

Torque-Vectoring Control for an Autonomous and Driverless Electric Racing Vehicle with Multiple Motors

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ABSTRACT

Electric vehicles with multiple motors permit continuous direct yaw moment control, also called torque-vectoring. This allows to significantly enhance the cornering response, e.g., by extending the linear region of the vehicle understeer characteristic, and by increasing the maximum achievable lateral acceleration. These benefits are well documented for human-driven cars, yet limited information is available for autonomous/driverless vehicles. In particular, over the last few years, steering controllers for automated driving at the cornering limit have considerably advanced, but it is unclear how these controllers should be integrated alongside a torque-vectoring system. This contribution discusses the integration of torque-vectoring control and automated driving, including the design and implementation of the torque-vectoring controller of an autonomous electric vehicle for a novel racing competition. The paper presents the main vehicle characteristics and control architecture. A quasi-static model is introduced to predict the understeer characteristics at different longitudinal accelerations. The model is coupled with an off-line optimization for the a-priori investigation of the potential benefits of torque-vectoring. The systematic computation of the achievable cornering limits is used to specify and design realistic maps of the reference yaw rate, and a non-linear feedforward yaw moment contribution providing the reference cornering response in quasi-static conditions. A gain scheduled proportional integral controller increases yaw damping, thus enhancing the transient response. Simulation results demonstrate the effectiveness of the proposed approach.

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INTRODUCTION

Torque-vectoring (TV) is the continuous control of the traction/braking torque levels on the left- and right-hand sides of a vehicle, which generates a direct yaw moment. TV is easily achievable in electric vehicles (EVs) with multiple motors, given the relatively precise torque controllability of electric motor drives. An extensive literature discusses the benefits of TV control for the cornering response of EVs with multiple motors [1, 2, 3, 4, 5, 6, 7, 8]. However, most of the existing implementations and published experiments mainly refer to passenger cars and small urban EVs, rather than advanced racing vehicles. Moreover, independently from the niche applications of TV and EV technologies, there is limited availability of papers assessing racing vehicles through objective characterizations and performance indicators, calculated or measured during repeatable maneuvers. In fact, most of the motorsport literature focuses on lap time simulations or measurements, or is rather generic, due to confidentiality issues, in the analysis of vehicle performance during objective tests.

[9] discusses control algorithms for a semi-active differential. [10] describes the implementation and testing of a TV system based on an automated fiddle brake. Individual brake pressures are applied to the driven wheels through an electro-hydraulic system. The resulting difference among the rear traction forces provokes a controllable yaw moment. [11] analyzes the steady-state cornering response of a Formula SAE vehicle during skid pad tests for different left-to-right wheel torque distributions on the rear axle. [12] presents a TV controller for an electric Formula SAE vehicle, aimed at permanently achieving neutral steering response through a proportional integral derivative (PID) formulation. In [13] the same methodology as in [12] is adopted for a PID controller on vehicle sideslip.

Within the literature focused on motorsport applications different from Formula SAE vehicles, [14] proposes a TV controller aimed at maximizing lateral tire forces. [15] deals with an extended lap time simulation method, combining a quasi-steady-state approach with a transient vehicle model. In [16] the linearization of a seven-degree-of-freedom model of a race car with a limited slip differential is carried out for multiple operating points. Eigenvalue and yaw rate response analyses are then used to establish the stability margin contour on the plot of lateral acceleration as a function of longitudinal

[9, 10, 11, 12, 13, 14, 15, 16, 17, 18] are a selection of relevant studies on motorsport vehicle dynamics with TV control. Many of the papers [9, 10, 11, 12, 13] study Formula SAE vehicles. For example,

acceleration (G-G diagram). [17] addresses the problem of minimum-time maneuvering in EVs with multiple motors. The proposed strategy is compared against a benchmark optimal solution. On similar subject, [18] presents a non-linear optimal controller to find the optimal left-to-right torque distribution profile for achieving minimum time during a lane change maneuver.

This study focuses on a driverless electric racing car prototype with multiple motors, developed for an EV competition. From a technological viewpoint, this is aimed at: a) demonstrating the feasibility of high speed track racing with autonomous EVs; and b) facilitating the development and testing of cutting edge automated driving systems for high performance vehicle applications.

The paper presents the EV control system architecture and its TV controller design, and critically analyzes its performance benefits. With respect to the available studies, the points of novelty are:

- i. The description of the control structure of the case study driverless EV for racing applications, with focus on the integration of the automated driving system with the TV controller.
- ii. The analysis of the EV cornering response in steady-state and transient conditions without TV, for a wide range of vehicle speeds, based on the objective comparison of the understeer characteristics. The effect of vehicle aerodynamics on the understeer characteristics is shown to be significant.
- iii. The implementation of a single-input single-output yaw rate controller for the computation of the reference yaw moment, with a reference yaw rate taking into account the measured (or estimated) sideslip angle.
- iv. The demonstration that the single-input single-output yaw rate controller in iii) is effective in managing consistent and robust vehicle response, and constraining sideslip also for varying tire-road friction conditions. This functionality does not need tire-road friction coefficient estimation, or a multi-variable feedback controller on yaw rate and sideslip angle.
- v. The critical analysis of the benefits associated with the implementation of the TV controller.

The paper is organized as follows. The case study automated driverless EV is introduced, together with its control structure, with focus on the TV controller, including feedforward and feedback contributions. The quasi-static model for the off-line design of the feedforward control action is presented. The design of the feedback part of the controller is discussed. Vehicle dynamics simulations show the performance benefits of the proposed TV control structure.

CASE STUDY ELECTRIC VEHICLE

Vehicle Hardware

Figure 1 shows the schematic of the autonomous EV including the sensors, while the main vehicle parameters are reported in Table 1. An aerodynamic package is included, which generates significant

aerodynamic downforce, in order to increase the longitudinal and lateral acceleration limits. The aerodynamic characteristics have a measurable impact on the level of understeer or oversteer. In other words, different from passenger cars, the cornering response changes with speed.

The EV consists of four on-board electric drivetrains (D1,..., D4), each of them including a permanent magnet electric motor and inverter, a single-speed transmission system, outer and inner constant velocity joints, and a half-shaft, connecting the transmission to the wheel. The vehicle is equipped with an electro-hydraulic braking system, decoupling the pressure levels at the master cylinders from those at the respective brake calipers. In particular, the pressure levels of the front and rear calipers can be independently controlled (i.e., this is a two-channel system), but no controllable pressure difference can be generated among the left- and right-hand side calipers. Hence, a yaw moment cannot be created through the friction brakes, but it can be generated by the electric motor drives.

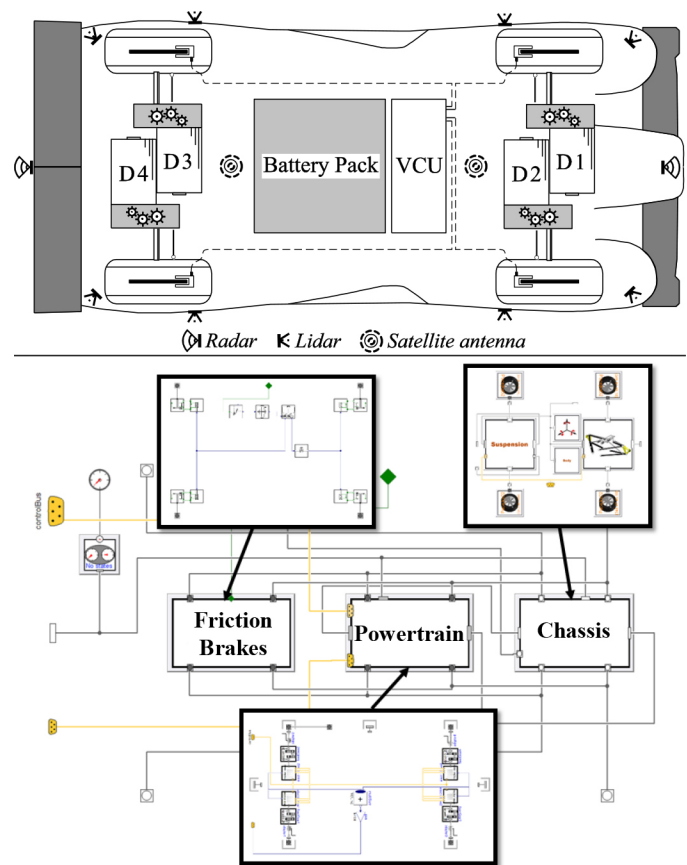


Figure 1. Schematic of the autonomous EV with the respective Dymola simulation model.

The vehicle includes a comprehensive set of sensors, such as cameras, radars, lidars, proximity sensors, and global navigation satellite system receiver, for the implementation of the automated driving function. Moreover, the EV is equipped with the typical sensors for stability control, such as the inertia measurement unit for

detecting the linear accelerations and angular velocities in all directions. For effective TV control, the value of sideslip angle is directly measured at a specific location. Through the yaw rate signal, the sideslip angle measurement can be referred to any other point on the longitudinal axis of the vehicle reference system. Sideslip angle measurement is particularly beneficial to control system performance in variable tire-road friction conditions.

Table 1. Main vehicle parameters.

Vehicle mass	1100 kg
Front semi-wheelbase	1.6 m
Rear semi-wheelbase	1.3 m
Front track width	1.35 m
Rear track width	1.52 m
Front tires	Michelin Pilot Sport 30/65-18
Rear tires	Michelin Pilot Sport 31/71-18
Powertrains	4 on-board 135 kW electric motors
Transmission gear ratio	6.25:1

Control System Hierarchy

Figure 2 summarizes the control structure running on the Vehicle Control Unit (VCU), including:

- The sensor fusion system, combining the measurements from the different sensors, and generating the inputs for the other parts of the control system. The outputs of the sensor fusion system include: i) the position and relative velocity of the ego vehicle (i.e., the case study EV) with respect to the surrounding track, obstacles and other vehicles. This information is used for the definition of the reference path and velocity profile, and the computation of the error between the reference path and actual vehicle trajectory; ii) the operating conditions of the ego vehicle for the purpose of TV control (e.g., longitudinal acceleration, lateral acceleration, yaw rate, sideslip angle, vertical load at each corner); and iii) the longitudinal slip of each tire, for the purpose of wheel slip control.
- The path generation layer, defining the reference path, i.e., the reference vehicle trajectory, including path re-planning based on the other vehicles and obstacles actually present on the race track. This layer is also responsible for defining the reference speed profile, $V_{ref}(t)$.
- The path tracking controller, based on the formulation presented in [19], with a steering angle control law given by the combination of feedforward and feedback contributions, respectively δ_{FFW} and δ_{FB} :

$$\begin{aligned}\delta &= \delta_{FFW} + \delta_{FB} \\ \delta_{FFW} &= l\kappa_{ref} - \alpha_F^{FFW} + \alpha_R^{FFW} \\ \delta_{FB} &= -K_{P,PT}e_{LA}\end{aligned}\quad (1)$$

κ_{ref} is the curvature of the reference path. e_{LA} is the look-ahead error of the path tracking controller. The feedforward contribution is based on the steady-state cornering response of a simplified vehicle model. The feedforward slip angles, α_F^{FFW} and α_R^{FFW} , are calculated from a brush tire model, with consideration of load transfers. α_F^{FFW} and α_R^{FFW} must generate the respective feedforward lateral tire forces on the front and rear axles, F_{yF}^{FFW} and F_{yR}^{FFW} . These are computed as functions of vehicle speed, u , and the curvature of the reference path, i.e., they are functions of the reference lateral acceleration $u^2\kappa_{ref}$:

$$\begin{aligned}F_{yF}^{FFW} &= \frac{mb}{l}u^2\kappa_{ref} - \frac{M_{z,sat}}{l} \\ F_{yR}^{FFW} &= \frac{ma}{l}u^2\kappa_{ref} + \frac{M_{z,sat}}{l}\end{aligned}\quad (2)$$

Eq. (2) is the result of the yaw moment and lateral force balance equations applied to the single-track vehicle model. The terms in $M_{z,sat}$ account for the TV-related yaw moment contribution, which alters the lateral tire force distribution among the front and rear axles. This control layer also generates the reference wheel torque demand, $T_{w,top}$ in order to track the reference velocity.

- The TV controller, defining the reference wheel torques, $T_{w,i}$, to achieve the target vehicle yaw rate and overall wheel torque demand. The subscript i indicates an individual wheel (see the Definitions/Abbreviations section). This control block will be discussed in detail in the remainder.
- The wheel slip controllers, aimed at preventing individual wheel locking or spinning in extreme braking or traction conditions. This block alters the individual wheel torque demands, thus generating the modified wheel torques, $T_{w,mod,i}$.
- The brake blending controller. In traction the reference motor torques are calculated by dividing $T_{w,mod,i}$ by the transmission gear ratio. In braking the brake blending controller calculates the torque split between the friction brakes and the electric motors, thus generating the front and rear friction brake pressures, $p_{b,F}$ and $p_{b,R}$, and the vector of the electric motor torque demands, $T_{EM,i}$.

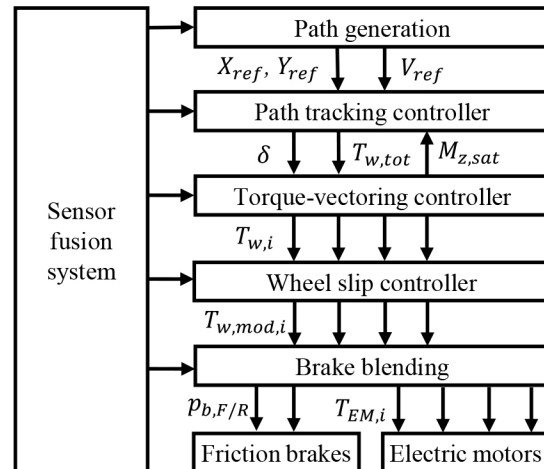


Figure 2. Simplified schematic of the EV control hierarchy.

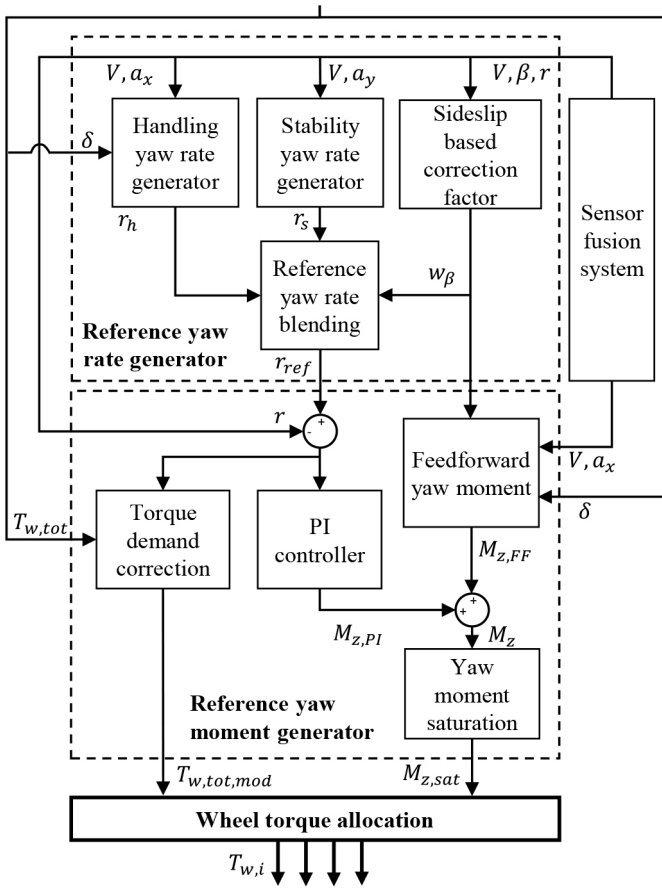


Figure 3. Simplified schematic of the TV control structure.

TORQUE-VECTING CONTROLLER

The TV controller is the focus of this study. Figure 3 is the simplified block diagram of the implemented control structure, including:

- The reference yaw rate generator, calculating the reference yaw rate from the inputs from the path tracking controller and the sensor fusion system.
- The reference yaw moment generator, outputting the reference yaw moment for EV TV control, and correcting the reference wheel torque demand in case of critical conditions.
- The wheel torque control allocator, responsible for the calculation of the individual reference wheel torques generating the reference yaw moment and total reference wheel torque.

Reference Yaw Rate Generator

The main contribution of the reference yaw rate is defined in a look-up table, which is a function of steering angle, vehicle speed and filtered longitudinal acceleration:

$$r_h = r_h(\delta, V, a_x) \quad (3)$$

r_h is the so-called handling yaw rate, i.e., a yaw rate that is suitable for the vehicle operating in high friction conditions. Different look-up tables for r_h can be defined, corresponding to driving modes, which

can be selected depending on the specific conditions and characteristics of the race track, and desired EV behavior, i.e., more or less conservative.

In order to achieve stable EV cornering response even in case of quickly variable tire-road friction conditions, a second reference yaw rate value is defined, i.e., the so-called stability yaw rate, r_s . r_s represents a reference yaw rate that is compatible with the available tire-road friction conditions, i.e., with the current level of measured lateral acceleration, a_y . In practice, r_s is the saturated value of r_h :

$$r_s = \begin{cases} r_{sat} \text{sign}(r_h) & \text{if } |r_{sat}| < |r_h| \\ r_h & \text{if } |r_{sat}| \geq |r_h| \end{cases} \quad (4)$$

where the saturation value, r_{sat} , is given by:

$$r_{sat} = \frac{a_y - \text{sign}(a_y)\Delta a_y}{V} \quad (5)$$

The offset Δa_y , which is also a function of a_y , is set to guarantee some conservativeness on r_s , i.e., to make sure that the vehicle does not experience a progressive increase of sideslip angle when tracking r_s at constant steering angle and torque demand.

The steady-state value of the reference yaw rate is calculated as the weighted average of the handling yaw rate and the stability yaw rate:

$$r_{ref,ss} = r_h w_\beta + r_s (1 - w_\beta) \quad (6)$$

The weighting factor, w_β , is a function of the sideslip angle of the rear axle, defined as:

$$\beta_R = \beta - \frac{br}{V} \quad (7)$$

w_β is formulated such that when $|\beta_R| < \beta_{act}$ the reference yaw rate is equal to the handling yaw rate, and when $|\beta_R| > \beta_{th}$ the reference yaw rate is equal to the stability yaw rate.

$$w_\beta = \begin{cases} 1 & \text{if } |\beta_R| < \beta_{act} \\ \frac{\beta_{th} - |\beta_R|}{\beta_{th} - \beta_{act}} & \text{if } \beta_{act} \leq |\beta_R| \leq \beta_{th} \\ 0 & \text{if } |\beta_R| > \beta_{th} \end{cases} \quad (8)$$

The thresholds β_{act} and β_{th} are important tuning parameters of the TV controller. They can be used to effectively constrain EV sideslip in variable friction conditions, even in absence of a tire-road friction coefficient estimator. In particular, in the remainder the following tunings of the sideslip-related contribution are considered: i) tuning A: $\beta_{act} = 4.5$ deg, $\beta_{th} = 7.5$ deg; ii) tuning B: $\beta_{act} = 9$ deg, $\beta_{th} = 13$ deg; iii) tuning C: $\beta_{act} = 20$ deg, $\beta_{th} = 25$ deg; and iv) tuning D: $\beta_{act} = 2$ deg, $\beta_{th} = 4$ deg.

The reference yaw rate is calculated by filtering $r_{ref,SS}$ i.e., $r_{ref}(s) = r_{ref,SS} \omega_r / (s + \omega_r)$, where the cut-off frequency ω_r is an important tuning parameter of the transient cornering response.

Reference Yaw Moment Generator

The reference yaw moment generator is based on a non-linear feedforward contribution, $M_{z,FF}$, and a feedback contribution, $M_{z,PI}$, which operate in parallel and are summed together:

$$M_z = M_{z,FF} + M_{z,PI} \quad (9)$$

Consistently with the scheme already used for r_h , in the on-line implementation of the controller $M_{z,FF}$ is computed from a look-up table (different for each driving mode), which is a function of δ , V , and a_x . This look-up table, outputting $M_{z,FF,LUT}$, is designed off-line with a quasi-static model, and is stored in the VCU. The purpose of $M_{z,FF}$ is to achieve the desired cornering response for the EV operating in steady-state with a high value of the tire-road friction coefficient (dry tarmac). The feedforward contribution brings this objective with a relaxed tuning of the feedback contribution of the controller, thus guaranteeing good stability properties and yaw rate sensor noise rejection. $M_{z,FF,LUT}$ is corrected with a factor w_{β,M_z} , which has a formulation similar to that of w_{β} in Eq. (8).

$$M_{z,FF} = w_{\beta,M_z} M_{z,FF,LUT}(\delta, V, a_x) \quad (10)$$

The purpose of w_{β,M_z} is to reduce the significance of the feedforward contribution when $|\beta_R|$ is larger than expected. High values of sideslip mostly happen when the tire-road friction coefficient is lower than that for dry tarmac. In these situations $M_{z,FF,LUT}$ is not relevant any more, as it is designed for EV operation on dry tarmac, which represents the majority of the EV running time.

$M_{z,PI}$ is based on a PI controller. The formulation includes an anti-windup term, to prevent issues with the integral contribution when M_z is saturated:

$$M_{z,PI} = K_p(r_{ref} - r) + \int K_I(r_{ref} - r) dt - \int K_w(M_z - M_{z,sat}) dt \quad (11)$$

$M_{z,sat}$ is the output of the dynamic saturation of the reference yaw moment. The yaw moment saturation is carried out based on the drivetrain torque limits and estimated tire-road friction limits. The latter are identified when $|\beta_R|$ goes beyond critical thresholds. Given the dependency of yaw and sideslip damping on vehicle speed, a vehicle speed-based gain scheduling scheme is used for the proportional and integral gains, i.e., $K_p = K_p(V)$ and $K_I = K_I(V)$. The integral term also includes a reset integrator condition, based on the reference yaw rate and its time derivative:

$$K_{I,reset} |r_{ref}| + |\dot{r}_{ref}| > K_{I,reset,th} \quad (12)$$

The reference yaw moment generator is also responsible for reducing the total wheel torque demand, $T_{w,tot}$, when significant values of the yaw rate error occur, thus originating $T_{w,tot,mod}$. This method is similar to the algorithms currently adopted on stability control systems based on the actuation of the friction brakes.

Control Allocation Algorithm

Because of the overactuated nature of the system in normal operating conditions, a control allocation criterion is employed for optimally distributing the wheel torques. In particular, under the hypothesis of small steering angles, the total reference wheel torques for the left- and right-hand side drivetrains are:

$$\begin{aligned} T_l &= \frac{T_{w,tot,mod}}{2} - \frac{2R_l M_{z,sat}}{t_F + t_R} \\ T_r &= \frac{T_{w,tot,mod}}{2} + \frac{2R_l M_{z,sat}}{t_F + t_R} \end{aligned} \quad (13)$$

The individual reference wheel torques are assigned according to the estimated vertical load distribution on the respective EV side:

$$\begin{aligned} T_{w,1} &= T_l \frac{F_{z,1}}{F_{z,1} + F_{z,3}} \\ T_{w,2} &= T_r \frac{F_{z,2}}{F_{z,2} + F_{z,4}} \\ T_{w,3} &= T_l \frac{F_{z,3}}{F_{z,1} + F_{z,3}} \\ T_{w,4} &= T_r \frac{F_{z,4}}{F_{z,2} + F_{z,4}} \end{aligned} \quad (14)$$

Especially when the EV is operating in proximity of its limits, it can happen that Eq. (14) provides wheel torques that are higher than the maximum or minimum wheel torques achievable on the individual wheels. To guarantee the generation of $T_{w,tot,mod}$ and $M_{z,sat}$ in any case, a set of rules identifies the saturation conditions on the individual wheels. If required, the wheel torque distribution of Eq. (14) is corrected a-posteriori, to provide the reference wheel torque demand and yaw moment. The details of this rule-based two-stage control allocation algorithm will be reported in future papers.

QUASI-STATIC MODEL

The TV controller requires the off-line computation of the look-up tables of $r_h(\delta, V, a_x)$ and $M_{z,FF,LUT}(\delta, V, a_x)$. The reference yaw rate and feedforward yaw moment values in the look-up tables must be consistent with each other, and compatible with the achievable EV cornering response, and with the powertrain torque and power limitations as well. According to the approach in [20-21], an eight-degree-of-freedom non-linear quasi-static model is used for the design of the look-up tables.

The degrees of freedom are the longitudinal, lateral, roll and yaw motions, and the rotation of the four wheels. The quasi-static approach assumes the time derivative of vehicle sideslip angle, $\dot{\beta}$, roll angle, $\dot{\phi}$, and wheel slip ratios, $\dot{c}_i$, to be zero. In this way the