

Acoustic Analysis of a Compact Muffler for Automotive Application

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ABSTRACT

A production muffler of a 2.2 liter compression ignition engine is analyzed using plane wave (Transfer Matrix) method. The objective is to show the usefulness of plane wave models to analyze the acoustic performance (Transmission Loss, TL) of a compact hybrid muffler (made up of reactive and dissipative elements). The muffler consists of three chambers, two of which are acoustically short in the axial direction. The chambers are separated by an impervious baffle on the upstream side and a perforated plate on the downstream side. The first chamber is a Concentric Tube Resonator (CTR). The second chamber consists of an extended inlet and a flow reversal 180-degree curved outlet duct. The acoustic cavity in the third chamber is coupled with the second chamber through the acoustic impedances of the end plate and the perforated plate. The effect of manufacturing defects in the form of leakages and weep holes (to reduce internal corrosion) which are commonly found in automotive mufflers is investigated by modelling it as perforations. The back pressure of the muffler is predicted by using the friction factor for perforated pipes in the flow resistance network of the muffler. The TL of the muffler is measured without mean flow inside a semi-anechoic room by using the two load method. The back pressure is measured on board the vehicle at different mass flow rates and temperatures. The predicted and experimental results are found to be in reasonable agreement. The effect of leaks, weep holes and plastic cover of glass wool are found to be significant.

CITATION: Neihguk, D., Munjal, M., Ram, A., and Prasad, A., "Acoustic Analysis of a Compact Muffler for Automotive Application," *SAE Int. J. Veh. Dyn., Stab., and NVH* 1(2):2017, doi:10.4271/2017-01-9550.

INTRODUCTION

The contribution of exhaust noise to the overall vehicle noise level is significant especially in the first few harmonics of the engine firing frequency. Coincidentally, the plane wave assumption is sufficiently valid in this frequency range provided the muffler dimensions are such that the cut off frequency is higher than the engine firing frequency of interest. Consequently, the well-known transfer matrix technique based on plane wave theory is adequate to initiate the design of a good muffler, if not to finalize the design. Ideally, this should be the first step of any muffler design program as transfer matrices of most of the acoustic elements are readily available [1, 2, 3]. However, muffler designers are more inclined towards the use of the finite element or boundary element methods right from the start of the design. This is not a cost effective approach because the turnaround time for design iterations is much higher compared to the plane wave approach, and often designers do not comprehend the significance of various design changes that are taken up to meet the desired performance targets. As a result, cost-effective and on-time solutions are delayed, hampering project timelines.

Herrin et al. [4] demonstrated the proper use of plane wave models for muffler design with two practical examples of industrial mufflers that have complicated three-dimensional geometries and multiple flow paths. They rightly argued that the use of plane wave method served the purpose of better understanding and helps the designer to develop intuition about the performance of the mufflers. The effect of adding or deleting acoustic elements can be quickly assessed in contrast to the finite element or boundary element method where any design changes necessarily call for a modified CAD geometry and a re-mesh to assess the effect of the changes. They also suggest some useful tips for modelling of mufflers using plane waves. However, they did not present the application of the same method to hybrid mufflers (made up of reactive as well as dissipative elements) as well as mufflers with curved ducts. Besides, the equally important back pressure performance was not discussed.

The complex mufflers with the perforated baffles as well as intermediate tubes connecting the consecutive chambers of the muffler can best be analyzed by means of the Integrated Transfer Matrix approach [5]. Recently, Verma and Munjal [6] successfully implemented this approach for a multiply connected three chamber

perforated baffle hybrid muffler configurations with and without a curved duct. They conducted a comprehensive parametric study and highlighted the importance of perforated baffles and absorptive materials for good acoustic performance. Incidentally, the muffler analyzed by them was acoustically long having a length to diameter ratio of 2.3 with offset inlet and outlet, and circular in shape.

The muffler investigated here is elliptical in shape with eccentricity of 0.6 and overall length to diameter ratio of 0.67. The degree of compactness is a geometrical constraint imposed by the packaging requirements in the underbody of the vehicle. Owing to the presence of an impervious baffle in the upstream and a perforated plate in the downstream side, which is backed by an acoustic cavity, the flow paths are linearly connected (each cross section of the muffler is characterized by two acoustic state variables, i.e. the acoustic pressure and the acoustic mass velocity). Therefore, a simple Transfer Matrix Method (TMM) [1] is ideally suited in place of the Integrated Transfer Matrix Method (ITMM) [5] used by Verma and Munjal [6]. The back pressure is predicted by using the friction factor for perforated duct [7]. The effect of glass wool is analyzed by comparing the acoustic performance of the hybrid and reactive configurations of the muffler.

MUFFLER GEOMETRY

The muffler geometry is shown in Figs. 1 and 2 where the first chamber is a Concentric Tube Resonator (CTR) and the second chamber is a combination of uniform pipes, extended inlet, and reversal end chamber with perforated end plate. The third chamber is coupled with the second chamber only through the impedance of the perforated plate and the rigid end plate. A pipe originating from the second chamber takes a 180 degree turn in the first chamber and passes through the second and third chamber to form an outlet pipe. In a nutshell, the compact muffler is an assembly of selected acoustic elements. In the hybrid configuration, the first and third chambers are filled with Silentex® glasswool with filling densities of 45 g/l and 200 g/l, respectively. The dimensions of the muffler are given in Table A1 of Appendix A.

1D Transfer Matrix Analysis with No Leaks

The compact muffler is notionally divided into 10 segments while retaining the benefit of their acoustic isolation between the annular chamber and the inner pipes except for the first chamber, as shown in Figs. 1 and 2. As a result, the acoustic field at each section is fully described by two state-variables, namely, the acoustic pressure (p) and the acoustic mass velocity (v). At the outlet pipe junction of section 6-7, the incoming acoustic mass velocity (v_{6a}) is divided into two components ($v_{4,7}$) and ($v_{3,6}$). (Here, the first subscript denotes the domain marked in circle in Figs. 1 and 2, and the second subscript denotes the section number). This is due to the presence of a reversal chamber and an outlet pipe downstream. Consequently, three 2×2 resultant transfer matrices $[P]$, $[Q]$ and $[R]$ are obtained which relate the state-variables of section 1 to section 6a, section 7 to section 10 of domain 4, and section 6 to section 10 of domain 3, respectively.

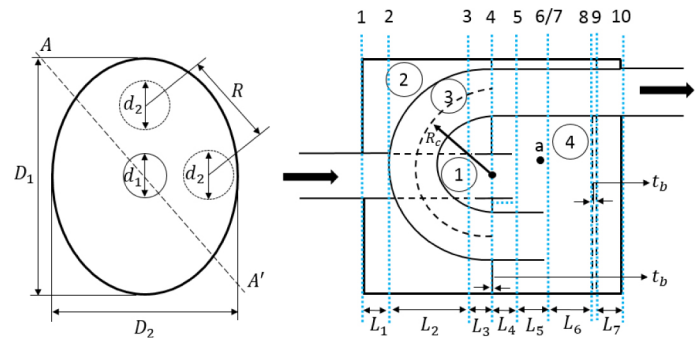


Figure 1. Front view and side view of Reactive muffler configuration with no Leaks.

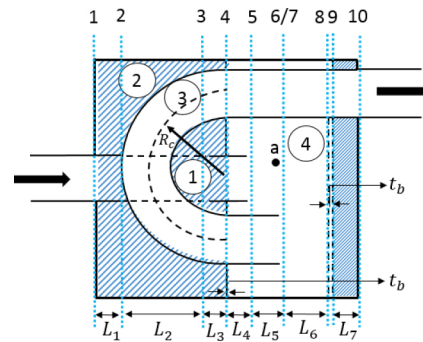


Figure 2. Hybrid muffler configuration with no Leaks.

The resultant transfer matrix $[P]$ of Eqns. (1-2) below is the product of five transfer matrices of three acoustic element types, namely, CTR [1] from section 1 to 4, extended inlet [1] from section 4 to 5 and annular pipe [1] section 5 to 6.

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = [P] \begin{bmatrix} p_{6a} \\ v_{6a} \end{bmatrix} \quad (1)-(2)$$

where $[P] = [T_{CTR}]_{1-4} [T_{Extended\ inlet}]_{4-5} [T_{pipe}]_{5-6}$

The state-variables in domain 3 and domain 4 are related to each other through section 6-7 by means of the mass conservation and momentum equations as given in Eqns. (3-5):

$$p_{6a} = p_{4,7} = p_{3,6} \quad (3)-(4)$$

$$v_{6a} = v_{4,7} + v_{3,6} \quad (5)$$

The end plate of the compact muffler located at section 10 is assumed to be rigid. Therefore, the acoustic mass velocity vanishes as given by Eqn. (6):

$$v_{4,10} = 0 \quad (6)$$

Similarly, the resultant transfer matrix $[Q]$ of Eqns. (7-8) below is the product of four transfer matrices of three acoustic element types, namely, uniform pipes from section 7 to 8 and section 9 to 10 of domain 4 with lengths L_6 and L_7 , respectively, perforated plate

transfer matrix $[T_{\text{perforated plate}}]$ of thickness t_b from section 8 to 9 given by Eqn. (9), and an interface transfer matrix $[T_{\text{int}}]$ given by Eqn. (10) which ensures the continuity of acoustic mass velocity at the interface of the air-absorptive media.

$$\begin{bmatrix} p_{4,7} \\ v_{4,7} \end{bmatrix} = [Q] \begin{bmatrix} p_{4,10} \\ v_{4,10} \end{bmatrix} \quad (7)-(8)$$

where $[Q] = [T_{\text{pipe}}]_{4,7-8} [T_{\text{perforated plate}}]_{4,8-9} [T_{\text{int}}] [T_{\text{pipe}}]_{4,9-10}$

$$[T_{\text{perforated plate}}] = \begin{bmatrix} 1 & Y\zeta \\ 0 & 1 \end{bmatrix} \quad (9)$$

$$[T_{\text{int}}] = \begin{bmatrix} 1 & 0 \\ 0 & \rho/\rho_w \end{bmatrix} \quad (10)$$

Here Y is the characteristic impedance of the perforated plate and ζ is the non-dimensional acoustic impedance of the perforated plate. ρ and ρ_w are the densities of air without and with absorptive medium. It may be noted that, the interface transfer matrix of Eqn. (10) would be reduced to an identity matrix for the reactive muffler configuration as given by Eqn. (10a)

$$[T_{\text{int}}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (10a)$$

Finally, the resultant transfer matrix $[R]$ of Eqns. (11-12) below relating the state-variables at section 6 to section 10 of domain 3 is the transfer matrix of a uniform pipe with diameter d_2 and equivalent length, $l_{\text{outlet}} = 2(L_4 + L_5) + \pi R_c + L_6 + t_b + L_7$ is given by

$$\begin{bmatrix} p_{3,6} \\ v_{3,6} \end{bmatrix} = [R] \begin{bmatrix} p_{3,10} \\ v_{3,10} \end{bmatrix} \quad (11)-(12)$$

where $[R] = [T_{\text{pipe}}]_{3,6-10}$

It may be noted that the effect of curved duct inside the annular cavity of the first chamber constituting the outlet pipe has been neglected in the 1D TMM by considering it as an equivalent straight pipe. Thus, we have a set of 10 equations and 12 variables, which is solved by using Gaussian elimination method in Matlab®. Hence, the desired transfer matrix relating the inlet and outlet state-variables can be written in the matrix form as given in Eqn. (B1) of Appendix B.

The required transfer matrix relating the inlet acoustic-state variables (p_1, v_1) and outlet acoustic-state variables ($p_{3,10}, v_{3,10}$) can be extracted from Eqn. (B1) as

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{3,10} \\ v_{3,10} \end{bmatrix} \quad (13)$$

where $[T]$ is the transfer matrix relating the upstream state variables to the downstream variables, obtained by premultiplying both sides of the matrix equation (B1) with the inverse of the coefficient matrix on the left hand side of the equation. Finally, the Transmission Loss can be calculated as follows [1]:

$$TL = 20 \log_{10} \left[\left(\frac{Y_3}{Y_1} \right)^{\frac{1}{2}} \frac{1}{2} \left| T_{11} + \frac{T_{12}}{Y_3} + T_{21} Y_1 + T_{22} \frac{Y_1}{Y_3} \right| \right] \quad (14)$$

where Y_1 and Y_3 are the characteristic impedance of the inlet pipe (pipe 1) and outlet pipe (pipe 3), respectively.

1D Transfer Matrix Analysis with Leaks

The presence of leaks and weep holes make the muffler multiply connected. Therefore, it necessitates the use of the ITMM [5] that relates the acoustic-state variables across the entire cross section of the muffler shell according to the direction of acoustic wave propagation. The robustness and flexibility of this approach in allowing the modeling of acoustic coupling between the muffler chambers, inner perforated pipes and annular regions enables the evaluation of the four-pole parameters of the muffler which in turn is used to calculate the Transmission Loss. In this study, the leaks between the inner pipes and flanges and also the weep holes are assumed as perforations in the baffle plate. As a result, the segmentation of the muffler yields 11 segments as shown in Figs. 3 and 4.

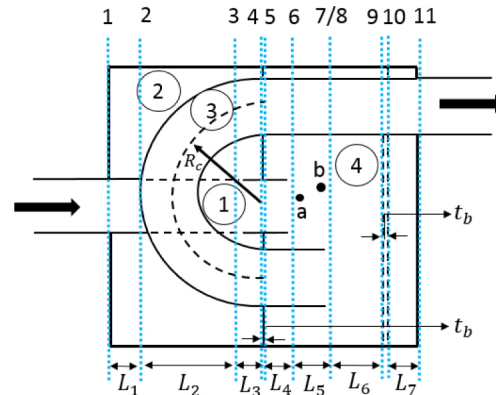


Figure 3. Reactive muffler configuration with Leaks.

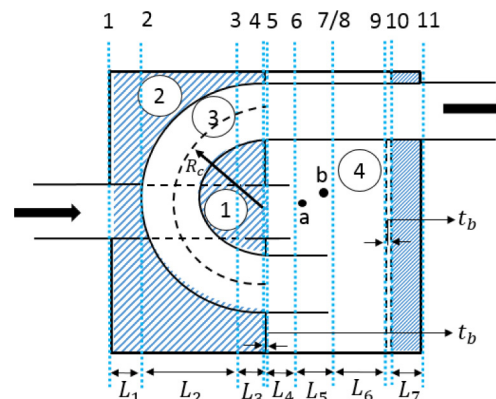


Figure 4. Hybrid muffler configuration with Leaks.

The corresponding relationships leading to two 4 x 4 transfer matrices $[P]$, $[Q]$ and two 2 x 2 transfer matrices $[R]$, $[S]$ are as follows:

$$\begin{bmatrix} p_{1,1} \\ v_{1,1} \\ p_{2,1} \\ v_{2,1} \end{bmatrix} = [P] \begin{bmatrix} p_{1,6} \\ v_{1,6} \\ p_{2,6} \\ v_{2,6} \end{bmatrix} \quad (15-18)$$

where $[P] = [T_{pipe}]_{1-4} [T_{perforated\ plate}]_{4-5} [T_{int}] [T_{pipe}]_{5-6}$

$$\begin{bmatrix} p_{3,8} \\ v_{3,8} \\ p_{4,8} \\ v_{4,8} \end{bmatrix} = [Q] \begin{bmatrix} p_{3,11} \\ v_{3,11} \\ p_{4,11} \\ v_{4,11} \end{bmatrix} \quad (19-22)$$

where $[Q] = [T_{pipe}]_{8-9} [T_{perforated\ plate}]_{9-10} [T_{int}] [T_{pipe}]_{10-11}$

$$\begin{bmatrix} p_{3,7} \\ v_{3,7} \end{bmatrix} = [R] \begin{bmatrix} p_{3,8} \\ v_{3,8} \end{bmatrix} \quad (23-24)$$

where $[R] = [T_{pipe}]_{3,7-8}$

$$\begin{bmatrix} p_a \\ v_a \end{bmatrix} = [S] \begin{bmatrix} p_b \\ v_b \end{bmatrix} \quad (25-26)$$

where $[S] = [T_{pipe}]_{6-7}$

The continuity of mass flux and conservation of momentum at sections 6, 7 and 8 can be written as

$$p_a = p_{2,6} = p_{1,6} \quad (27-28)$$

$$v_a = v_{2,6} + v_{1,6} \quad (29)$$

$$p_b = p_{3,7} = p_{4,8} \quad (30)-(31)$$

$$v_b = v_{3,7} + v_{4,8} \quad (32)$$

By assuming the end plates as rigid termination we get a vanishing acoustic mass velocity at both the ends,

$$v_{2,1} = 0 \quad (33)$$

$$v_{4,11} = 0 \quad (34)$$

The leaks in the first baffle are modeled as a perforated baffle plate with a porosity of 0.45 percent and 4 mm holes distributed throughout baffle with a thickness of 1.2 mm. By moving progressively along the direction of wave propagation, the global

transfer matrix of the muffler including leakage can be obtained by assembling Eqns. (15-34) into a 20 x 20 matrix as given in Eqn. (B2) of Appendix B. Finally, the matrix is solved to obtain a 2 x 2 matrix relating the state variables $p_{1,1}$, $v_{1,1}$ to $p_{3,11}$, $v_{3,11}$ by using the Gaussian elimination method and the Transmission Loss is computed from Eqn. (14).

3D FEM Analysis

The muffler is divided into different domains which are then discretized into small finite elements. By using the acoustic pressure as the independent variable, the wave propagation is solved in the frequency domain using the time harmonic pressure acoustic mode in COMSOL [8]. The governing equation is the modified version of the 3D Helmholtz equation:

$$\nabla \cdot \left(-\frac{\nabla p}{\rho} \right) - \frac{\omega^2 p}{\rho c^2} = 0 \quad (35)$$

where ρ , c and ω are the density, speed of sound and angular frequency, respectively. The following boundary conditions are used to simulate the system [8]:

1. At the solid boundaries, which include the outer walls of the muffler, the baffles between the chambers and the walls of the inner pipes, sound hard (wall) boundary conditions are used:

$$\left(-\frac{\nabla p}{\rho} \right) \cdot n = 0 \quad (35a)$$

here n is the unit normal vector.

2. At the inlet boundary, a combination of an incoming and outgoing plane waves is assumed:

$$\left(-\frac{\nabla p}{\rho} \right) \cdot n = \frac{i\omega}{\rho c} p - \frac{2i\omega}{\rho c} p_0 \quad (35b)$$

where $p_0 = 1$ Pa is the applied pressure at entrance to the inlet pipe and i is the imaginary unit.

3. At the outlet boundary, an outgoing plane wave radiation condition with a vanishing reflection coefficient (anechoic termination) for normally incident waves is used:

$$\left(-\frac{\nabla p}{\rho} \right) \cdot n = \frac{i\omega}{\rho c} p \quad (35c)$$

The Transmission Loss (TL) is expressed in terms of the ratio of the incident acoustic power at the inlet, $W_{incident}$ using the applied pressure, p_0 and the transmitted acoustic power at the outlet, $W_{transmitted}$ using the computed pressure, p_c at the outlet as

$$TL = 10 \log_{10} \left(\frac{W_{incident}}{W_{transmitted}} \right) \text{ dB}$$

where

$$W_{incident} = \frac{p_0^2}{2\rho c} \text{ and } W_{transmitted} = \frac{p_c^2}{2\rho c}$$

Here p_0 is the pressure associated with the incident wave, and p_c is the pressure associated with the wave transmitted into an anechoic termination.

Mesh Independence Study

Two mesh sizes with maximum mesh element of one-fifth and one-tenth of the minimum wavelength and a minimum mesh element of 11.2 mm and 5.6 mm are used as given in Table 1. As shown in Fig. 5, it is found that the result is invariant of the two mesh sizes. Consequently, the subsequent analysis is performed using Mesh 1.

Table 1. Mesh parameters.

Layer	Mesh 1	Mesh 2
Maximum element size [m]	$\lambda_{min}/5$	$\lambda_{min}/10$
Minimum element size [m]	$11.2 \times E-03$	$5.6 \times E-03$
Maximum element growth rate	1.5	1.5
Curvature factor	0.6	0.6
Resolution of narrow regions	0.5	0.5

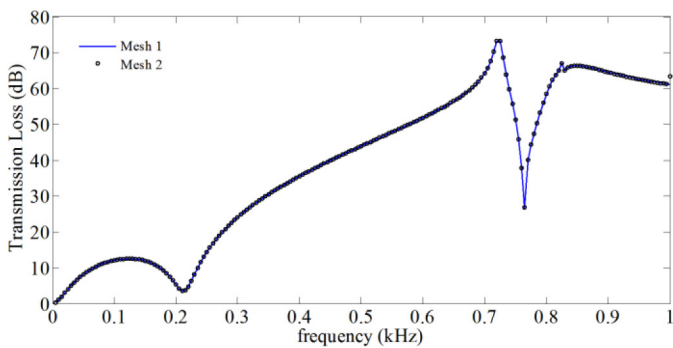


Figure 5. Mesh independent study.

TRANSMISSION LOSS AND BACK PRESSURE MEASUREMENT

The Transmission Loss measurement is carried out inside a semi anechoic room using the two load method as per ASTM E2611-09 at a temperature of 299.4 K where the two loads are achieved by using a rigid termination and anechoic termination. The muffler is fixed between the upstream and downstream impedance tube and the microphone readings are recorded in a four channel data acquisition system. The schematic layout and actual test set up of Transmission Loss measurement is shown in Figs. 6 to 8. The back pressure measurement is carried out on-board the vehicle where the gas flow rate is directly recorded from the Engine Management System (EMS) Electronic Control Unit (ECU) using INCA software. Exhaust gas temperatures and static pressures are recorded using two thermocouples and two pressure probes fitted downstream and upstream of the muffler. The thermocouple works on the Negative Temperature Coefficient (NTC) principle with a range of 0 to 1000°C. The pressure probes are connected to the GE UNIK 5000 transducers for sensing the static pressures. In this way, the exhaust

mass flow rate, exhaust gas temperature and back pressure are recorded simultaneously. In this experiment, an IPETRONIK data logger has been used.

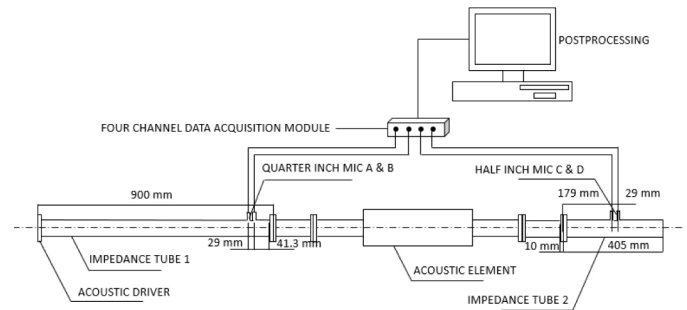


Figure 6. Schematic layout of Transmission Loss measurement set up [reproduced with permission from M/s Sharda Motors Ltd.]



Figure 7. Transmission Loss measurement set up with rigid load termination [reproduced with permission from M/s Sharda Motors Ltd.]



Figure 8. Transmission Loss measurement set up with anechoic termination [reproduced with permission from M/s Sharda Motors Ltd.]

Results

The acoustic impedance of perforated pipe for the reactive muffler configuration of Fig. 1 in stationary medium is modeled by using Elnady et al.'s [9] expression. For the hybrid muffler configuration of Fig. 2 in stationary medium, the Kirby and Cummings' [10] expression has been used. The present study uses the texturized fiber glass roving manufactured by the SilentexTM process and the Delany and Bazley model [11] of complex characteristic impedance and wave number extended to low frequencies by Mechel [12]. The Transmission Loss measurement is conducted at a temperature of 299.4 K using the two-load method as per ASTM E2611-09. The frequency range of interest here is limited to the cutoff frequency below which only the plane waves will be transmitted. For a muffler having major axis D_1 of 0.280 m and speed of sound c_0 of 347 m/s, the cut off frequency is given by [1] Eqn. (37). Therefore, all the TL results are plotted here up to frequency of 1 kHz.

$$f_{co} = \frac{1.84 \times c_0}{D_1 \times \pi} = \frac{1.84 \times 347}{0.280 \times \pi} = 725 \text{ Hz}$$