

Torque Ripple Description and Its Suppression through Flux Linkage Reconstruction

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ABSTRACT

Description of PMSM torque in high accuracy is critical and previous work for its further research. However, the traditional linear torque model fails to describe its non-ideal characteristics of practical working. This paper presents a generalized torque model of PMSM based on flux linkage reconstruction. In synchronous rotating space coordinates, flux linkage were reconstructed through Fourier series expansion and bivariate polynomial. Based on this model, a precise PMSM torque ripple description and corresponding suppression method were developed. Current feed-forward compensation and the rotor field oriented control were applied in torque ripple suppression. Simulation and experimental results both show that the model not only accurately describes the nonlinear variation of PMSM torque in different working conditions, but also can be used to suppress PMSM torque ripple effectively.

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INTRODUCTION

As a multi-variable, strong coupling and strong nonlinear mechanical and electrical system, Permanent Magnet Synchronous Motor (PMSM) is affected by high saturation of iron core, harmonic wave of air-gap field and the stator slotting. The traditional PMSM linear model cannot reflect these non-ideal factors [1, 2]. Thereby, it is pivotal to build an accurate PMSM model for torque ripple suppression and PMSM control.

In the past, several methods of torque modeling have been proposed. In [3], a torque model was deduced through using bivariate polynomial to reconstruct co-energy, but it did not reflect the periodic variation of torque with angle. In [4], based on the idea of magnetic circuit, the torque model was deduced by air-gap field distribution, but its precondition is that the magnetic conductivity of iron must be assumed infinite. In [5, 6], nonlinear time-varying equivalent magnetic circuit method was proposed. It could calculate the torque variation with respect to the rotor angle. In [7], a method of building PMSM model based on M-function was proposed, in which both direct axis and quadrature axis (d-q axis) inductances and permanent magnet flux linkage were obtained via FEA. The equation of voltage and torque with respect to working current were given. Although it considered the influence of the saturation along with motor and rotor magnet field harmonic, the given modeling was a look-up table based on M-function, not an analytical model.

In this paper, it presents a generalized torque model of PMSM based on flux linkage reconstruction, which considers the high saturation of iron core, harmonic wave of air-gap field and the stator slotting. In

synchronous rotating space coordinates, flux linkage are reconstructed through Fourier series expansion and bivariate polynomial. The accuracy of torque model is verified by simulation and experiments. Furthermore, based on this model, methods of PMSM torque ripple suppression are explored and verified through simulation and experiment.

GENERALIZED FLUX LINKAGE MODEL

Flux Linkage Reconstruction

Figure.1 shows synchronous rotating coordinate and stationary coordinate used in this paper. Two kinds of generalized coordinates are introduced:

$$Z_{dq} = [i_d, i_q, \theta_r]; Z_{abc} = [i_a, i_b, i_c, \theta_r]$$

The transformation of Z_{abc} and Z_{dq} can be realized through Clark and Park transformation.

The steady point of the analysis begin with numerical FEA of the given PMSM. Using PMSM finite element model, the numerical results of flux linkage in different steady working points (i_d, i_q) have been obtained. In addition, three-phase flux linkage are transformed into d-q axis flux linkage (ψ_d, ψ_q). At every steady working points, ψ_d and ψ_q change with respect to electric angle θ_r . Therefore, Fourier series decomposition is suitable for analytical description of d-q axis flux linkage.

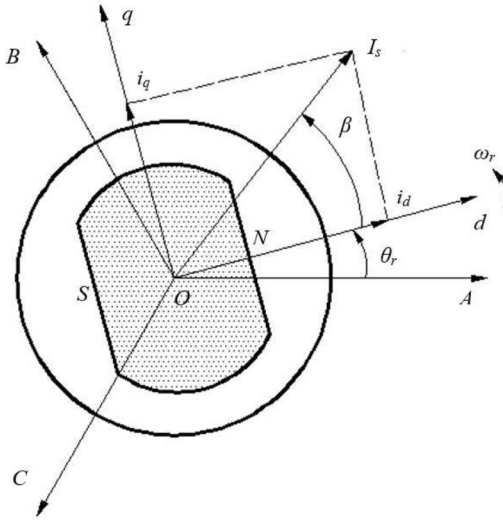


Figure 1. space reference coordinate

Many papers [8, 9, 10] have proved that the flux linkage in the synchronous rotating coordinate ψ_d and ψ_q only include the $6k$ -th ($k = 1, 2, 3, \dots$) harmonic components when excited with three-phase symmetry currents. Using N order Fourier series expansion, the d-axis flux linkage ψ_d^m in the m -th working points (i_d^m, i_q^m) can be represented as:

$$\psi_d^m(\theta_r) \approx \frac{a_0^m}{2} + \sum_{n=1}^N (a_n^m \cos 6n\theta_r + b_n^m \sin 6n\theta_r) \quad (1)$$

Define coefficients:

$$a_n^m = \frac{2}{T} \int_{-T/2}^{T/2} \psi_d^m(\theta_r) \cos 6n\theta_r d\theta_r$$

$$b_n^m = \frac{2}{T} \int_{-T/2}^{T/2} \psi_d^m(\theta_r) \sin 6n\theta_r d\theta_r$$

Where, ($n=1, 2, 3, \dots$), Fourier coefficients a_n^m and b_n^m mean the cosine component and sinusoidal component of d-axis flux linkage in the m -th working points, and $T = 2\pi$. According to the Eq. (1), the d-axis flux linkage only have the $6k$ -th harmonic components except the DC component.

Different working points (i_d, i_q) have different d-axis flux linkage ψ_d^m and corresponding Fourier coefficients a_n^m and b_n^m . Using M order polynomial, these Fourier coefficients can be described as function of working currents as follows:

$$a_k = f_A^k(i_d, i_q) = \alpha_k \cdot X(i_d, i_q) \quad (2)$$

$$b_k = f_B^k(i_d, i_q) = \beta_k \cdot X(i_d, i_q) \quad (3)$$

Where,

$$\alpha_k = [\alpha_0^k, \alpha_1^k, \dots, \alpha_{(M+2)(M+1)/2-1}^k]$$

$$\beta_k = [\beta_0^k, \beta_1^k, \dots, \beta_{(M+2)(M+1)/2-1}^k]$$

: polynomial coefficients of the k -th Fourier coefficients.

$$X(i_d, i_q) = [1, i_d, i_q, \dots, i_d^M, i_d^{M-1}i_q, \dots, i_d^{M-1}i_q^{M-1}, i_q^M]^T$$

: polynomial vector of working current.

By the means of Fourier series expansion and bivariate polynomial, d-axis flux linkage ψ_d^m can be represented with respect to d-q axis currents i_d, i_q and electric angle θ_r as follows:

$$\bar{\psi}_d(i_d, i_q, \theta_r) = \begin{bmatrix} C(\theta_r) & S(\theta_r) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} X(i_d, i_q) \quad (4)$$

Where,

$$C(\theta_r) = [\cos 0\theta_r, \dots, \cos 6k\theta_r, \dots, \cos 6N\theta_r]$$

$$S(\theta_r) = [\sin 0\theta_r, \dots, \sin 6k\theta_r, \dots, \sin 6N\theta_r]$$

: The vectors of cosine and sinusoidal component, where dimension is $(N+1)$.

$$A = [\alpha_0, \dots, \alpha_k, \dots, \alpha_N]^T; B = [\beta_0, \dots, \beta_k, \dots, \beta_N]^T$$

: The constant matrices, where dimension is $(N+1)(M+1)(M+2)/2$.

The accuracy and complexity of the generalized flux linkage model are related to the size of those above matrices. So it should be chosen appropriately.

In the same way, q-axis flux linkage can also be analytically expressed with respect to d-q axis currents i_d, i_q and electric angle θ_r . The reconstructed flux linkage $\bar{\psi}_d, \bar{\psi}_q$ are shown as follows:

$$\begin{cases} \bar{\psi}_d(i_d, i_q, \theta_r) = \begin{bmatrix} C(\theta_r) & S(\theta_r) \end{bmatrix} \begin{bmatrix} A_d \\ B_d \end{bmatrix} X(i_d, i_q) \\ \bar{\psi}_q(i_d, i_q, \theta_r) = \begin{bmatrix} C(\theta_r) & S(\theta_r) \end{bmatrix} \begin{bmatrix} A_q \\ B_q \end{bmatrix} X(i_d, i_q) \end{cases} \quad (5)$$

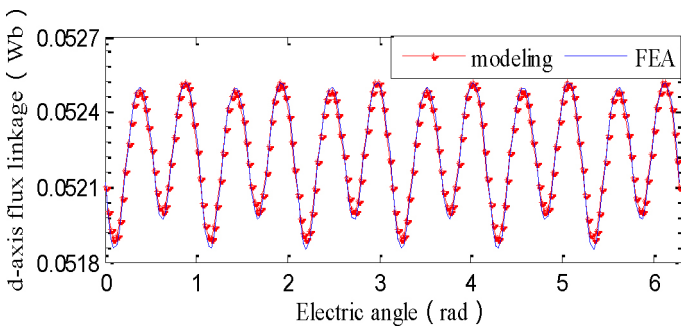
Where, A_d, B_d, A_q and B_q are constant matrices.

Flux Linkage Comparison with FEA

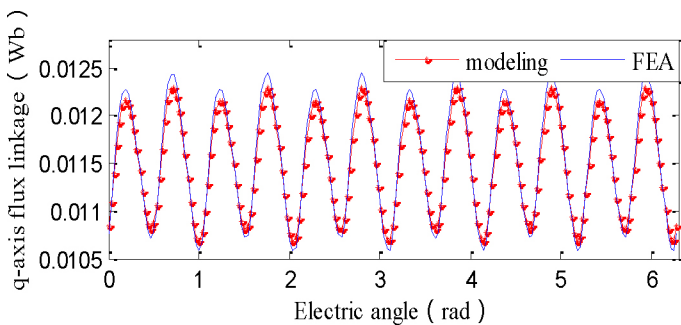
Considering the practical working condition of PMSM, we set amplitude of phase current range from 0A to 300 A, interval of 25A. Meanwhile, we set torque angle range from 60 degree to 165 degree, interval of 15 degree.

As for this generalized flux linkage model of PMSM, considering the accuracy and complexity of model, the number of Fourier series equals 6 (N=6) and the number of polynomial equals 5 (M=5). As a result, the dimensions of constant matrices A_d , B_d , A_q and B_q are 7×21 .

The reconstructed flux linkage is given by Eq. (5). Figure.2 and Figure.3 show the comparison of d-q axis flux linkage between reconstruction and FEA at working points (I_s, β) : (50A, 105°), (250A, 120°).

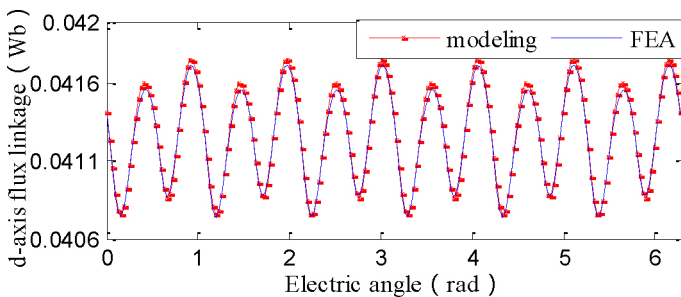


a.

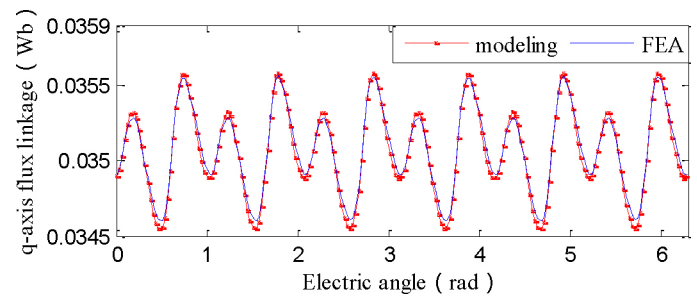


b.

Figure 2. Comparison of flux linkage in (50A, 105°). Figure.2 (a): Flux linkage of d-axis. Figure.2 (b): Flux linkage of q-axis



a.



b.

Figure 3. (cont.) Comparison of flux linkage in (250A, 120°). Figure.3 (a): Flux linkage of d-axis. Figure.3 (b): Flux linkage of q-axis

According to the comparison of results, the amplitudes and phases of d-q axis flux linkage are almost identical with FEA. The d-q axis flux linkage only include the 6k-th harmonic components. There is a subtle difference in the peaks and troughs of wave due to the order of polynomial. Furthermore, d-axis flux linkage is more accurate than q-axis flux linkage. The accuracy is decided by the order of Fourier series and polynomial. In the range of permissible error, the variation of flux linkage with respect to electric angle can be accurately described by the method of flux linkage reconstruction.

GENERALIZED TORQUE MODEL

Torque Equation Derivation

According to the theory of electric machinery, torque equation can be deduced. [1] has proposed that co-energy changes with the stator current and rotor position in PMSM. When co-energy is changed due to tiny angular variation, PMSM will generate torque. That is to say, torque equation can be derived by co-energy and virtual displacement method from electromechanical energy conversion.

According to the definition of co-energy calculation, keeping angle constant, co-energy is obtained by the integral of flux linkage with respect to currents. The co-energy expression in d-q frame is represented as follows:

$$\begin{aligned}
 W_c^{dq}(i_d, i_q, \theta_r) &= \int_0^{i_d} \psi_d(i_d', 0, \theta_r) di_d' + \int_0^{i_q} \psi_q(i_d, i_q', \theta_r) di_q' \\
 &= [C(\theta_r) \quad S(\theta_r)] \left(\begin{bmatrix} A_d \\ B_d \end{bmatrix} X_1(i_d, 0) + \begin{bmatrix} A_q \\ B_q \end{bmatrix} X_2(i_d, i_q) \right)
 \end{aligned} \quad (6)$$

Where,

$$\begin{aligned}
 X_1(i_d, 0) &= \int_0^{i_d} X(i_d', 0) di_d'; \\
 X_2(i_d, i_q) &= \int_0^{i_q} X(i_d, i_q') di_q'
 \end{aligned}$$

: the integral of the polynomial vector on d-q axis current.

Figure 3.