

A Hybrid Approach to Model the Temperature Effect in Tire Forces and Moments

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ABSTRACT

Tire is an integral part of any vehicle which provides contact between the vehicle and the surface on which it moves. Forces and moments generated at the tire-road interaction imparts stability and control of motion to the vehicle. These forces and moments are functions of many variables such as slip, slip angle, contact pressure, inflation pressure, coefficient of friction, temperature, etc. This paper deals with the effect of temperature on the lateral force, the longitudinal force and the self-aligning moment. The analysis is done at different tire surface temperatures such as 20°C, 40°C, and 60°C. Since the experimental set up with the mounted tire is complex and expensive, we use a hybrid approach in which we take the results from the experiments done by the researchers on a sample piece of tire rubber at various temperatures. Then, we do the steady state analysis in ABAQUS considering the variation of coefficient of friction, slip speed and the elastic modulus of rubber with temperature. The steady state numerical results from ABAQUS at different surface temperatures are compared with the modified PAC2002 tire model to capture the temperature effect. After validating the variations of steady state forces and moments from ABAQUS with the modified PAC2002, we use these steady state tire models to do the transient analysis in order to capture the effect of temperature on the transient response of tire forces and moments for different driving conditions of acceleration, braking and double lane change using MSC ADAMS/CAR.

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INTRODUCTION

A vehicle tire is one of the most vital parts in a vehicle dynamic system. Designing, modeling and fabrication of tires depends on many known as well as uncertain factors as they have to support vehicle weight, provide cushioning effect to surface irregularities, provide sufficient traction for driving and braking, and provide steering control and directional stability. To analyze various effect of tires on the performance of vehicle, most of the tire related effects can be captured in terms of longitudinal force, lateral force, and self-aligning moment. However, accurate computation of the tire forces and moments, and their dependence on various factors such as tire temperature, inflation pressure, contact pressure, the rolling surface, vehicle path, etc., are difficult to predict. In this paper, we focus on the modeling of temperature effect on the tire forces and moment under the acceleration, braking, and lane change conditions of a moving vehicle.

To model the coupled effect of various factors on forces and moments, there exist mainly three classes of modeling techniques, phenomenological/analytical model, empirical model, and numerical models. Initial development of analytical models were mainly based on different simplifications of the tire behavior such as the string based tire models, ring based tire models, and beam-on-elastic foundation based tire models [1,2]. Most of these tire models consider the tread as a pre-stressed string or a ring and the sidewalls as elastic foundations supporting the tread structure. Loo [1] developed an analytical tire

model based on the flexible ring under tension with radially arranged linear springs and dampers to represent the tread band of the tire. The developed model was used to characterize the vertical load displacement of tire and its free rolling resistance. Allen et al. [3] mentioned the importance of coupling the tire model with vehicle operating condition. Rotta [4] employed a circular membrane model to represent the sidewall softening effect to analyze the deformation of a tire on a flat surface. To further improve the flexible ring type tire model, Rhyne and Cron [5] proposed the inclusion of bending and shear deformations. Although, these analytical models (also called as phenomenological models) have proved to be valuable to study the general dynamic behavior of tires, they are difficult to be used as a design tool for the tire encompassing various effects. Consequently, many tire companies specify the tire parameters with respect to an empirical tire model, known as the magic formula. The parameters associated with the magic formula can be found by comparing its value with experimental results. Pacejka [6] and his colleagues have established such magic formulas based on measurement data to predict the longitudinal force, cornering force and aligning moment under different loading conditions.

To capture the tire deformation and its coupling effect with road surface, finite element modeling or FEM, of tire were introduced in early 1970 [7]. As FEM formulation and computer speed improved, more accurate FEM models were obtained. Fiala [8] developed a homogeneous tire-road model using three dimensional finite

elements. Later, the effective use of three dimensional elements was described by Chang et al [9]. To include the surface effect accurately, Faria et al [10] included contact region in the modeling. Pottinger compared stress distribution of solid and pneumatic tires in the contact region. Danielson et al [11] analyzed three dimensional tire models in cylindrical coordinate system to obtain the forces. Luchini and Popio [12] employed the transient dynamic rolling analysis to obtain the time-histories of strains including the rolling effect. To include the thermal effect, Cho et al [13] predicted the rolling resistance and the temperature distribution of a three dimensional periodic patterned tire. Based on the iterative method, they modeled the coupled effect of hysteresis loss associated with viscoelastic rubber element and the temperature. They concluded that the rolling resistances as well as peak temperature rise are strong function of tire tread pattern. Tang et.al [14] developed a finite element approach to compute the temperature field of a steady-state rolling tire and track using 3D thermal analysis software RADTHERM and FEM software ABAQUS under different loading and velocity conditions. They found that the temperature fields changes due to change in rolling as well as loading conditions. However, they did not describe the corresponding effect on the forces and moment of tire. In order to compute tire forces and moments, Korunovic et al [15] developed FEM model to study the variation of rubber friction with slip speed and contact pressure without considering temperature. They also compared the numerical results with experimental results. Bibin et al [16] used FEM and incorporated the variation of rubber friction with slip speed at the contact point by using the experimentally measured value from the literature. They showed the variation of tire forces and moments for a given tire pressure and normal load. However, the influence of temperature on forces and moments were not discussed.

To experimentally analyze the effect of temperature on tire forces and moment, Higgins et al [17] investigated the friction process of styrene butadiene rubber which is a standard tire rubber material. To perform the experiments, they used a model tire rubber compound consisting of 46 % styrene/butadiene. Subsequently, they obtained the variation of elastic modulus of tire material with temperature using Tritex 2000 DMA (Dynamic Mechanical Apparatus) system as shown in Fig. 1. Such variation can be considered directly into the material property of tire during finite element simulation. To experimentally understand the variation of coefficient of friction with temperature, Persson [18] studied the variation of the coefficient of friction of a tire tread rubber block sliding on a hard substrate with temperature due to local heating. Based on his experiments, he described the variation of friction with temperature for a particular slip velocity, and then also explained the variation of coefficient of friction with sliding velocity for a particular temperature $T=18^{\circ}\text{C}$. Assuming that the profile of coefficient of friction versus slip velocity remains similar at all temperatures and using the values of the coefficient of frictions versus temperature at a particular slip velocity, we have extrapolated the variation of coefficient of friction with sliding velocities at temperatures $T=20^{\circ}\text{C}$, 40°C , and 60°C , respectively, as shown in Fig. 2. Angrick et al. [19] performed experiments to study the dependence of the cornering stiffness, relaxation length, and lateral coefficients of friction on either core or surface temperature of tire.

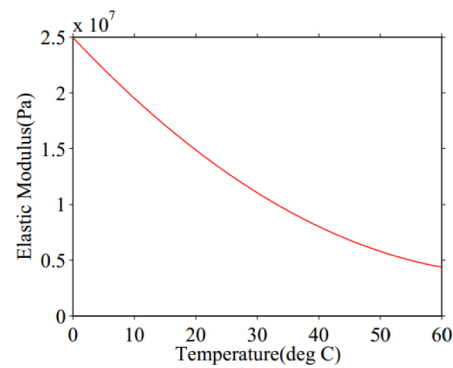


Figure 1. Variation of elastic modulus with temperature [16]

In a nutshell, while the analytical model are limited in capturing various factors associated with the changes in tire material with temperature, etc., the finite element based numerical methods are rarely used to do full vehicle dynamic analysis. As full vehicle dynamics requires multibody analysis of different components of vehicle subjected to any change in operating conditions, the modification of empirical model based on magic formula have been explored. Mizuno et al [19] included temperature effect in magic formula by including additional terms associated with the friction and specific heat and compared the computed lateral force with experiments. Recently, Singh and Shivramakrishnan [20] modified the terms associated with cornering stiffness and peak grip level in magic formula to include the influence of temperature, pressure, and normal load. They also compared the results with experimental values. As there may be many limitations for considering various factors in experiments, the above modification in the magic formula should also be compared with corresponding numerical model using FEM. Therefore, we develop FEM model to include the influence of temperature on tire forces and moment by modifying the elastic properties as well as coefficient of friction from Figs. 1 and 2. Subsequently, we use the modified magic formula given by Mizuno et al [20] and compute the related constants based on numerical results. Finally, to discuss the performance of modified tire model under the influence of temperature on the driving, braking, and lane change conditions, we perform transient analysis in multibody software, ADAMS.

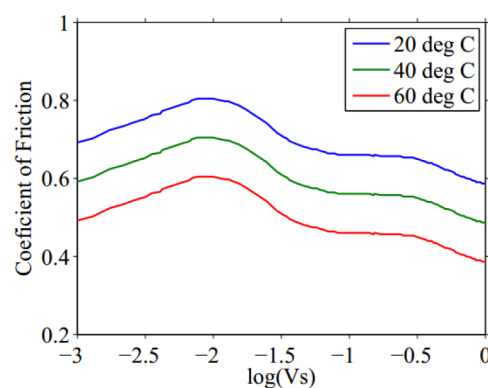


Figure 2. Variation of the coefficient of friction with temperature at different temperatures

THE MAGIC FORMULA PAC2002 TIRE MODEL WITH TEMPERATURE EFFECTS

In this section, we present mathematical details of empirical model, also known as magic formula or PAC2002 model [6], to capture the steady state tire forces and moments under the influence of temperature [20]. The magic formula to find the steady state forces and moments are expressed as [6]

$$Y(X) = y(x) + S_v \quad (1)$$

$$y(x) = D \sin [C \arctan \{Bx - E(Bx - \arctan(Bx))\}] \quad (2)$$

$$x = X + S_h \quad (3)$$

where, D , C , B , E are the main parameters of the magic formula. These parameters are tuned to fit the magic formula curve with the tire measurement data/numerical results. The values of these parameters give an idea about the tire-road interaction surface, properties of tire, etc. For example, the parameter D is the peak factor, which gives an idea of the coefficient of friction at the interaction. C is the shape factor, B is the stiffness factor, E is the curvature factor. The product of parameters BCD , which is the slope of magic formula curve, gives the stiffness of the tire. (S_h) is the vertical shift and (S_v) is the horizontal shift of the curve due to side slip in the horizontal and vertical directions, respectively. We use this formula to compute the longitudinal force (F_x), lateral force (F_y) and self-alignment moment (M_z). Table 1 presents the relationship between dependent variable, $Y(X)$, and independent variable, X , of magic formula to compute the tire forces and moment.

Table 1. Independent and Dependent Variables of Magic Formula.

Independent quantity (X)	Longitudinal slip (k)	slip angle (α)	slip angle (α)
Dependent quantity (Y)	Longitudinal Force (F_x)	Lateral Force (F_y)	Pneumatic trail (t) ($M_z = -t \times F_y$)

To include more variables in magic formula, Pacejka modified the basic form of the magic formula to obtain PAC2002 tire model by defining new parameters which captures the effect of camber angle, etc. [6]. Subsequently, other researchers modified PAC2002 model to include the effect of pressure, temperature, etc. [20, 21]. In this paper to capture the temperature related effect on the longitudinal force, lateral force and cornering moment, we use the modified model proposed by Mizuno et al [20] which was obtained by modifying PAC2002 model to find lateral force. In the subsequent section, we present the details of the modified PAC2002 model and describe the procedure to include temperature effects.

Lateral Force at Pure Slip

Lateral force, F_y , is developed due to the existence of a side slip angle under the cornering condition. The side slip angle or slip angle (α) is defined as the angle between the direction of travel of the tire and the

direction of wheel plane. The temperature effects on lateral force are modeled under the assumptions that the lateral force varies proportional to the tire surface temperature, and its maximum value is dependent on different proportionality constant with respect to the temperature [20]. The expression of lateral force can be written as

$$F_y(\alpha, T) = D_y(T) \sin [C_y \arctan \{B_y(T) \alpha_y - E_y(B_y(T) \alpha_y - \arctan(B_y(T) \alpha_y))\}] + S_{vy} \quad (4)$$

where,

$$\alpha_y = \alpha + S_{Hy} \quad (5)$$

$$C_y = p_{cy1} \cdot \lambda_{cy} \quad (6)$$

$$D_y = \mu_y \cdot F_z \cdot \zeta_2 \quad (7)$$

$$D_y(T) = D_y \cdot \{1 + \frac{\partial \mu}{\partial T} (T - T_m)\} \quad (8)$$

$$\mu_y = (p_{dy1} + p_{dy2} \cdot df_z) \cdot (1 - p_{dy3} \cdot \gamma^2) \lambda_{\mu y} \quad (9)$$

$$E_y = (p_{ey1} + p_{ey2} \cdot df_z) (1 + p_{ey5} \gamma^2 - (p_{ey3} + p_{ey4} \cdot \gamma_y) \cdot \text{sgn}(\alpha_y)) \cdot \lambda_{E_y} \quad (10)$$

$$B_y(T) = \frac{K_y(T)}{c_y \cdot D_y(T)} \quad (11)$$

$$K_y(T) = K_y \cdot \{1 + \frac{\partial c_p}{\partial T} (T - T_m)\} \quad (12)$$

$$K_y = p_{ky1} \cdot F_{z0} \cdot \sin[2 \arctan(\frac{F_z}{p_{ky2} \cdot \lambda_{Fz0} \cdot F_{z0}})] \cdot (1 - p_{ky3} \cdot |\gamma_y|) \cdot \zeta_3 \cdot \lambda_{ky} \cdot \lambda_{Fz0} \quad (13)$$

$$S_{Hy} = (p_{hy1} + p_{hy2} \cdot df_z) \lambda_{Hy} + p_{hy3} \cdot \gamma_y \cdot \zeta_0 + \zeta_4 - 1 \quad (14)$$

$$S_{vy} = F_z \cdot \{ (p_{vy1} + p_{vy2} \cdot df_z) \lambda_{vy} + (p_{vy3} + p_{vy4} \cdot df_z) \cdot \gamma_y \} \cdot \zeta_4 \cdot \lambda_{\mu y} \quad (15)$$

$$\gamma_y = \gamma \cdot \lambda_{\gamma y} \quad (16)$$

where, T_m is the ambient average tire surface temperature, D_y , E_y , K_y , etc., are the parameters of steady state tire lateral force model at T_m . $D_y(T)$, $B_y(T)$ and $K_y(T)$ are the parameter values at any temperature T . The rate of change of the peak parameter $D_y(T)$ and cornering

stiffness parameter K_y (T) with temperature can be correlated with the corresponding change in the coefficient of friction and the specific heat, respectively, as [20]

$$\frac{\partial \mu_y}{\partial T} = \frac{d(D_y(T))/dT}{D_y} \quad (17)$$

$$\frac{\partial c_p}{\partial T} = \frac{d(K_y(T))/dT}{K_y} \quad (18)$$

where, $d(D_y(T))/dT$ is equal to the gradient of peak value of lateral force with respect to temperature. Since the coefficient of friction, μ_y , is correlated with change in peak value due to change in temperature, we capture the effect of temperature change in numerical modeling by varying the coefficient of friction, μ_y , corresponding to different temperatures. Taking μ_y from the experimental results [18] and developed empirical relation at different temperature as shown in Fig. 2, we compute the peak value of lateral force under the load of 4 kN from the numerical results using ABAQUS. Although, we will present the detail discussion on numerical modeling later, we present the variation of peak value of lateral force at different temperatures in Table 2. Consequently, the value of $d(D_y(T))/dT$ is found as 20.56 N. Similarly, to compute $d(K_y(T))/dT$, which is defined as the rate of change of lateral force with respect to change in temperature corresponding to the slip angle of $\pm 1^\circ$ [19], we numerically compute lateral forces for $\mu_y(T)$ corresponding to different temperatures when the slip angle is fixed at $\pm 1^\circ$ as mentioned in Table 3. After computing the gradient of forces with respect to equivalent change in temperature for $+1^\circ$ and -1° separately, we take average value of the gradient of K_y (T). The value of $d(K_y(T))/dT$ is found as 7.78 N/deg. In the subsequent sections, we compute describe the computation of longitudinal force and aligning torque by following the same approach.

Table 2. Variation of Peak Lateral Force with Temperature from ABAQUS.

Temperature (degree Celsius)	Peak value of Lateral force(N) (for -ve slip angles)	Peak value of Lateral force(N) (for +ve slip angles)
20	-2615.65	2393.38
40	-2203.61	1977.23
60	-1799.22	1565.01

Table 3. Variation of Lateral Force at $\alpha = \pm 1^\circ$ corresponding to Different Temperatures obtained from ABAQUS.

Temperature (degree Celsius)	Lateral force(N) (at $\alpha = -1^\circ$)	Lateral force(N) (at $\alpha = +1^\circ$)
20	-1247.28	1026.01
40	-1114.58	890.867
60	-945.005	706.167

Longitudinal Force at Pure Slip

To compute longitudinal force, we first define the slip speed or slip velocity at the tire-road interface which produces shear force in the longitudinal direction called as longitudinal force, F_x . This slip velocity is due to the speed difference between tangential velocity of tire at the contact patch ($R_e \omega_w$) and the velocity of the vehicle ($R_0 \omega_w$). The slip ratio (k) of the slip velocity to the velocity of the vehicle can be expressed as

$$k = -\frac{V_x - R_e \omega_w}{V_x}, \quad (19)$$

$$V_x = R_0 \omega_w \quad (20)$$

where, V_x is the longitudinal velocity of the vehicle, R_e is the effective rolling radius and ω_w is the angular velocity of the wheel. Following the similar procedure as mentioned by Mizuno et al [20] to obtained modified PAC2002 model for lateral forces only, here, we also modify the PAC 2002 model to study the effect of temperature on longitudinal forces and self-alignment moment. The modified PAC2002 model for longitudinal force can be written as,

$$F_x(k, T) = D_x(T) \sin [C_x \arctan \{B_x(T) k_x - E_x(B_x(T) k_x - \arctan(B_x(T) k_x))\}] + S_{Vx} \quad (21)$$

where,

$$k_x = k + S_{Hx} \quad (22)$$

$$C_x = p_{Cx1} \cdot \lambda_{Cx} \quad (23)$$

$$D_x = \mu_x \cdot F_z \cdot \zeta_1 \quad (24)$$

$$D_x(T) = D_x \cdot \{1 + \frac{\partial \mu_x}{\partial T} (T - T_m)\} \quad (25)$$

$$\mu_x = (p_{Dx1} + p_{Dx2} \cdot df_z) \cdot \lambda_{\mu x}$$

$$E_x = (p_{Ex1} + p_{Ex2} \cdot df_z + p_{Ex3} \cdot df_z^2) \cdot (1 - p_{Ex4} \cdot \text{sgn}(k_x)) \cdot \lambda_{Ex} \quad (26)$$

$$B_x(T) = \frac{K_x(T)}{c_x \cdot D_x(T)} \quad (27)$$

$$K_x = F_z \cdot (p_{Kx1} + p_{Kx2} \cdot df_z) \cdot (\exp(p_{Kx3} \cdot df_z)) \cdot \lambda_{Kx} \quad (28)$$

$$K_x(T) = K_x \cdot \{1 + \frac{\partial c_p}{\partial T} (T - T_m)\} \quad (29)$$

$$S_{Hx} = (p_{Hx1} + p_{Hx2} \cdot df_z) \cdot \lambda_{Hx}$$

