

# Study on Energy Loss due to Cornering Resistance in Over-Actuated Vehicles using Optimal Control

Sriharsha Bhat, Mohammad Mehdi Davari, and Mikael Nybacka  
KTH Royal Institute of Technology

## ABSTRACT

As vehicles become electrified and more intelligent in terms of sensing, actuation and processing; a number of interesting possibilities arise in controlling vehicle dynamics and driving behavior. Over-actuation with in-wheel motors, all wheel steering and active camber is one such possibility, which facilitate the control strategies that push boundaries in energy consumption and safety. Optimal control can be used to investigate the best combinations of control inputs to an over-actuated system. This paper shows how an optimal control problem can be formulated and solved for an over-actuated vehicle case, and highlights the translation of this optimal solution to a real-world scenario, enabling intelligent means to improve vehicle efficiency.

This paper gives an insight into Dynamic Programming (DP) as an offline optimal control method that guarantees the global optimum. Therefore the optimal control allocation to minimize an objective function and simultaneously fulfill the defined constraints can be achieved. As a case study the effects of over-actuation on the cornering resistance were investigated in two different maneuvers i.e. step steer and sine with dwell, where in both cases the vehicle assumes to be in steady state situation. In this work the cornering resistance is the main objective function and maintaining the reference trajectory is the constraint which should be fulfilled. A parameter study is conducted on the benefits of over-actuation, and depending on the type of over-actuation about 15% to 50% reduction in cornering resistance were observed during step steer and sine with dwell maneuver respectively. From a second parameter study that focused on COG position from a safety perspective, it is more beneficial for the vehicle to be designed to under-steer than over-steer. Finally, a method is described to translate the offline optimal results to vehicle implementable controllers in the form of both feed-through lookup-tables and rule-based feed-forward control.

**CITATION:** Bhat, S., Davari, M., and Nybacka, M., "Study on Energy Loss due to Cornering Resistance in Over-Actuated Vehicles using Optimal Control," *SAE Int. J. Veh. Dyn., Stab., and NVH* 1(2):2017, doi:10.4271/2017-01-1568.

## INTRODUCTION

Two interesting trends are seen in the vehicle industry, the first is a drive towards highly efficient and low emission vehicles due to the problem of rising carbon emissions. Second, safety targets have led to advances in vehicle intelligence, with exciting technologies in sensing, actuation, driver assistance and autonomy. In order to meet fuel efficiency and emissions targets, manufacturers have spent effort in electrification and hybridization of transport. Another challenge to be met is that harmful particles in our urban environments need to be reduced. Electric vehicles in urban transport offer new vehicle design possibilities that can offer improvements in efficiency, intelligence and safety. Over-actuated vehicles are one such design possibility in controlling vehicle dynamics and driving behavior, and offer an opportunity for utilizing vehicle intelligence to improve vehicle efficiency and performance.

An over-actuated system has more control inputs than degrees of freedom. In the vehicular context, over-actuation could be performed by individual wheel steering, active camber, torque vectoring with in-wheel motors and independent braking. In this work, an over-actuated vehicle with in-wheel motors, front and rear wheel steering,

as well as active camber is considered. Such a system has the potential to push boundaries in energy consumption and safety thanks to better utilization of tire forces, according to prior work performed by Jonasson, Jerrelind et. al and Edren [1,2,3]. Over-actuation also allows for fault tolerant control, as seen in work by Wanner et. al [4,5]. From a rolling loss perspective, a study performed by Davari et. al using a nonlinear tire model highlighted the effect of active camber in reducing losses[6].

As a definition optimal control is the method of finding the best possible controls for a given system subject to constraints and following a specified objective. Intelligence in the form of optimal control can be used to improve efficiency. Techniques in optimal control can be utilized to find the best control combinations with a defined objective (for example minimum energy consumption, or minimum deviation from the defined path) in an over-actuated vehicle for known maneuvers. One such technique is Dynamic programming (DP), a numerical method of solving optimal control problems that utilizes Bellman's Principle of Optimality to find the global optimum solution [7]. DP is iterative, and splits the problem's state variables into smaller pieces in a grid, and recursively calculates

the minimum at each stage in the state trajectory by iterating over control inputs. Thanks to the Principle of optimality, if the optimal solution is found for one section of the trajectory, it is a subset of the global optimum, which means fewer states to be iterated in the next step. Therefore significantly fewer iterations are needed compared to a brute force approach, where one would need to consider every single trajectory.

In the context of automotive systems, DP has been primarily employed in the context of energy management. Optimization of hybrid powertrains was performed by Lee et al [8], while rule based control strategies based on DP were employed for the same purpose by Lin et al [9]. Electric vehicle charging was optimized considering state-of-charge and price by Mody and Steffen [10]. Gear-shifting scheduling was optimized for trucks considering clutch energy loss by Škugor et al [11], and DP was one of several methods used in a study on semi-active suspension control by Siahkalroudi [12]. The present work focuses on vehicle dynamics control instead, and considers losses during cornering.

DP can only be formulated in an offline case, since its iterative nature means it can only perform optimization for a known reference trajectory where predicted optimal control inputs are defined for each time instant. In addition, it is not possible to obtain instantaneous optimal inputs from DP. However, this offline formulation and solution offers a benchmark, enabling one to observe the performance limits of a particular system, as well as generate sub-optimal rule based real-time control strategies based on the optimal solution. In this paper, as a case study DP is applied to a model representing the vehicle dynamics of an over-actuated vehicle and the optimal control inputs in terms of wheel steering and camber angles are derived for known driving maneuvers considering a customized objective that minimizes tire cornering resistance (as a measure of efficiency). Following the optimal solution, a method to translate the optimum to a vehicle-implementable case is presented, leading to a cost saving vehicle controller.

The aim of this paper is to describe how an optimal control problem can be formulated and solved for an over-actuated vehicle case, and highlight the translation of this optimal solution to a real-world scenario, enabling intelligent means to improve vehicle efficiency.

## MODELS, METHODS AND RESULTS

### Vehicle Model and Dynamic Programming Formulation

The dpm() function [13] in Matlab is an implementation of the Dynamic Programming algorithm which can be used in an optimal control formulation. The dpm() function in level-set mode is used in this work [14]. The level set mode utilizes level set functions to represent the region of feasible solution states, reducing the number of iterated states without sacrificing optimality therefore allowing for smaller grid sizes and faster processing.

The method of solving an offline vehicle dynamics control problem to minimize an objective is depicted in Figure 1. An over-actuated vehicle is idealized with an extended bicycle model considering front

and rear wheel steering and camber, and its states and dynamics make up the DP state grid. A reference driving maneuver in terms of front wheel steering angle and constant vehicle longitudinal velocity is defined, and this steering profile is used to calculate reference state values analytically using a bicycle model (considering only front-wheel steering). These reference values are used as state and input constraints in the optimizer, and an objective is defined. The DP solver minimizes the objective function subject to the constraints and optimal system inputs are derived (in the form of steering and camber angles). For realistic optimization results, the objective function must be monotonically increasing (for a feasible minimum to found), and linked to the input or state variables. Pre-defined performance parameters are derived from the optimal control, and comparisons can be made between different objectives and vehicle parameters.

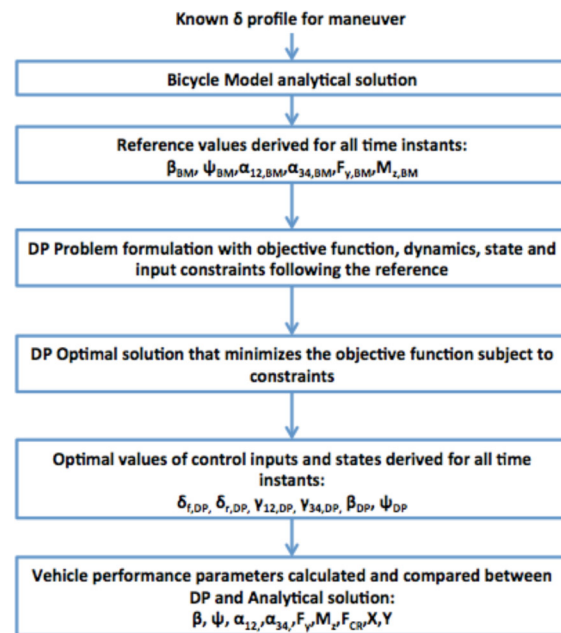


Figure 1. Method of solving an optimal vehicle dynamics control problem using DP.

### Vehicle State Variables

An idealized vehicle is represented by the parameters in Table 1. Reference states are calculated analytically considering a bicycle model with only front wheel steering. The states considered are side slip,  $\beta_{BM}(\cdot)$  and yaw rate  $\dot{\psi}_{BM}(\cdot)$  for all time instants between initial time  $t_i$  and final time  $t_f$ .

Table 1. Parameters of the simulated vehicle.

| Type                             | Symbol           | Value                 |
|----------------------------------|------------------|-----------------------|
| Vehicle mass including 2 Persons | $m$              | 1000 kg               |
| Yaw Moment of Inertia            | $J_z$            | 2000 kgm <sup>2</sup> |
| Wheelbase                        | $l$              | 3 m                   |
| C.G. Position Ratio              | $\lambda$        | 0.5                   |
| C.G Distance from Front Axle     | $f$              | 1.5 m                 |
| C.G Distance from Rear Axle      | $b$              | 1.5 m                 |
| Tire Cornering Stiffness         | $C_{12}, C_{34}$ | 50 kN/rad             |
| Camber Stiffness                 | $G_{12}, G_{34}$ | 10 kN/rad             |

## Control Inputs to be Optimized

Steering and camber angles -  $\delta_f, \delta_r, \gamma_{12}, \gamma_{34}$

## Optimal States to be Iterated

Side slip  $\beta_{DP}(\cdot)$  and yaw rate  $\psi_{DP}(\cdot)$  for all time instants between  $t_i$  and  $t_f$

## Constraints

The limits of the state grid, the terminal value constraints of the final state, and the initial values of the states are assigned from the results of the analytical solution of the bicycle model. The limits of the input grid are assigned from the user-defined steering profile for the maneuver, given by the steering angle  $\delta_{in}$  at each time instant. The camber angles are assigned limits according to physical actuator limits, and the maximum and minimum values range between 0.08 and -0.08 radians.

## Objective Function

The cornering resistance  $F_{CR}$  is used as a measure to track efficiency, with a lower cornering resistance denoting less motion resistance to the vehicle during cornering. Physically, the cornering resistance is the component of the force generated by the tire that opposes tire longitudinal motion. Therefore a reduction in cornering resistance gives a reduction in energy loss. For the optimization model, the cornering resistance is simplified as Equation 1:

$$F_{CR} = C_{12}\alpha_{12}^2 + C_{34}\alpha_{34}^2 \quad (1)$$

In order to ensure safety and path following, the deviation of the lateral force between the DP model and the reference model shown in Equation 2 is minimized.

$$\Delta F_y = (F_{12,DP} \cos \delta_f + F_{34,DP} \cos \delta_r) - (F_{12,BM} \cos \delta_{in} + F_{34,BM}) \quad (2)$$

The objective function  $J_1$  in Equation 3 aims to minimize the cornering resistance thereby improving vehicle efficiency and reducing energy loss, while maintaining the same lateral force, and therefore the same vehicle trajectory as the reference case.

$$J_1 = F_{CR} + |\Delta F_y| \quad (3)$$

## Performance Parameters

In addition to the optimal inputs, states and vehicle trajectory, two performance metrics in Equations 4 and 5 were set to quantify the reduction in cornering resistance (therefore a measure of efficiency improvement) and the deviation from reference path (a measure of safety). A higher value of relative cost ( $\Delta F_{CR}$ ) meant a greater reduction in cornering resistance, while a higher value of path deviation ( $\Delta XY$ ) meant a sacrifice in safety.

$$\text{Relative Cost } \Delta F_{CR} = \frac{\sum F_{CR,DP} - \sum F_{CR,BM}}{\sum F_{CR,BM}} \times 100 \quad (4)$$

$$\text{Path Deviation } \Delta XY = \max \left( \left| \sqrt{x_{DP}^2(\cdot) + y_{DP}^2(\cdot)} - \sqrt{x_{BM}^2(\cdot) + y_{BM}^2(\cdot)} \right| \right) \quad (5)$$

The structure of the Dynamic Programming formulation with the definition of the objective, constraints, system dynamics and performance parameters are depicted in Figure 2.

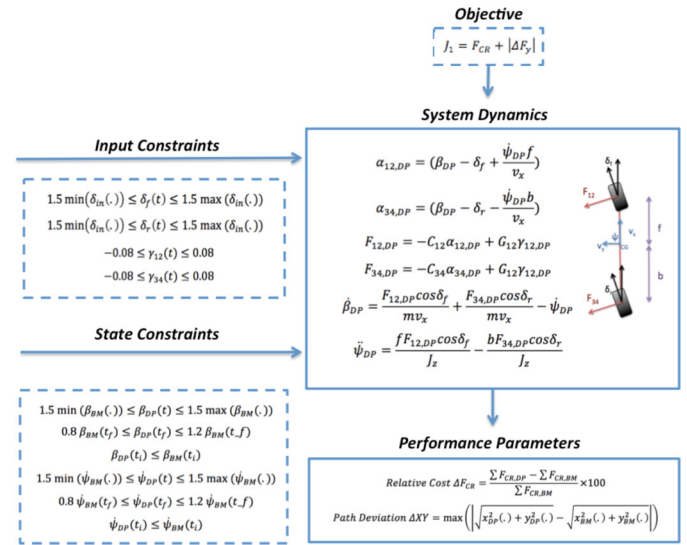


Figure 2. DP formulation for an over-actuated vehicle idealized by a bicycle model.

## Verification

In order to verify the optimization model, DP was used to calculate the results of the analytical solution of the bicycle model backwards, therefore validating the optimization method with a ground truth. In this case, the control inputs  $\delta_r, \gamma_{12}, \gamma_{34}$  were set to 0, and only  $\delta_f$  was predicted by DP to follow the reference states. The predicted values of  $\delta_f$  overlapped with the steering reference, meaning that the optimization model is representative of reality.

A comprehensive investigation into the applicability of DP for optimal vehicle dynamics control has been performed by the author [15] and is closely related to the current work.

## Optimization Results

Detailed simulations were performed for two different driving maneuvers at a longitudinal velocity of 10 m/s: Step (to observe both steady state and sudden dynamic behavior) and Sine with Dwell (To observe applicability in a real lane change scenario). In the case of the Step maneuver depicted in Figure 3, the reference steering angle was instantaneously increased to 0.2 rad from 0, and the optimal solution minimizing both cornering resistance and path deviation was found. From the solution, a steady state camber angle of 0.08 rad is