

An Efficient Input Mobility Mapping Computational Method

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ABSTRACT

The input mobility is a crucial structural parameter regarding vibro-acoustic design of industrial objects. Whatever the frequency range, the vibrational power input into a structure -and consequently the average structural-acoustic response- is governed by the input mobility. When packaging structure-borne noise sources, the knowledge of the input mobility at the source connection points is mandatory for noise control. The input mobility is classically computed at the required points as a specific Frequency Response Function (FRF). During an industrial design process, the choice of connection points requires an a priori knowledge of the input mobility at every possible location of the studied structure-borne source, i.e. a mapping of the input mobility. The classical FRF computation at every Degree Of Freedom (DOF) of the considered structure would lead to consider millions of load cases which is beyond current computational limits.

This paper presents how to efficiently compute the full map of band-averaged input mobility over a Finite Elements mesh. The proposed method is based upon a modal decomposition of the structural response and analytical frequency integration; consequently, it only requires the modal basis as it is currently computed. The frequency band average allows optimizing the storage size. Since the input mobility is a real quantity, associated to each DOF of a structure, modal display tools can be used without modification.

Thus, a meaningful map is provided, allowing efficient structural dynamics analysis over a broad frequency range.

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INTRODUCTION

Structural input mobility is a central concept regarding structure-borne noise [1, 2, 3], mostly in a mid-high frequency approach. Nevertheless, it can also advantageously be used at low frequencies. Whatever the frequency range, the vibrational power input into a structure -and consequently the average structural-acoustic response- is governed by its input mobility. It was shown in [4] that -above a given frequency- the response in an observation area, distant from the source area, only depends on the input power in the source area and not on the manner dynamic forces act on the structure. As a consequence, using input mobilities to control the vibro-acoustic response doesn't require a full vibro-acoustic modeling of the studied system, which allows early design stage applications.

Input mobility is especially important for heterogeneous structures such as automotive structures, where local curvature and 3D assemblies may drastically change the steel panels' stiffness, introducing a frequency dependent behavior. At low frequencies, the car body may show a beam framework behavior with low mobility and high spatial variations. At high frequencies (typically higher than 2 kHz), the input mobility tends to the single metal sheet mobility that is some orders of magnitudes higher and rather constant spatially since it only depends on the material and its thickness. In this case,

the structural behavior becomes similar to a thin flexural flat plate that is known to be uniform in space and frequency, requiring no specific modelling effort.

Source behavior also widely changes from low to high frequencies: forces are generally highly correlated at low frequencies and tend to be uncorrelated at high frequencies. This is mostly a consequence of the phase shifting (relatively to the source extend) within the observed frequency band that tends to increase when enlarging the bandwidth as in $1/n^{\text{th}}$ octave bands analysis. Low frequencies issues generally require the knowledge of the full mobility matrix between the source's multiple connection DOFs. On the other hand, high frequency problems only consider single DOF excitations.

At first order, whatever the frequency range of interest, the knowledge of the input mobility at any tentative source packaging location is mandatory for structure-borne noise control. The input mobility is classically computed at the required points as a specific FRF. During an industrial design process, the choice of connection points requires an a priori knowledge of the input mobility at every possible location of the studied structure-borne source, i.e. a mapping of the input mobility over the entire analyzed structure. The classical FRF computation at every DOF of the structure would lead to consider millions of load cases which is far beyond current computational limits.

The methodology presented here is an efficient way to tackle this limitation when using large finite element models. Based on a modal approach of the structural response, it allows a fast computation of the input mobility at every structural DOF. After presenting the theoretical developments, some applicative results are shown for a trimmed body model of a passenger car.

MODAL IMPLICATIONS ON MOBILITY

Structural Dynamic Response Modal Computation

Using a finite element approach of a structural dynamic problem ([5, 6, 7]) described by a mass matrix $\mathbf{M}$ and a stiffness matrix $\mathbf{K}$, and assuming a proportional damping, a dynamic displacement vector over a set of observation DOFs, $\mathbf{X}$, may be written using generalized coordinate as:

$$\mathbf{X} = \sum_{i=1,M} \alpha_i \Phi_i, \quad (1)$$

where Φ_i is the i^{th} mass normalized modal shape, such that, $\Phi_i^T \mathbf{M} \Phi_i = 1$, and $\alpha_i = \frac{\Phi_i^T \mathbf{F}}{\omega_i^2 - \omega^2}$ is the associated modal amplitude, with $\hat{\omega}_i^2 = 4\pi^2 f_i^2 (1 + j\eta_i)$ where f_i is the i^{th} modal angular frequency such that $\Phi_i^T \mathbf{K} \Phi_i = 4\pi^2 f_i^2$ and η_i is a modal damping factor, and $\mathbf{F}$ is a vector of applied forces on a set of excitation DOFs.

The mobility matrix, $\mathbf{Y}$, is defined as the square matrix obtained when observation nodes are also the excitation nodes:

$$\mathbf{V} = j\omega \mathbf{X} = \mathbf{Y} \mathbf{F} \quad (2)$$

The mobility is a square symmetric complex matrix, of which real part is a definite positive matrix (See [4], [8])

Structural Power Input

The input power provided by a deterministic load $\mathbf{F}$, may be written in a matrix form as:

$$\Pi = \text{Re}\{\mathbf{V}^* \mathbf{F}\} = \text{Re}\{\mathbf{F}^* \mathbf{Y}^* \mathbf{F}\} \quad (3)$$

Above a given frequency, it can be assumed that the vibratory response of the structure is proportional to the input power whatever the manner it is introduced. (See Ref [4])

The input power may be expressed using an eigenvalue decomposition of $\text{Re}\{\mathbf{Y}\}$ (See Ref [4]) such that

$$\text{Re}\{\mathbf{Y}\} = \mathbf{u} \mathbf{u}^T = \sum_j y_j \mathbf{u}_j \mathbf{u}_j^T \quad (4)$$

where y_j is named a principal mobility generally sorted in descending order, $\mathbf{u}_j$ is the associated direction describing the principal mobility pattern. The power input may then be written:

$$\Pi = \sum_j y_j f_j^2 \quad (5)$$

where $f_j = \mathbf{u}_j^T \mathbf{F}$ is the projection of the force vector over the i^{th} principal mobility vector.

It is interesting to notice that this decomposition provides uncorrelated loads effects: the power appears as a sum of positive values involving the square of the projected force. At low frequency, i.e. when modal density is low, the decomposition exhibits few principal excitation patterns that fully control the power input for a given set of excitation nodes. When increasing frequencies, those patterns tend to locate around single points. For high frequencies, the decomposition of $\text{Re}\{\mathbf{Y}\}$ tends to a sum over single DOFs power input.

Although, the physical concept of a single DOF power input is only valid for high frequencies when considering distributed uncorrelated loads, it always makes sense for loads associated to a single DOF, such as the loads considered when computing structural FRFs.

Single DOF Input Mobility

The structural single DOF input mobility, Y_k , is the velocity response of a given degree of freedom, k -denoting either one translation or one rotation of any node of the considered mesh-, to a unit force applied to the same degree of freedom. Using equations (1) and (2), one gets:

$$Y_k = j\omega \sum_{i=1,M} \frac{\Phi_{ik}^2}{\omega_i^2 - \omega^2} \quad (6)$$

Band Averaged Single DOF Power Input

There are many reasons to consider band averaged quantities. First of all, reality can only be observed over a finite time period, limiting the analysis bandwidth. Broad band analysis often allows further physical simplifications steps and reduces the amount of information required to control the system's response. It also appears to be more robust to systems uncertainties. Then, let us now consider averaged dynamic quantities over a frequency band, $B = [f_1, f_2]$. Depending on the application, the analysis can be conducted using constant bandwidth bands or 1/nth octave bands. In some cases, a single wide range analysis may also be relevant.

From equations (3) and (6), one deduces the input power at a single DOF integrated over the frequency band B :

$$\langle \Pi_k \rangle_B = \int_{f_1}^{f_2} \text{Re}\{F^* Y_k F\} df = \sum_{i=1,M} \int_{f_1}^{f_2} |\Phi_{ik}^T F|^2 \text{Re}\left\{\frac{j\omega}{\omega_i^2 - \omega^2}\right\} df \quad (7)$$

Let us introduce the force spectrum associated to the band B , computed from the force spectral density:

$$\langle F^2 \rangle_B = \int_{f_1}^{f_2} |F|^2 df$$