

Nonlinear Time-Varying Dynamic Interactions of Hypoid Gear-Shaft-Bearing Systems

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ABSTRACT

Nonlinear interaction between time-varying hypoid gear mesh and bearing support is investigated in this study. Mesh parameters are time-varying due to complex tooth profile of hypoid gear. Bearing stiffness is formulated based on real geometry and instantaneous orbital position of rolling elements. Linear model is firstly analyzed to study the modal frequency and mode shape variations under different stiffness ratio between gear mesh and bearing support. Then, nonlinear analysis is conducted to compare the differences between linear and nonlinear dynamic response based on specific nonlinear conditions of geared rotor system. It is found that the coupling between hypoid gear mesh and bearing support can be either strong or weak depending on the ratio between mesh stiffness along line-of-action (LOA) and bearing stiffness in radial direction. Parametric studies indicate that dynamic mesh force is sensitive to bearing clearance for certain stiffness ratio. Spectrum analysis further reveals complex nonlinear behavior due to loss of contact between meshing gear teeth. Dynamic force changes on actual bearing locations due to bearing clearance are evaluated. It is found that bearing radial clearance has influence on structure-borne noise transmission for a complete hypoid gear transmission system due to its effect on gear dynamic response and actual bearing loads.

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1. INTRODUCTION

Gear dynamic response due to combined excitation of transmission error and external torque is a main source of structure borne noise of transmission. Therefore, it is necessary to develop an accurate geared shaft-bearing model for analyzing the underlying mechanism of complex interaction between different components especially when there exist clearance type nonlinearity in either gear mesh or bearing support. Therefore, an accurate prediction of the severity of system dynamic problems can be obtained at an early design stage.

Gear backlash nonlinearity has been extensively studied in the past several decades. Özgüven and Houser [1] concluded a series of modeling methods for nonlinear dynamic analysis of spur gear mesh. Comparin and Singh [2] investigated response stability of an impacting gear pair using analytical method. Kahraman and Blankenship [3] further analyzed mesh interface of gear pair and observed period doubling bifurcation and broad band chaotic response in experimental tests. Walha et al [4] applied nonlinearity and time-varying mesh stiffness in a two-stage geared rotor system and solved the model by decomposing into several linear systems. Cheon-Jae and Parker [5] proposed an analytical modeling technique for a planetary gear system with backlash. The results showed good correlation with numerical solution. Sébastien and Jacquenet [6] studied the interaction between gear mesh and hydrodynamic journal bearings. Their work further showed the influence of time-varying mesh stiffness in nonlinear dynamic response. However, much less work can be found in hypoid gear dynamic analysis. Cheng and Lim

[7] put forward a mathematical framework for hypoid and bevel gear dynamic analysis based on tooth contact analysis (TCA). Mesh coupling consist of effective mesh stiffness, mesh damping, mesh point and line-of-action (LOA). Backlash was modeled as clearance type nonlinear function. Wang [8] extended the model by incorporating multi-point mesh during operation and studied the load sharing effect of engaging gear pairs. Later, Wang [9] derived asymmetric mesh model considering different mesh parameters of convex and concave side of meshing teeth. More recently, Peng [10] developed a coupled multi-body dynamic model for hypoid geared shaft-bearing system. Instead of analyzing pure vibration or relative motion of meshing gear pair, the model was developed to incorporate large rotational motion of pinion and gear body. Time-varying mesh parameters were formulated based on instantaneous rotation angle of pinion or gear shaft. The model can thus be used for either steady state or transient dynamic analysis.

Bearing radial clearance is another nonlinearity commonly seen in geared shaft bearing system which can interact with gear backlash. El-Saeidy and Fawzi MA [11] evaluated response spectrum of simple spur gear box considering back lash and bearing deadband clearance using FFT. Kahraman and Singh [12] studied nonlinear interaction and route to chaos of a spur geared rotor-bearing system with multiple clearances. Later, Gurkan and Ozguven [13] conducted parametric analysis on the nonlinear coupling effect and discovered its relationship with gear mesh and bearing stiffness ratio. Housheer et al [14] investigated the combined effect of bearing internal radial

clearance and surface waviness on nonlinear response of balanced rotor supported by ball bearings. Tiwari et al [15] focused on the in-depth analysis of response instability due to variation of bearing radial clearance and it was found that sub-harmonic and chaotic behavior in response are mostly dependent on bearing nonlinearity. Theodossiades and Natsiavas [16] formulated and compared several analytical modeling methods of gear mesh that incorporate both bearing clearance and gear backlash in analyzing periodic and chaotic phenomenon in dynamic response of load dependent gear mesh. The analysis was further validated by comparing with experimental and numerical study. Later, Guo and Parker [17] introduced bearing clearance into planetary gear dynamic analysis and investigated root cause of softening and hardening effect in response. However, bearing clearance type nonlinearity has not been considered in hypoid gear analysis by far due to complex geometry of tooth profile. In addition, most nonlinear bearing stiffness applied are based on time-invariant function without taking into account the changing of orbital position of rolling elements which has been proved to have certain effect on hypoid gear dynamic response [18, 19].

Therefore, the purpose of this study is to further extent the previous work on hypoid gear dynamics by studying the coupling effect between gear backlash and bearing radial clearance nonlinearities. First, time-invariant model is studied under different nonlinear conditions through which good correlation with related research work is demonstrated. In addition, more complicated coupling influences that cause softening or hardening are found. Then time-varying mesh parameters and bearing stiffness functions are introduced. It is shown that time-varying parameters tend to have significant effect on nonlinear dynamic response in both frequency and time domain. Finally, actual load on bearing is also investigated through parametric study to analyze system sensitivity to different nonlinear conditions. Results demonstrate specific load distribution among bearings which is expected to provide potential guidelines in future bearing design and selection in a hypoid geared rotor system.

2. NONLINEAR GEARED ROTOR SYSTEM MODELING

A generic 14-DOF coupled multi-body dynamic model of a hypoid geared rotor system applied in this study is illustrated in Fig. 1. The system consists of an engaging gear pair, driver, load and supporting shaft-bearing elements. The pinion/gear body is assumed as rigid body with lumped stiffness and damping as the mesh interface. The general modeling process is as follows:

1. The coupling between pinion and gear is simplified as a spring-damper element based on quasi-static loaded tooth contact analysis (LTCA) which applies a semi-analytical and finite element method [20, 21]. Major mesh parameters include mesh point, LOA, mesh stiffness and kinetic transmission error.
2. The dynamic model of hypoid geared rotor system is then built by considering coupling effect from shaft-bearing support. Bearing stiffness function can be either assumed as simple support for basic parametric study or time-varying model for more accurate prediction of dynamic performance.

3. In order to evaluate actual dynamic load on bearings, a combined quasi-static shaft-bearing model is further incorporated. The kinematic relationship between reference point and bearing support locations are shown. Thus bearing load due to dynamic mesh force excitation can be derived.

A. Gear Mesh Modeling and Tooth Contact Analysis

The gear pair geometry and mesh mechanism is illustrated in Fig. 2 where the contact area is divided into N cells with $r_i(r_{ix}, r_{iy}, r_{iz})$ and $n_i(n_{ix}, n_{iy}, n_{iz})$ as position and normal vector of an individual cell and f_i is the cell contact force in normal direction. Therefore, the overall combined mesh force can be written as:

$$F_m = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (1)$$

$$F_s = \sum_{i=1}^N n_{is} f_i, \quad (s = x, y, z) \quad (2)$$

The coordinates of mesh LOA vector $L_m(n_x, n_y, n_z)$ can be expressed as:

$$n_s = F_s / F_m, \quad (s = x, y, z) \quad (3)$$

Position of effective mesh point $R_m(r_x, r_y, r_z)$ calculated as:

$$r_x = (M_z + F_y r_y) / F_y \quad (4)$$

$$r_y = (M_x + F_z r_z) / F_z \quad (5)$$

$$r_z = (M_y + F_x r_x) / F_x \quad (6)$$

where $M_{x/y/z}$ is the moment derived from combined load vectors of each contact cell in contact area about coordinate axis.

Transmission error (TE) is the source of internal excitation of gear mesh which describes the deviation of gear body rotational motion from ideal position. Translational TE is calculated by projecting loaded or unloaded angular TE along LOA. Then, the effective gear mesh stiffness can be formulated as:

$$k_m = \frac{F_m}{e_L - e_0} \quad (7)$$

where e_L and e_0 are loaded and unloaded translational TE respectively. The above time-varying mesh parameters generated from loaded TCA represent the coupling between engaging gear pair and will be used in coupled multi-body dynamic model of geared rotor system.



B. Multi-body Dynamic Model of Hypoid Gear Shaft-Bearing System

The complete geared rotor system shown in Fig. 1 is represented as a 14 degree-of-freedom (DOF) lumped parameter model. The equation of motion can be written in matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} + [G]\{\dot{x}\} + [G_a]\{x\} = \{F\} \quad (8)$$

$$[M] = \text{diag}[I_D, M_p, M_p, M_p, I_{px}, I_{py}, I_{pz}, M_g, M_g, M_g, I_{gx}, I_{gy}, I_{gz}, I_L] \quad (9)$$

where I_D and I_L stand for mass moment of inertia of driver/engine and load respectively. In addition, M_l and I_{lm} ($l=p, g; m=x, y, z$) are the body mass and moment of inertia of pinion/gear. Accordingly, the system displacement vector is shown as:

$$\{x\} = \{\theta_D, x_p, y_p, z_p, \theta_{px}, \theta_{py}, \theta_{pz}, x_g, y_g, z_g, \theta_{gx}, \theta_{gy}, \theta_{gz}, \theta_L\}^T \quad (10)$$

The torsional motion of pinion/gear body, engine and load are large rotational displacements [10]. Notice that the coordinate system of the complete geared rotor system is different from that of tooth contact analysis illustrated in Fig. 2. Therefore, coordinate transition is need when applying mesh parameters into system dynamic analysis.

The stiffness matrix consists of input (k_D) and output (k_L) shaft torsional stiffness and lumped shaft-bearing support stiffness on pinion ($[K_p]$) and gear ($[K_g]$) shafts. Damping matrix $[C]$ is derived from component support damping model. The system stiffness matrix can be expressed as:

$$[K] = \begin{bmatrix} k_D & & \\ & [K_p] & \\ & & [K_g] \\ & & & k_L \end{bmatrix} \quad (11)$$

The excitation force vector is comprised of internal and external excitations which can be defined as:

$$\{F\} = \{T_D, h_p \cdot F_m, -h_g \cdot F_m, -T_L\}^T + \{F_{ext}\} \quad (12)$$

where T_D and T_L are torque at driving and load end. Coordinate transition vector h_l ($l=p, g$) is used for projecting mesh force onto pinion or gear local coordinates. It is represented by normal vector and rotational radii of effective mesh point:

$$h_l = \{n_{lx}, n_{ly}, n_{lz}, \lambda_{lx}, \lambda_{ly}, \lambda_{lz}\} \quad (13)$$

The mesh force F_m acts at effective mesh point along effective LOA and is derived from dynamic transmission error (δ) and static transmission error (e_0):

$$F_m = k_m \cdot f(\delta - e_0) - c_m \cdot (\dot{\delta} - \dot{e}_0) \quad (14)$$

where k_m is the effective mesh stiffness calculated from loaded TCA and c_m is the mesh damping coefficient. Dynamic transmission error δ is expressed as:

$$\delta = h_p^T \cdot \{x_p, y_p, z_p, \theta_{px}, \theta_{py}, \theta_{pz}\} - h_g^T \cdot \{x_g, y_g, z_g, \theta_{gx}, \theta_{gy}, \theta_{gz}\}. \quad (15)$$

where $f(\delta - e_0)$ is the clearance type nonlinear function comprised of three stages depending on the relationship between $\delta - e_0$ and backlash b_c :

$$f(\delta - e) = \begin{cases} \delta - e_0 - b_c, & \delta - e_0 \geq b_c, \\ 0, & -b_c < \delta - e_0 < b_c, \\ \delta - e_0 + b_c, & \delta - e_0 \leq -b_c, \end{cases} \quad (16)$$

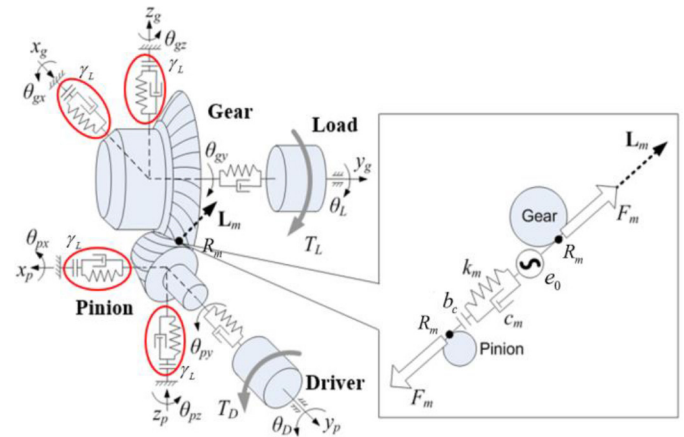


Fig. 1. 14-DOF lumped parameter model of hypoid geared rotor system

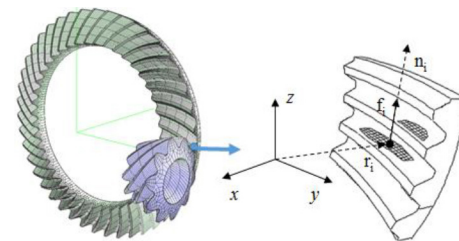


Fig. 2. Illustration of loaded tooth contact analysis

C. Nonlinear Time-Varying Bearing Stiffness Model

The clearance type nonlinear rolling element bearing model is illustrated in Fig. 3 where $\{F_{bxm}, F_{bym}, F_{bzm}, M_{xm}, M_{ym}\}$ is the load acting on the bearing assuming it can rotate freely around z-axis of the local coordinate system. Accordingly, $\{\delta_{xm}, \delta_{ym}, \delta_{zm}, \beta_{xm}, \beta_{ym}\}$ is the

translational and rotational displacement of bearing model. Based on previous studies [18], bearing stiffness matrix can be generated using partial differential methods assuming rigid raceways as:

$$[K_b] = \begin{bmatrix} \frac{\partial F_{bim}}{\partial \delta_{jm}} & \frac{\partial F_{bim}}{\partial \beta_{jm}} \\ \frac{\partial M_{bim}}{\partial \delta_{jm}} & \frac{\partial M_{bim}}{\partial \beta_{jm}} \end{bmatrix} \quad (17)$$

Bearing stiffness variation is caused by changing orbital position of rolling elements in raceway during operation. Assuming that interaction between rolling elements and inner raceway is pure motion, the position of rolling elements in raceway can be expressed as:

$$\psi_s(t) = \frac{1}{2} \left(1 - \frac{r_b}{r_d} \cos(\alpha_0) \right) \omega_z t + \frac{2\pi(s-1)}{Z}, s = 1, 2, \dots, Z, \quad (18)$$

where r_b is element radius, r_d is the pitch radius of raceway, α_0 is unloaded contact angle and ω_z is the shaft speed.

Radial clearance type nonlinearity is also considered in bearing stiffness model (Fig. 3) which can be described using non-linear displacement functions $f_{bq}(x_{lq})$ where $l = p, g$; $q = x, z$. Therefore, the equation of motion in radial translational direction can be rewritten as:

$$M_{lq} \ddot{x}_{lq} + c_{lqt} \dot{x}_{lq} + k_{lqt} f_{bq}(x_{lq}) = n_{lq} F_m \quad (19)$$

$l = p, g; \quad q = x, z$

where the detailed expression of non-linear function based on quasi-static contact analysis can be formulated as [12]:

$$f_{bx}(x_{lq}) = \begin{cases} \sum_j \left\{ (x_{lq} \cos \psi_j - \gamma_L) \right\}^n \cdot \cos \psi_j, & x_{lq} > \gamma_L \\ 0, & -\gamma_L < x_{lq} < \gamma_L \\ -\sum_j \left\{ (|x_{lq}| \cos \psi_j - \gamma_L) \right\}^n \cdot \cos \psi_j, & x_{lq} < -\gamma_L \end{cases} \quad (20)$$

where γ_L is bearing radial clearance, ψ_j is the angular position of j th rolling element, Z is the total number of rolling elements under load conditions, n is the power coefficient defined by Hertzian contact: for ball bearing $n = 1.5$, for roller bearing $n = 10/9$. According to previous studies [12, 13], the bearing nonlinear displacement function can be approximated by piecewise linear function similar to gear backlash nonlinear function shown in Eq. 16 without losing much accuracy.

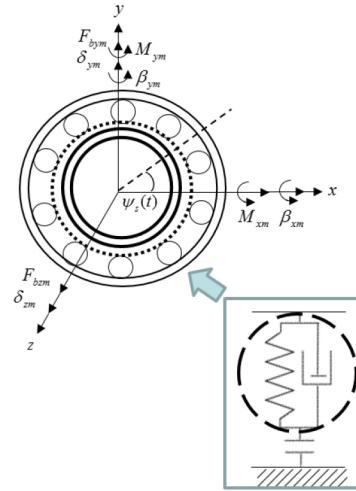


Fig. 3. Stiffness model of nonlinear bearing

D. Combined Quasi-Static Shaft-Bearing Model

The configuration of combined shaft-bearing model used is illustrated in Fig. 4. Shaft finite element model is built from static beam element and is combined with bearing stiffness model to synthesize effective lumped support stiffness matrix relative to lumped point. Hypoid gear pair which is modeled as rigid body is mounted at the reference point.

For computational simplicity and convenience, it is assumed that the bearing stiffness is not influenced by operating loads. Therefore, the load displacement relation of the combined shaft-bearing system can be written in matrix form as:

$$\begin{Bmatrix} [\Delta_R]_{5 \times 5} \\ [\Delta_S]_{5 \times n} \end{Bmatrix} = \begin{bmatrix} K_{RR} & K_{RS} \\ K_{SR} & K_{SS} \end{bmatrix}_{n \times n}^{-1} \begin{Bmatrix} [F_R]_{5 \times 5} \\ 0 \end{Bmatrix}_{5 \times n} \quad (21)$$

The 5 by 5 matrix $[\Delta_R]$ stands for the displacement at reference point in five DOFs except the torsional displacement around rotation axis. The 5 DOF matrix $[\Delta_S]$ stands for the displacement at other nodes. The external force expressed as $[F_R]$ is acting on the reference point. For calculation simplicity, the force on reference point is set to be unit load. Therefore, the equivalent time-varying lumped stiffness matrix can be written as:

$$[K_{RR}]_{5 \times 5} = [\Delta_R]^{-1} \quad (22)$$

The stiffness is calculated at each mesh frequency and specific roller orbit position. An example of x-direction translational stiffness of lumped support on pinion is illustrated in Fig. 5. The stiffness value in Fig. 5 (a) is dependent on different position of rolling elements as shown in Fig. 5 (b).