\rho}{1+\rho} (\tau_{hsL2} + \tau_{hsR2}) \right) \quad (4)$$

$$\dot{\omega}_{m,TV} = \frac{1}{J_{m,TV}} \left(\tau_{TV} - \frac{1}{g_1 g_2 (1+\rho)} (\tau_{hsL2} - \tau_{hsR2}) \right) \quad (5)$$

$$\dot{\theta}_{hsL2} = \frac{\rho}{1+\rho} \omega_{m,erad} + \frac{1}{g_1 g_2 (1+\rho)} \omega_{m,TV} - \omega_{w,L2} \quad (6)$$

$$\dot{\theta}_{hsR2} = \frac{\rho}{1+\rho} \omega_{m,erad} - \frac{1}{g_1 g_2 (1+\rho)} \omega_{m,TV} - \omega_{w,R2} \quad (7)$$

$$\dot{\omega}_{w,i1} = \frac{1}{J_w} (\tau_{hsf} - R_w F_{x,ij}) \quad i \in \{L, R\} \quad (8)$$

$$\dot{\omega}_{w,i2} = \frac{1}{J_w} (\tau_{hs,ij} - R_w F_{x,ij}) \quad i \in \{L, R\} \quad (9)$$

In the equations above, the halfshaft torques are calculated as linear spring-damper systems

$$\tau_{hsf} = K_{hsf}\theta_{hsf} + b_{hsf}\left(\frac{2G_{m,ps}}{G_{fd,ps}}\omega_{m,ps} - \omega_{w,L1} - \omega_{w,R1}\right) \quad (10)$$

$$\tau_{hs,L2} = K_{hs,L2}\theta_{hs,L2} + b_{hs,L2}\left(\frac{\rho}{1+\rho}\omega_{m,erad} + \frac{1}{g_1g_2(1+\rho)}\omega_{m,TV} - \omega_{w,L2}\right) \quad (11)$$

$$\tau_{hs,R2} = K_{hs,R2}\theta_{hs,R2} + b_{hs,R2}\left(\frac{\rho}{1+\rho}\omega_{m,erad} - \frac{1}{g_1g_2(1+\rho)}\omega_{m,TV} - \omega_{w,R2}\right) \quad (12)$$

One of the benefits of the proposed TVerAD is that the torque vectoring functionality is motor torque based rather than clutch based – the torque vectoring functionality is achieved by applying a motor torque in the desired direction instead of slipping and locking a clutch. It has been shown that clutch-based torque vectoring has several drawbacks due to clutch capacity estimation and, most notably, friction heating dynamics[8]. Therefore, it is expected that this approach will offer faster and smoother torque vectoring control.

Integration with CarSim

The eAWD drivetrain model developed above is integrated with a high fidelity vehicle dynamics model from the CarSim[9] software suite. The idea is to have the CarSim vehicle model in the loop with the eAWD drivetrain model and the proposed control system. In this way, we may validate the performance of the control system on the high fidelity vehicle dynamics model.

In order to make the model interaction as smooth as possible, it is chosen to remove the wheel dynamics from the drivetrain model and let CarSim handle the wheel dynamics and the tire model. In return, the CarSim model outputs the resulting wheel speed to the drivetrain model. The drivetrain model uses the wheel speeds and the motor control inputs to calculate and output the halfshaft torques of each wheel to CarSim. This interaction is seen pictorially in Figure 5.

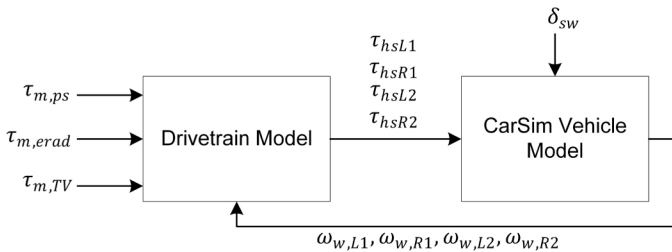


Figure 5. Interaction between drivetrain model and CarSim vehicle model

Note that since the front axle uses an open differential, the halfshaft torque of each front wheel (R1, L1) is equal to the lumped halfshaft torque defined in eq. (10).

$$\tau_{hsL1} = \tau_{hsR1} = \tau_{hsf} \quad (13)$$

Linear System Analysis

Before beginning the controls development, we first present a linear system analysis of the TVerAD. The purpose of this analysis is to determine the cross coupling effects between the two motor torque control inputs and the resulting tire force output. By knowing the magnitude of the cross coupling between the input-output pairs, we may attack the control problem in a much more efficient and effective manner.

Equations (4)–(7) and (9) represent the dynamics of the TVerAD. The TVerAD is first coupled with the standard three-state planar vehicle model described by

$$\dot{U} = \frac{1}{m}(F_{xL2} + F_{xR2} + mV\omega_y) \quad (14)$$

$$\dot{V} = \frac{1}{m}(F_{yL1} + F_{yR1} + F_{yL2} + F_{yR2} - mU\omega_y) \quad (15)$$

$$\dot{\omega}_y = \frac{1}{I_y}\left(a(F_{yL1} + F_{yR1}) - b(F_{yL2} + F_{yR2}) + \frac{T}{2}(F_{xR2} - F_{xL2})\right) \quad (16)$$

Where the tire forces are transformed from the tire frame to the vehicle frame through the transformation matrices assuming a small tire steer angle δ

$$\begin{bmatrix} F_{x,i1} \\ F_{y,i1} \end{bmatrix} = \begin{bmatrix} 1 & -\delta \\ \delta & 1 \end{bmatrix} \begin{bmatrix} F_{xt,i1} \\ F_{yt,i1} \end{bmatrix} \quad i \in \{L, R\} \quad (17)$$

$$\begin{bmatrix} F_{x,i2} \\ F_{y,i2} \end{bmatrix} = \begin{bmatrix} F_{xt,i2} \\ F_{yt,i2} \end{bmatrix} \quad i \in \{L, R\} \quad (18)$$

The longitudinal and lateral tire forces are calculated using the linear tire model commonly used in the vehicle dynamics literature

$$F_{xt} = C_x\sigma \quad (19)$$

$$F_{yt} = C_a\alpha \quad (20)$$

Thus, the TVerAD coupled with the planar vehicle model results in a nine state system. The overall system can be linearized about an operating condition using Jacobian linearization as cast in the state space framework as

$$\dot{x} = Ax + B_i u_i + B_d u_d \quad (21)$$

$$y = Cx + D_i u_i + D_d u_d \quad (22)$$

Where the inputs are separated as control inputs, u_p , and disturbances, u_d

$$x = [\omega_{m,erad}, \omega_{m,TV}, \theta_{hsL2}, \theta_{hsR2}, \omega_{w,L2}, \omega_{w,R2}, U, V, \omega_y]^T \quad (23)$$

$$u_i = [\tau_{erad}, \tau_{TV}]^T \quad (24)$$

$$u_d = \delta \quad (25)$$

Physically speaking, the ideal input-output pairing for the TVeRAD is for the main traction motor to provide the total rear axle traction force, F_{xr} , and for the small TV motor to distribute tire forces from one side to the other, thus creating a tire force difference on the rear axle, ΔF_{xr} . Note that due to the mechanical constraint of the TVeRAD, we cannot provide two independent tire forces at the rear axle. Instead, the TVeRAD can remove torque from one wheel and add the same quantity to the other wheel, thus creating a force difference between the left and right side of the rear axle.

Choosing the operating condition as high speed cornering, the desired plant to analyze the system coupling can be derived as

$$G_p = \begin{bmatrix} \frac{F_{xr}}{\tau_{erad}} & \frac{F_{xr}}{\tau_{TV}} \\ \frac{\Delta F_{xr}}{\tau_{erad}} & \frac{\Delta F_{xr}}{\tau_{TV}} \end{bmatrix} \quad (26)$$

In order to avoid excessive equation lengths, the numerical values of the plant are not presented. However, the frequency response of eq. (26) is given in Figure 6. As can be seen from the figure, the off-diagonal elements are low gain relative to the diagonal elements, thus implying that the control inputs are fairly decoupled from one another.

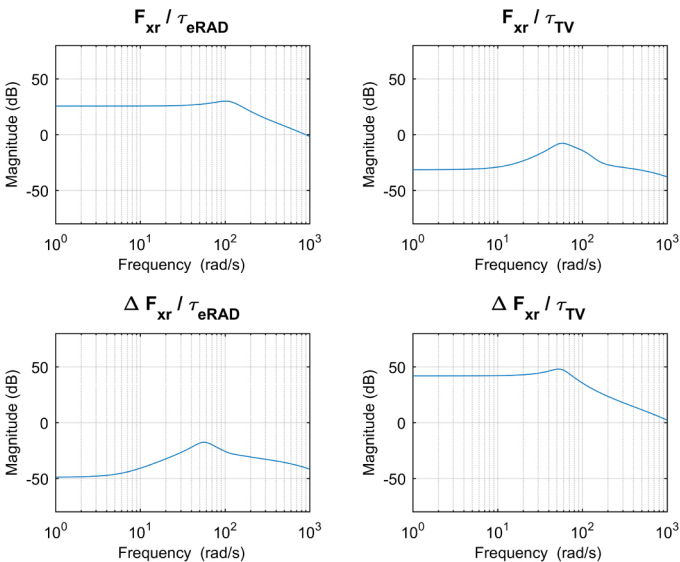


Figure 6. Frequency response of TVeRAD with planar vehicle model

To further analyze the coupling of the input-output pairs, the Relative Gain Array (RGA) of eq.(26) is calculated. It has been shown that the RGA offers a good measure interactions of input-output pairings[10, 11]. The relative gain, λ , measures the ratio of the magnitude between input-output pairs. Therefore, the relative gains of the RGA can tell us which output should be paired with an input. In practice, it is ideal to pair an output with an input if the relative gain of that input-output pairing is near unity within the vicinity of the bandwidth frequency[11]. The relative gains of each element can be computed and evaluated at a frequency ω as

$$\lambda_{ij}(\omega) = \frac{g_{ij}(\omega)}{\hat{g}_{ji}(\omega)} = [G_p(\omega)]_{ij} [G_p^{-1}(\omega)]_{ji} \quad i \in \{1,2\}, j \in \{1,2\} \quad (27)$$

In eq.(27), $[G_p]_{ij}$ represents the ij element of G_p . The resulting relative gains as a function of frequency is given in Figure 7. It is clear from the figure that the diagonal elements are always near unity whereas the off-diagonal elements are always near zero for all frequencies. Therefore, it is clear that the following input-output pairing should be used for control:

- τ_{erad} input should control the F_{xr} output
- τ_{TV} input should control the ΔF_{xr} output

Therefore, since the two input-output pairs are decoupled, we may treat them as two separate single-input single-output (SISO) control loops during the control design.

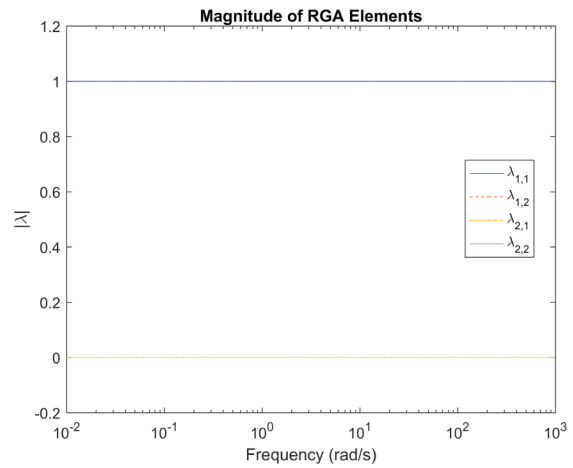


Figure 7. RGA magnitudes of the plant as a function of frequency