

Experimental and Simulation Studies on Instability of a Two Wheeler Vehicle

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ABSTRACT

Two and three wheeler vehicles are largely used in many developing and under developing countries because of their lower cost, better fuel economy and easy handling. Although, the construction of them is simpler than the four wheeler vehicle, they pose some problems related to instability. Wobbling is the main cause of instabilities in two wheeler and three wheeler vehicles. In this study, a mathematical model was proposed and developed to determine wobble instability of a two wheeler. Nonlinear equations were formulated by using kinematics and the D'Alembert's principle with the help of multi body formalism. The non-linear equations found in the study were linearized with respect to rectilinear and upright motion, considering no rolling. It led to formation of matrix. The real part of the Eigen value of the matrix was found to be negative, implication of whose was an asymptotic stable motion. It was observed that, the above real part of Eigen value was a function of different parameters such as Tire stiffness, Frame compliance, Steering angle and Rider's way of handling. These parameters play an important role in determination of wobble frequency. Further, a model was developed using the Lotus Suspension Software, and dependency of above parameters on wobble frequency was studied. Further, experiments were conducted to verify the dynamics behaviour of these parameters included in the proposed mathematical model. The simulation and experimental results are presented in this paper.

Keywords: Two wheeler, wobble, steering angle, tyre stiffness

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INTRODUCTION

In this ultramodern world, use of automotive vehicles is one of the important needs of the humanity and exhibits larger demand in the global market. Three types of vehicles are available in the world for passengers that are (i) two wheeler (ii) three wheeler, and (iii) four wheeler. Depending on the easiness of handling most of the people in the world prefer two wheeler vehicles, even if a short distance has to be reached. Generally, four wheeler vehicles are useful for a long travel, which is due to its load bearing capacity and high comfort. Examples of four wheeler used for passenger applications are buses and cars. Three wheelers are useful for public transport for shorter distances, which may be generally used up to a few kilometres, at an economical cost. While developed countries mostly use four wheelers, developing countries rely upon two and three wheel vehicles. It is highly essential that safety of passengers has to be taken in consideration. This urges for the study of stability issues in these vehicles. Many studies have been made on the stability of two, three and four wheeler vehicles till today. As long as two wheeler vehicles are concerned, there are many factors which have a great impact on the stability of two wheeler vehicles. Road camber alters the magnitude and direction of the tyre force and moment vectors relative to the wheels. Evangelou et al [1] studied the influence of road camber on Motorcycle stability. They made computer simulations that exploited a high-fidelity motorcycle model whose parameter set was based on a Suzuki GSX-R1000 sports machine. In

order to study camber-induced stability trends for a range of machine speeds and roll angles, they studied the machine dynamics as the vehicle travelled over the surface of a right circular cone. They reported that, positive camber, which is often introduced by road builders to aid drainage and enhance the friction limit of four-wheeled vehicle tyres, prove to be detrimental to the stability of two-wheeled vehicles. Ghosh and Mukhopadhyay [2] studied the stability of a two wheeler in a dynamic environment such as braking and cornering. They used a simplified model to analyse the effects of forward speed and braking force on the roll instability during cornering of a two-wheeler. From the investigation results, they have indicated that braking during cornering compromised the stability. It was found that instability further increased if mass of two wheeler vehicle was increased and front trail was decreased. Gaillet al [3], studied the behaviour of Anti-Lock Braking Systems (ABS) and Vehicle Stability Control (VSC) for two wheeler vehicles by the help of different analysis and real time data acquisition system. They carried out tests with five different brake systems with and without ABS to check that the critical driving situation being detected by VSC. By means of studying accident figures as well as the technical possibilities for the implementation of VSC on motorcycles, possible safety gains were determined. Sharp [4], developed of a suitable mathematical model of a motorcycle in free control and used the model to show typical stability characteristics and how these characteristics depended on various parametric values. He concluded that, the fixed control characteristics of the motorcycle would be

unimportant, and the steady state response to steering torque would be probably of secondary importance. He has also concluded that, a conventional motor cycle had a small stable speed range below which its instability was a low frequency oscillatory one and above which it would be a slow divergence.

In this study, Wobble instability of a two wheeler was investigated. For this, a mathematical model was proposed and developed to determine wobble instability. Various nonlinear equations were formed by using multi body formalism and the dynamic relations between different parameters. The equations were further linearized using MATLAB to obtain a matrix. The real and complex part of the Eigen values of the matrix were also studied using various plots. Furthermore, simulation was also done using Lotus Suspension Software, and the results were compared with the experimental results obtained in a two wheeler. The first section of this paper describes the mathematical model used for the numerical analysis of stability along with the results derived from it. The next section describes the test motorcycle used for the investigation and test experiment results, followed by the simulation results obtained from the Lotus Suspension Software. Then, the results drawn from the mathematical model, analysis and simulation are presented. In the last section, the conclusions drawn are mentioned. Also, future works that can be done on the instability of two wheeler vehicles are presented.

MATHEMATICAL MODEL

Model Description

Figure 1(a), 1(b) and 1(c) depict the view of a motorcycle with a passive rider in upright position, rolling position and cornering position receptively. Let a motor cycle is moving at a roll angle " ϕ " of mainframe, with a steer angle " δ " of handlebar about the steering axis. The handlebar creates a rake angle " ϵ " with respect to vertical line in the upright position, projecting a caster length " t_c " in the ground. "A" is taken as the point of reference. It is located exactly below the centre of gravity of the mainframe, which moves forward with a velocity " u " and in lateral direction with velocity " v ". The line of joining the plane of symmetry and road plane moves over the road surface and shows a yaw angle " ψ ". The time rate of change of yaw angle is denoted by " r ". The rider has a mass of " m_r " and makes an angle of " ϕ_r " with longitudinal axis. Lateral shift " y_m " of centre of the gravity of mainframe and " y_r " for the rider lead to joint offset of " y_{mr} " for the system of mainframe and rider. The steering torque " M_δ " and the lean torque " $M\phi_r$ " are acted upon the rider upper body and the main frame, respectively. These constitute the input vector of the system. Besides the mass of the rider " m_r ", the model consists of the masses of the mainframe " m_m ", which includes the mass of lower part of the rider and the rear wheel. The model includes the mass of the front upper frame " m_f " and of the front sub frame " m_s ", including the front wheel.

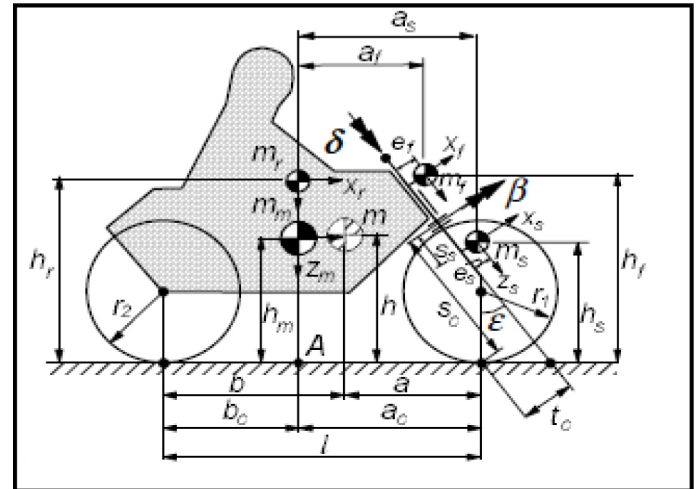


Figure 1(a). View of a motor cycle with rider in upright position

For the analysis, the D'Alembert principle and multi-body formalism were applied to formulate non-linear kinematic equations. Mass properties and geometric data were measured from the motorcycle taken for experimentation, which were further used for the numerical analysis. The spring and damper characteristics of the motorcycle for the passive rider model was taken from literature, e.g. [5], and other rider parameters were obtained from an anthropometric model [6, 7], which corresponded to the 50th percentile male configuration. The rider's mass was found to be 88 kg which matches approximately with the rider, taken for test runs, which is discussed in later section.

For the dynamic analysis, deviations from rectilinear path were avoided. The differential equations were kept linear with forward speed " u " considered as a constant parameter. The equations were formulated using the Lagrange equations [8]:

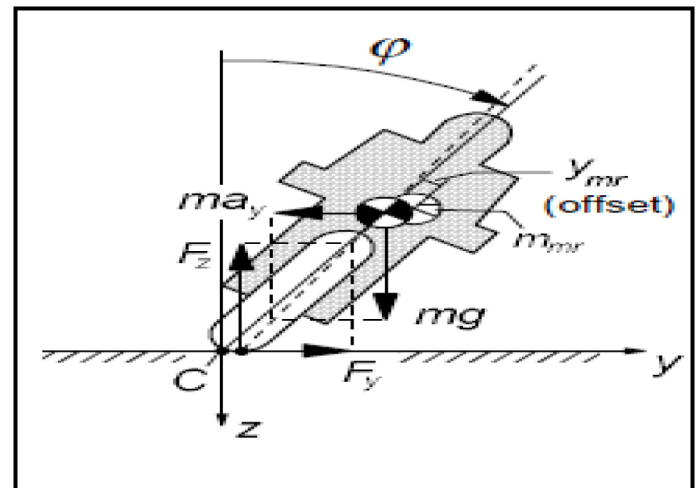


Figure 1(b). View of motor cycle with rider in rolling position

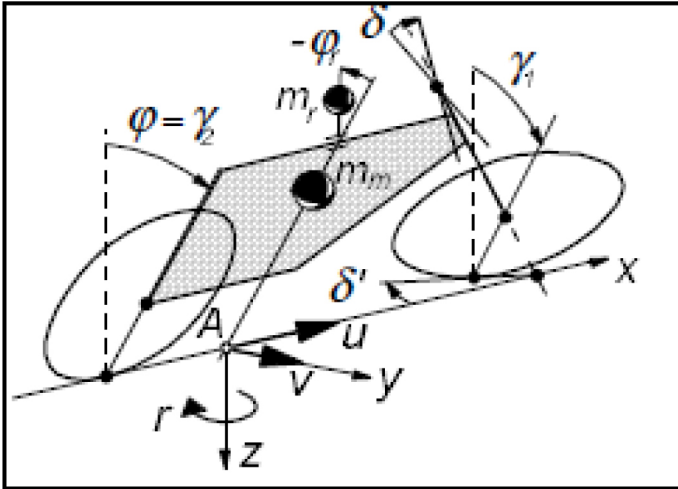


Figure 1(c). View of a motor cycle in cornering position

$$\frac{d}{dt} \frac{\partial \tau}{\partial u} - r \frac{\partial \tau}{\partial v} = Q_u \quad (1)$$

$$\frac{d}{dt} \frac{\partial \tau}{\partial v} + r \frac{\partial \tau}{\partial u} = Q_v \quad (2)$$

$$\frac{d}{dt} \frac{\partial \tau}{\partial r} - v \frac{\partial \tau}{\partial u} + u \frac{\partial \tau}{\partial v} = Q_r \quad (3)$$

$$\frac{d}{dt} \frac{\partial \tau}{\partial \phi} - \frac{\partial \tau}{\partial \phi} + \frac{\partial U}{\partial v} = Q_\phi \quad (4)$$

Where apparently,

$$Q_u = \sum F_x = F_{x1} \cos \delta - F_{y1} \sin \delta + F_{x2}$$

$$Q_v = \sum F_y = F_{x1} \sin \delta - F_{y1} \cos \delta + F_{y2}$$

$$Q_r = \sum M_z = a F_{x1} \sin \delta + a F_{y1} \cos \delta + M_{z1} - b F_{y2} + M_{z2}$$

$$Q_\phi = \sum M_\phi = - (k_{\phi 1} + k_{\phi 2})$$

$k_{\phi 1}$ and $k_{\phi 2}$ are the damping coefficients at the front and rear axles.

The front and rear tyre forces and moments were determined and designated by indices by 1 and 2, with slip angles α_1 , α_2 and camber angles γ_1 , γ_2 . Geometric calculations followed by matrix transformations were done to determine a unit vector s directed according to wheel spin axis.

$$s = \begin{bmatrix} -\delta \cos \varepsilon + \beta \sin \varepsilon \\ 1 - \phi (\delta \sin \varepsilon + \beta \cos \varepsilon) \\ \phi + \delta \sin \varepsilon + \beta \cos \varepsilon \end{bmatrix}$$

The ground steer angle δ_s and camber angle α_1 were determined from the components of unit vector,

$$\tan \delta' = \frac{-\delta \cos \varepsilon + \beta \sin \varepsilon}{\cos \phi - \sin \phi (\delta \sin \varepsilon + \beta \cos \varepsilon)} \quad (5)$$

$$\sin \alpha_1 = \sin \phi + \cos \phi (\delta \sin \varepsilon + \beta \cos \varepsilon) \quad (6)$$

Equation (5) and Equation (6) are non-linear in nature.

Approximating the nonlinear equations to linear form results in,

$$\delta' = \delta \cos \varepsilon - \beta \sin \varepsilon \quad (7)$$

$$\alpha_1 = \sin \phi + \cos \phi (\delta \sin \varepsilon + \beta \cos \varepsilon) \quad (8)$$

For the non-steered rear wheel,

$$\gamma_2 = \phi \quad (9)$$

At the steady state, the slip angles are assumed to be small and given by following relations

$$\alpha_1 = \delta' - (v + a_c r) / u \quad (10)$$

$$\alpha_2 = (-1/u) * (v - b_c r) \quad (11)$$

For the dynamic non steady state situation, the linearized model was applied, obtained by linearizing equation (10) and equation (11). The slip angles then become the time rate of changes of δ and β .

$$\alpha_1 = \delta \cos \varepsilon - \beta \sin \varepsilon - (v + a_c r - t_c \delta - s_c \beta) \quad (12)$$

$$\alpha_2 = \frac{-1}{u} (v - b_c r) \quad (13)$$

The transient response of the side force “ F_y ” along with aligning torque “ M_z ” is taken into consideration. However, the non-lagging part of the response is disregarded. Further, the overturning couple “ M_x ” is assumed to respond instantaneously to change in camber. For the transient responses, the relaxation length “ σ ” is used as parameter in first order differential equations. These equations describe the responses of transient slip angles α' and γ' which in steady state conditions become equal to the input slip and camber angles α and γ .

$$\frac{1}{u} \sigma_{\alpha i} \alpha_i' + \alpha_i = \alpha_i \quad (14)$$

$$\frac{1}{u} \sigma_{\gamma i} \gamma_i' + \gamma_i = \gamma_i \quad (15)$$

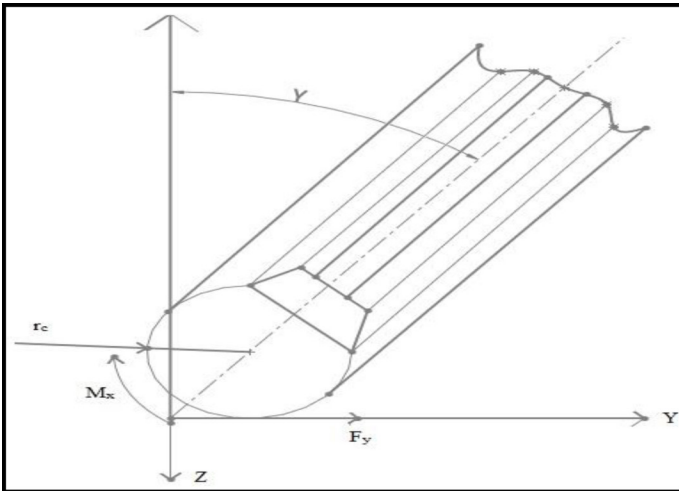


Figure 2. Tyre forces and moments acting at contact points

The lateral tyre forces or side force F_{yi} , the overturning couples M_{xi} and self-aligning moments M_{zi} are derived, by figure 2, for $i=1,2$,

$$F_{yi} = C_{Fai} \alpha_i' + C_{F\gamma i} \gamma_i' \quad (16)$$

$$M_{zi} = -C_{Mai} \alpha_i' + C_{M\gamma i} \gamma_i' - r_{ci} (F_{xi} \gamma_i') \quad (17)$$

$$M_{xi} = -C_{Mx\gamma i} \gamma_i' \quad (18)$$

All the necessary formulations were done by taking into consideration, the tyre forces and moments along with kinematic and geometrical relation between different dynamic parameters. The formulations led to formation of 14 nonlinear equations which were functions of 14 variables that are, $\phi, \phi_p, \delta, \beta, v, r, \dot{\phi}, \dot{\phi}_r, \dot{\delta}, \dot{\beta}, \alpha_1', \gamma_1', \alpha_2'$ and γ_2' . Subsequent linearization were done in MATLAB with respect to the upright, rectilinear motion, with no roll, code of which is given in [Appendix A](#).

$$\dot{s} = (A) (s) \quad (19)$$

Where,

$$s = f(\phi, \phi_p, \delta, \beta, v, r, \dot{\phi}, \dot{\phi}_r, \dot{\delta}, \dot{\beta}, \alpha_1', \gamma_1', \alpha_2', \gamma_2')^T$$

$A = A(u)$, where A is a 14×14 matrix.

$\dot{\phi}, \dot{\phi}_r, \dot{\delta}$ and $\dot{\beta}$ were the derivatives of ϕ, ϕ_p, δ and β with respect to time respectively.

By Lyapunov Theorem [9], the motion was found to be asymptotically stable at some conditions where the real parts of all the Eigen values of matrix "A" was negative. Further, various graphs were plotted in MATLAB by varying the dynamic parameters to study their influence on instability.

Modelling Results

Stability in Base Line Configuration Model

Figure 3 (a) depicts the variation of real part of Eigen values with speed of travel in base line configuration model which includes air drag, includes a rigid rider, zero forward acceleration and no control. The extent of stability of wobble was determined by the measure of a negative value of the real part of the Eigen value i.e. more the negative the real part of the Eigen value is more the stable is the motion.

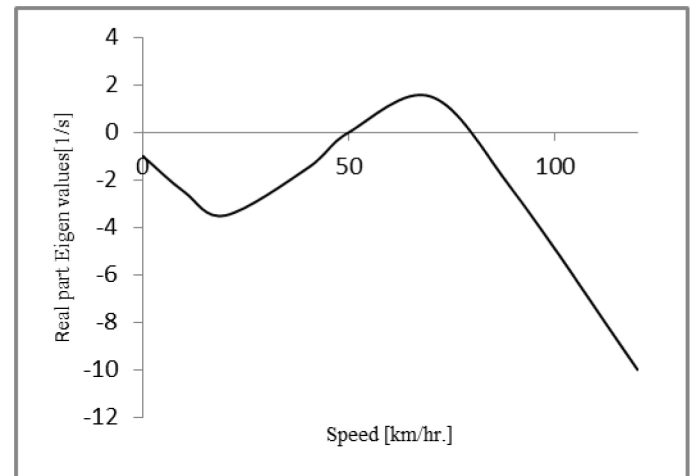


Figure 3 (a). Variation of the real part Eigen values with speed of travel

It can also be observed from Figure 3 (a), that steering oscillation or wobble is unstable in a range of moderate speed between 40km/h. and 70km/h as the real part of the Eigen values becomes positive.

Figure 3 (b) illustrates the variation of the imaginary part of the Eigen values with speed of travel in base line configuration model. The measure of the imaginary part of the Eigen value is the direct indication of frequency of steering oscillation.

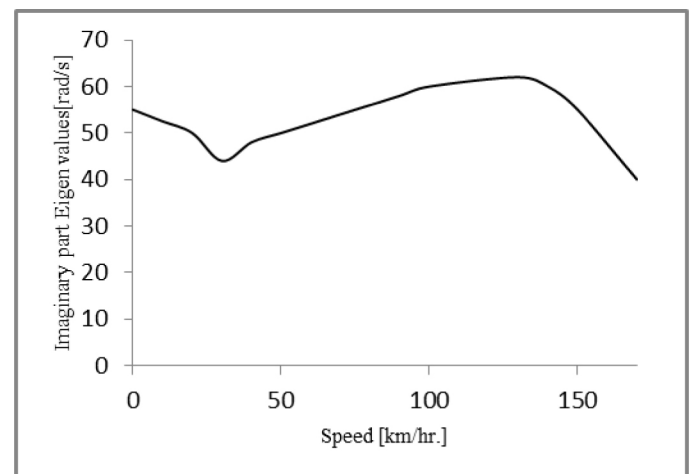


Figure 3 (b). Variation of the imaginary part the Eigen values with speed of travel

The frequency in this unstable range as can be observed from Figure 3 (b) is about 55 rad/s. It can also be found that, the real part of Eigen values becomes less negative at a lower speed range between 0 and