

A Study on Hypoid Gears NVH Robustness

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ABSTRACT

Hypoid gears transmission error (TE) is a metric that is usually used to evaluate their NVH performance in component level. The test is usually done at nominal position as well as out of positions where the pinion and gear are moved along their own axis and also along offset direction to evaluate sensitivity of the measured TE to positional errors. Such practice is crucial in practical applications where the gear sets are inevitably exposed to off position conditions due to a) housing machining and building errors, b) deflections of housing, bearings, etc. under load and c) thermal expansions or contractions of housing due to ambient temperature variations. From initial design to development stage, efforts should be made to design the gear sets to be robust enough to all combinations of misalignments emanated from all three mentioned categories. This study is one such effort a) to evaluate main parameters affecting TE response of a hypoid gear set at various misalignment conditions and temperature variations under various loads as well as b) quantifying topography effects on TE response under such load and misalignment conditions. Approximating surface corrections/modifications from nominal through a bi-cubical polynomial, the effects of different topography errors on loaded TE (LTE) is evaluated. Looking at overall LTE response under various micro-geometric modifications one can see how significant micro-geometry of gears are in controlling LTE behavior.

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INTRODUCTION

In recent years the so called End of Line Testers (EOLT) are used to screen axles or other drive units when they are final and before customer deliveries. EOLT's are usually utilized to evaluate a drive unit for NVH performance through exposing the unit to similar load and speed conditions that they will be experiencing in a real vehicle condition. Certain load or speed variations are applied and certain accelerations or vibrations or sometimes noise levels are measured, processed and correlated to vehicle conditions. This by itself has grown to be an effective technique on predicting NVH performance of drive units on EOLTs to reduce both varieties and levels of NVH issues experienced in vehicles[1, 2, 3, 4, 5].

There are various methods of generating screening data of an axle from the instrumentations utilized to screen an axle, as well as many methods to analyze and process the data. One popular approach is measurement of torsional acceleration of the output shaft when the input shaft is turning at constant speed and sweeping output torque from a minimum to a maximum. This is to represent a range at which quiet behavior of drive unit is desired [1]. Fig. 1 shows one sample of such data collection in chart format when the unit ran for drive side only (analysis of coast side is exactly the same and is skipped here). In this Figure each horizontal line is 10 dB and the chart goes from zero to 100 dB on vertical axis. As seen drive side torsional

acceleration goes significantly up when pinion torque sweeps from low load region of about 5 Nm to about 25 Nm where it makes its peak, then goes smoothly down to about 150 Nm and again gradually goes back up to about 190 Nm where the max test load is.

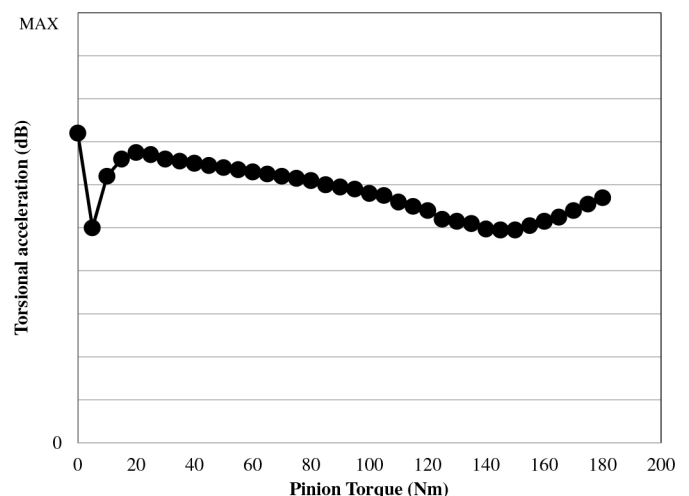


Figure 1. Sample EOLT torque sweep torsional acceleration for drive side of an axle.

This is just a sample of real world torsional acceleration measurement of an axle; depending on the tested axle especially gear set design and robustness of the housing and bearings which hold the gear set in place; this response can vary significantly both in shape and level of dB.

As it was stated in the abstract, the goal of this study is to see whether this response is predictable and hence subject to optimization and parameter study in design stage. If detail design parameters of the gear sets including both macro and micro level information are available and stiffness behavior of both housing and bearing to load is also in hand one could try to simulate the testing condition of an EOLT in design stage. This can potentially lead to a model capable of predicting EOLT results in design stage; correlating such model to actual EOLT data could make the model usable for design optimization or at least problem detection in early stages of gear, bearing or housing design. This study is one such effort toward developing such model.

Next sections of this study are aimed at developing base modules needed for such model which in short are as follows:

- i. First, a previously developed hypoid gear contact model used in this study is reviewed; this model is computationally enhanced for LTE calculation done in this study,
- ii. Second, potential misalignments coming from a) housing machining error, b) deflections of housing and bearings and c) thermal expansion and contraction are applied to calculate hypoid gear LTE response under different combinations of these misalignments,
- iii. Third, LTE signature response of a nominal/theoretical hypoid gear design to certain combination of misalignments is developed,
- iv. Fourth, micro-geometry deviations of hypoid gears from nominal/theoretical are introduced in form of a bi-cubical polynomial through ten different independent surface coefficients each of which representing a specific surface deviations like pressure angle error, etc. LTE response to some of these ten deviations is studied to show how sensitive LTE is to deviating from nominal/theoretical design. This deviation can happen as a result of a) teeth cutting errors, b) heat treating distortions and c) hard finishing deviations which could be grinding or lapping. Focus of this study is on lapped gear sets where micro-geometry of final gear set are always slightly different from theoretical designed surfaces.

HYPOID GEAR CONTACT MODEL (HGCM)

Development of a computationally efficient contact model for hypoid gears is one of the major building blocks for this NVH study. Not many models have been developed for different analysis of hypoid gears mainly due to complexity of the hypoid gear geometry [6,7]. The hypoid gear contact model (HGCM) used in this study is the model developed by Kolivand and Kahraman [8,9]. It combines three sub models of cutting simulation, unloaded tooth contact analysis (UTCA) and loaded tooth contact analysis (LTCA) which are briefly explained as follows.

Cutting Simulation Model

In general hypoid gear surfaces have implicit forms as envelope of the family of cutter surfaces. Any point on the hypoid gear cutter surfaces of either face-milling or face-hobbing cutting methods is described by two independent curvilinear variables s and t while each of the cutter surface families are described by their corresponding cradle angles φ_c , hence each family has the form [10,11]:

$$\mathbf{r} = f(\psi(\varphi_c), \kappa, s, t, \varphi_c) \quad (1)$$

Where $\psi(\varphi_c)$ is the vector of machine settings and is a function of cradle angle φ_c in case of applying higher order motions and is a constant vector otherwise and κ is the vector of cutter dimensions. Eq. (1) represents family of cutter surfaces seen from blank and its envelope is the gear surface. The envelope of $\mathbf{r}$ with three independent variables s , t and φ_c is given mathematically as [8,10,11]:

$$\mathbf{n} \cdot \partial \mathbf{r} / \partial \varphi_c = 0, \quad (2a)$$

$$\mathbf{n} = (\partial \mathbf{r} / \partial s) \times (\partial \mathbf{r} / \partial t) / |(\partial \mathbf{r} / \partial s) \times (\partial \mathbf{r} / \partial t)| \quad (2b)$$

This equation is equivalent to the fundamental equation of meshing [12]. Using Eq. (2), each point on a generated surface can be found by solving a system of two implicit nonlinear equations for two unknowns of s and t [13, 14, 15].

Unloaded Tooth Contact Model

Here ease-off approach is used to calculate unloaded tooth contact analysis (UTCA). Following Stadtfeld's proposed ease-off [12] Kolivand and Kahraman [8,16] developed a detailed formulation for construction of ease-off including determining the instantaneous potential contact lines/curves [16]. For UTCA, the goal is to calculate (i) Contact Point Path (CPP) and contact pattern on each of the gear surfaces, and (ii) the transmission error function between axes of contacting surfaces. Ease-off in this study is defined as the deviation of real gear surface from the conjugate of its real mating pinion surface [8,17,18]. Figure 2 shows a sample ease-off topography of a face hobbed hypoid gear set on ring gear which also includes toprem (top relief modification) at gear tip.

Loaded Tooth Contact Analysis

Actual number of tooth pairs in contact depends on the gear contact ratio, the roll angle of the pinion and the amount of applied torque. For loaded tooth contact analysis (LTCA), at each time step $k \in [1, n_s]$ all the tooth pairs that geometrically are likely to share the torque are taken into account with their respective separation distances extracted from ease-off topography [8,9]. Potential contact lines of all contacting tooth pairs are computed and discretized into a finite number of segments $\mathbf{e}_l^k$ where $k \in [1, n_s]$ and $l \in [1, n_l]$. The length of and separation at each segment $\mathbf{e}_l^k$ along each contact line are computed and used as input for the LTCA model. The load distribution, mesh stiffness and loaded transmission error are computed by satisfying compatibility and equilibrium conditions

simultaneously in the gear load distribution model [19]. The total compliance matrix used in both equations is sum of (i) bending and shear compliances estimated from shell theory [20], (ii) base rotation compliance approximated by the regression based equation of Stegmüller [21] and (iii) Hertzian compliance based on Weber equation [22]. Figure 3 is a sample mesh stiffness calculation for the same hypoid gear set of Figure 2.

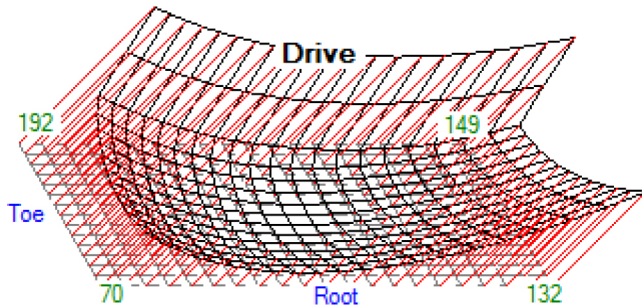


Figure 2. Sample hypoid ease-off (on ring gear) topography, including toprem.

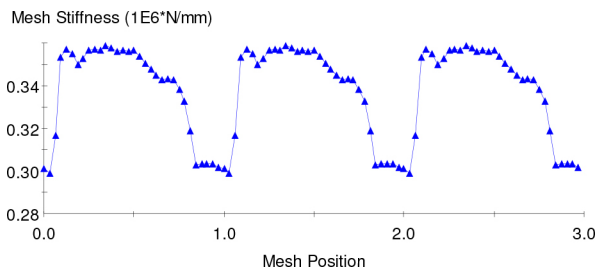


Figure 3. Sample mesh stiffness curve.

MISALIGNMENTS

Spatial relative position between pinion and ring gear in a hypoid gear set is usually defined through four independent parameters, three of them measured in distance and one in angle. These four are i) pinion offset, E, ii) pinion cone position, P, iii) gear cone position, G, and iv) pinion and gear axes relative angle, A. These four are sufficient to cover any kind of error realized between the two axes. Any other misalignments not exactly matching with one of these four can be introduced through certain combinations of them. Given this, any relative misalignment between the two axes can be captured with a deflection vector Δ_s as,

$$\Delta_s = E\bar{l} + P\bar{j} + G\bar{k} + A\bar{l} \quad (3)$$

where,

Δ_s : misalignments vector coming from any of misalignments sources. Subscript 's' is the source that any specific misalignment vector is coming from, which will be explained in next section,

$\bar{l}$: is in pinion offset direction or direction of common normal to pinion and gear axes,

$\bar{j}$: is along pinion axis,

$\bar{k}$: is along gear axis,

$\bar{l}$: is not really a vector per-se, it is just used to commonize this parameter as a vector with vectorial representation of other three parameters,

In general there are three independent sources that cause misalignments which are as explained in the following sections. These misalignments can be superimposed on each other to get overall misalignments Δ as follow:

$$\Delta = \Delta_B + \Delta_D + \Delta_T \quad (4)$$

where,

Δ : overall superimposed misalignments,

Δ_B : misalignments caused by machining or build errors,

Δ_D : misalignments cause by loading (deflections),

Δ_T : misalignments caused as a result of expansions or contractions because of temperature changes.

Build or Machining Error, Δ_B

This is caused by machining error in the housing where the bearing seats are; as well as other sources like differential case machining error where ring gear usually gets welded or bolted. When assembling hypoid gear sets, pinion and ring gear cone positions are usually shim-able, hence there is always potential for build errors through calling wrong shims and hence positioning one or both of the members off intended axial positions. Section 1 of table 1 shows in matrix form lower and upper levels for all the four independent parameters; this type of misalignments are random in nature as there is no direction or relation between these four; neither this is load or temperature dependent. Extensive measurements in production lines however may reveal some trends of machining errors inherent to each individual line [23,24].

Deflection, Δ_D

This is caused by deflection of all components that hold hypoid ring and pinion together. Main components are usually housing, bearings and differential case. Due to major difference in directions of the applied load when operating on drive vs. coast side, the deflection vectors between the two sides are quite different. Since this study is an attempt to predict loaded TE at various misalignments, deflection or in general misalignment vectors at a range of applied load from a minimum, Q^l , to a maximum, Q^u , is needed. For simplicity deflections at two different load levels of Q^l and Q^u are calculated and at any given load Q^i , deflection is calculated through linear interpolation or extrapolations between the two. Table 1 section 2 shows a matrix with two levels of load at which all four parameters are calculated.

Table 1. Misalignments range from all three sources.

	Min.	Max.
1. Housing machining or building error		
E(mm)	E_B^l	E_B^u
P(mm)	P_B^l	P_B^u
G(mm)	G_B^l	G_B^u
A(min)	A_B^l	A_B^u
2. Deflections (Drive or Coast)	Q^l	Q^u
E(mm)	E_D^l	E_D^u
P(mm)	P_D^l	P_D^u
G(mm)	G_D^l	G_D^u
A(min)	A_D^l	A_D^u
3. Thermal expansion or contraction	T^l	T^u
E(mm)	E_T^l	E_T^u
P(mm)	P_T^l	P_T^u
G(mm)	G_T^l	G_T^u
A(min)	A_T^l	A_T^u

Thermal Expansion or Contraction, Δ_T

Since axles usually operate under different ambient conditions, attention should be paid to analyze and test them under different temperatures. Any considerable change in temperature causes expansion or contraction of housing, bearing seats, pinion stem ring gear interface to differential case, etc. These expansions and contractions are also another source of misalignments, Table 1. section 3 summarizes such misalignment matrix when temperature changes from lower limit of T^l to upper limit of T^u ; for any given temperature T^l , expansion/contraction is calculated through linear interpolation or extrapolations between these two.

As stated misalignments from build or machining error (going forward will be referred in short as Δ_B), have random nature, so any combination of four parameters being at lower or upper limit is a possibility in real world production condition and hence it worth to be modeled. Having four parameters with two levels (lower and upper), we will have to consider 16 different cases to cover full factorial conditions (being at nominal is skipped here to minimize number of calculations, otherwise instead of $2^4=16$, $3^4=81$ should have been considered). These 16 different cases are at one single temperature condition; if influence of temperature at lower and upper limits is of interest total of 32 (16 for T^l and 16 for T^u) cases should be analyzed. Table 2 lists all 32 cases with their relevant build conditions; in addition it includes two more interesting cases of:

- Case 33: nominal condition, where all misalignment errors from build, Δ_B , are at zero and temperature is at T^l ,
- Case 34: Sum of squared, where sum of squared of all other 33 cases is calculated. This is used as an overall one single curve representing LTE response of a design to a misalignment condition.

For each of these 34 cases, misalignment matrix of deflection, Δ_D and thermal expansion/contraction, Δ_T is superimposed (simply added as in Eq. 4) to the misalignments caused by build errors, Δ_B , to reach full misalignment condition.

Next, LTE of a sample 240 mm face hobbled hypoid gear set of table 3 is analyzed to evaluate the design response to different misalignment conditions of table 2 (with Δ_D and Δ_T also superimposed).

LTE ROBUSTNESS TO MISALIGNMENTS

Using HGCM [8], LTE response of the design of table 3 was analyzed at all 34 (33 + SS) cases of table 2 under different input/pinion loads sweeping from 5 Nm all the way to 200 Nm in 21 equally spaced load steps. This is total of $33 \times 21 = 693$ contact analysis, and is where HGCM computational efficiency is of major advantage as it runs each analysis in less than 30 seconds, hence total of about 6 hrs. of computational time. The LTE analysis was limited to drive side only to reduce computational time. Figure 4 show results of such analysis in one single chart, where all 34 curves are drawn across the torque range. In this chart "MAX" LTE is at 100 urad and each block is 10 urad.

Table 2. Misalignments full factorial matrix.

Case #	T	E	P	G	A
1	T^l	E_B^l	P_B^l	G_B^l	A_B^l
2	T^l	E_B^l	P_B^l	G_B^u	A_B^l
3	T^l	E_B^l	P_B^u	G_B^l	A_B^l
4	T^l	E_B^l	P_B^u	G_B^u	A_B^l
5	T^l	E_B^u	P_B^l	G_B^l	A_B^l
6	T^l	E_B^u	P_B^l	G_B^u	A_B^l
7	T^l	E_B^u	P_B^u	G_B^l	A_B^l
8	T^l	E_B^u	P_B^u	G_B^u	A_B^l
9	T^l	E_B^l	P_B^l	G_B^l	A_B^u
10	T^l	E_B^l	P_B^l	G_B^u	A_B^u
11	T^l	E_B^u	P_B^l	G_B^l	A_B^u
12	T^l	E_B^u	P_B^l	G_B^u	A_B^u
13	T^l	E_B^u	P_B^u	G_B^l	A_B^u
14	T^l	E_B^u	P_B^u	G_B^u	A_B^u
15	T^l	E_B^l	P_B^l	G_B^l	A_B^u
16	T^l	E_B^l	P_B^l	G_B^u	A_B^u
17	T^u	E_B^l	P_B^l	G_B^l	A_B^l
18	T^u	E_B^l	P_B^l	G_B^u	A_B^l
19	T^u	E_B^u	P_B^l	G_B^l	A_B^l
20	T^u	E_B^u	P_B^l	G_B^u	A_B^l
21	T^u	E_B^l	P_B^u	G_B^l	A_B^l
22	T^u	E_B^l	P_B^u	G_B^u	A_B^l
23	T^u	E_B^u	P_B^u	G_B^l	A_B^l
24	T^u	E_B^u	P_B^u	G_B^u	A_B^l
25	T^u	E_B^l	P_B^l	G_B^l	A_B^u
26	T^u	E_B^l	P_B^l	G_B^u	A_B^u
27	T^u	E_B^u	P_B^l	G_B^l	A_B^u
28	T^u	E_B^u	P_B^l	G_B^u	A_B^u
29	T^u	E_B^l	P_B^u	G_B^l	A_B^u
30	T^u	E_B^l	P_B^u	G_B^u	A_B^u
31	T^u	E_B^u	P_B^u	G_B^l	A_B^u
32	T^u	E_B^u	P_B^u	G_B^u	A_B^u
33-Nom	T^l	0	0	0	0
34-SS		SUM OF SQUARED			