

Admissible Shape Parameters for a Planar Quasi-Static Constraint Mode Tire Model

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ABSTRACT

Computationally efficient tire models are needed to meet the timing and accuracy demands of the iterative vehicle design process. Axisymmetric, circumferentially isotropic, planar, discretized models defined by their quasi-static constraint modes have been proposed that are parameterized by a single stiffness parameter and two shape parameters. These models predict the deformed shape independently from the overall tire stiffness and the forces acting on the tire, but the parameterization of these models is not well defined. This work develops an admissible domain of the shape parameters based on the deformation limitations of a physical tire, such that the tire stiffness properties cannot be negative, the deformed shape of the tire under quasi-static loading cannot be dominated by a single harmonic, and the low spatial frequency components must contribute more than higher frequency components to the overall tire shape. It is hoped that this admissible region will guide the development of efficient and accurate tire models to support future vehicle development.

KEYWORDS: Terrain Surface, Road Surface, Tire Force Estimation, Planar Tire Model, Constraint Mode

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INTRODUCTION

The drive for improved automotive designs is intensified with the introduction of autonomous and hybrid/electric vehicles. The advances of these new vehicles often bring with them added cost and weight, which are anathema to the automotive industry. The introduction of these advances, coupled with an ever shortening timeframe for vehicle development, demands a fast-paced, highly efficient, iterative design environment. One critical area of efficient simulation is the interaction between the tire and the pavement surface through which all the vehicle forces must ultimately be reacted.

It is clear that highly efficient models that capture the fundamental physical properties of tires are critical to simulate vehicle responses so that informed design decisions can be made. High fidelity, but computationally intensive models that provide very accurate information after the parts have been released for production are of no use. Simulation results arising from overly simplified models that lack the fidelity to make accurate chassis load predictions cannot be trusted and have little value. Computationally efficient, planar tire model that accurately predicts the lower-frequency, but not necessarily low amplitude, tire shapes have been developed by considering the quasi-static constraint models of the tire. These low-frequency models are not appropriate for predicting higher-frequency dynamics such as those that would be required for noise, vibration and harshness (NVH) performance predictions, but are

appropriate for ride, handling, and some durability predictions. These models can predict relatively large deformations, approaching, but not including, the point of rim strike. Although the relationship between the radial deformation and radial load is modeled as both linear and quasi-static, the nonlinear stiffness relationship between the vertical spindle load and the vertical tire deflection that exists in real tires is captured by the model via the unidirectional geometric boundary condition at the tire-road interface. Specifically, as the tire deflection increases, more tire segments come into contact with the road surface and the change in boundary conditions of the problem.

The goal of this work is to develop a set of parameter constraints on this class of tire models such that the physical properties of the tire, and a reasonable quasi-static tire shape, are enforced. In this work, an admissible region is developed for the tire parameters, within which the vehicle engineer can tune the parameters to best match the desired force-deflection relationships and remain physically reasonable. Higher-frequency dynamics such as those that would be required for NVH performance predictions are outside the scope of this work. The emphasis is intentionally placed on the physical properties and the shape of the deformed tire. The tire contact patch imposes unidirectional geometric boundary conditions on the tire shape, but the majority of the tire is non-contacting and must adopt a smooth shape. That is, the tire model parameterization must be developed such that the quasi-static deformed tire shape does not exhibit the

higher frequency dynamics that are typically associated with free-vibration normal modes. Defining the parameter constraints such that these higher order dynamics are omitted is one of the major contributions of this work. This results in the development of an admissible region for the tire shape parameters.

The remainder of this work is developed as follows. Some general background on constraint mode tire models is presented. The constraints on the tire shape parameters are developed based on the required physical properties of a tire. First, constraints on shape parameters are developed such that the stiffness (bending, shear and radial) cannot be negative. Second, due to the quasi-static assumption, the tire shape should be smooth. Finally, a boundary case for the admissible parameter set is explored and discussed relative to experimental results.

BACKGROUND

Efficient tire models are a key element in vehicle development and a considerable research has been conducted on the subject. A comprehensive review of the current state of the art is presented by Willumeit and Boehm [1] and more recently by Umsrithong [2]. Although a complete review of recent developments is not presented herein, the results from several of these works have direct relevance to the objectives of this study. Researchers using Finite Element (FE) models have demonstrated that spindle responses can be predicted accurately, but may be computationally intensive and require a great number of physical parameters that are difficult or costly to obtain. 3-D tire models using an FE model for the tread and a lookup table for the sidewall membrane elements [3, 4], 1-D FE model for tires traversing large obstacles [5], FE models with contact patch descriptions to predict vertical force and displacement [6], and most recently detailed FE models using Mooney-Rivlin material properties [7] have all been developed. Similarly, finite difference approaches have been used including the use of a rigid ring to calculate belt vibrations [8] or a flexible ring to simulate the quasi-static response [9, 10, 11].

Ferris first suggested that a static constraint mode tire model can be used to capture the tire enveloping and bridging properties with an axisymmetric and circumferentially isotropic model [12]. This reduced representation of the tire model is founded on the seminal component mode synthesis work originated by Hurty [13] and Gladwell [14] where the order of the stiffness matrix is reduced via a Guyan static reduction [15]. This constraint mode tire model was expanded by Ma to include shear effects in the derivation and to reduce the required number of parameters from five dependent parameters to two parameters that define the constraint modes (the shape) and an independent parameter that defines the overall tire stiffness [16, 17]. The assumption that the low-frequency deformation of the tire shape can be adequately defined by the quasi-static constraint modes is given credence by Gillespie's study [18]. The work by Takayama et al. support the use of a linear and planar model to predict the response of a tire encountering a cleat [19]. This quasi-static, planar, low-frequency model is used in the remainder of this work. The remainder of this section provides an overview of the constraint mode tire method [17] that provides a framework for the admissible shape parameters developed in this work.

Planar Quasi-Static Constraint Mode Tire Model

Consider the tire to be a ring that is inextensible in total circumference, but radially flexible, and supported by an elastic foundation parameterized by the radial stiffness, κ . The assumed axisymmetric, planar, and circumferentially isotropic tire is divided into N segments. Each segment is modeled as an Euler elastic beam of length $2\pi R/N$ and notional cross-sectional area, A (the model itself is planar). The vector $\{\mathbf{u}\}$ defines the radial deflection of all tire segments and varies linearly with the radial forces according to the stiffness matrix $[\mathbf{K}]$. The six physical properties (E , I , G , A , κ , and R) are grouped into a tire modeling stiffness parameter (k_0) and two tire modeling shape parameters (α_1 and α_2), and the distributed force (f) is considered a discrete force (F_n), as defined in Equation (1), Equation (2), Equation (3), Equation (4).

$$k_0 = 6N^3EI(2\pi R)^{-3} - 2NGA(2\pi R)^{-1} + \kappa(2\pi R)/N \quad (1)$$

$$k_0\alpha_1 = -4N^3EI(2\pi R)^{-3} + NGA(2\pi R)^{-1} \quad (2)$$

$$k_0\alpha_2 = N^3EI(2\pi R)^{-3} \quad (3)$$

$$F_n = f(2\pi R)/N \quad (4)$$

The resulting stiffness equation for the n^{th} tire segment is expressed as Equation (5).

$$\mathbf{K}^0 \alpha^S \mathbf{r}^{S-S} + \mathbf{K}^0 \alpha^T \mathbf{r}^{S-T} + \mathbf{K}^0 \mathbf{r}^{S-S} + \mathbf{K}^0 \alpha^T \mathbf{r}^{S+T} + \mathbf{K}^0 \alpha^S \mathbf{r}^{S+S} = \mathbf{F}^S \quad (5)$$

The round shape of a physical tire results in a circulant stiffness matrix $[\mathbf{K}]$, as shown in Equation (6), that comprises two parts: the single model stiffness parameter k_0 and the model shape matrix $[\alpha]$ (defined by the two model shape parameters α_1 and α_2) that defines the relative deformations of the tire when acted upon by geometric constraints.

$$[\mathbf{K}] = k_0 \begin{bmatrix} 1 & \alpha_1 & \alpha_2 & 0 & \cdots & \alpha_2 & \alpha_1 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & \cdots & 0 & \alpha_2 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \vdots & 0 & 0 \\ 0 & \alpha_2 & \alpha_1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_1 & \alpha_2 & 0 & 0 & \cdots & \alpha_1 & 1 \end{bmatrix} = k_0 [\alpha] \quad (6)$$

The radial deformations of the tire $\{\mathbf{u}\}$, are categorized into active and omitted sets, represented by superscripts 'a' and 'o' respectively, where the active constraints are exclusively comprised of those degrees of freedom in contact with the road surface. The degrees of freedom are reordered and written as vector $\{\mathbf{u}\}$ such that the active degrees of freedom occupy the first positions in the vector, followed by the omitted degrees of freedom, via an orthogonal sorting matrix, $[\mathbf{S}]$, as shown in Equation (7).



$$\{u\} = [[S^a] \quad [S^o]] \begin{Bmatrix} \{u^a\} \\ \{u^o\} \end{Bmatrix} = [S] \{\hat{u}\}, \quad \{\hat{u}\} = [S]^T \{u\} \quad (7)$$

The stiffness matrix and the force vector are similarly reordered

$$[\hat{K}] = [S]^T [K] [S], \quad \{\hat{F}\} = [S]^T \{F\} \quad (8)$$

The reordered stiffness equation is then partitioned according to the active and omitted degrees of freedom as given in Equation (9).

$$[\hat{K}] \{\hat{u}\} = \begin{bmatrix} [\hat{K}^{aa}] & [\hat{K}^{ao}] \\ [\hat{K}^{oa}] & [\hat{K}^{oo}] \end{bmatrix} \begin{Bmatrix} \{\hat{u}^a\} \\ \{\hat{u}^o\} \end{Bmatrix} = \begin{Bmatrix} \{\hat{F}^a\} \\ \{0\} \end{Bmatrix} = \{\hat{F}\} \quad (9)$$

Given a set of active constraints, $\{\hat{u}^a\}$, the radial deformations of the tire in their original order $\{u\}$, is written as Equation (10).

$$\{u\} = \left[[S^a] - [S^o]([S^o]^T [\alpha] [S^o])^{-1} ([S^o]^T [\alpha] [S^a]) \right] \{\hat{u}^a\} \quad (10)$$

Note that this mode shape is completely independent of the overall tire stiffness and the forces acting on the tire. Rather, the shape is purely a function of the geometric boundary conditions (captured by the active constraint vector, $\{\hat{u}^a\}$, and the corresponding orthogonal sorting matrix, $[S]$) and the two shape parameters α_1 and α_2 . Deducing the set of active constraints is a recursive process defined by Ma [17] and not repeated here.

ADMISSIBLE REGION FOR SHAPE PARAMETERS

Clearly the physical tire stiffnesses must be positive and the constraints on the physical stiffnesses must be enforced on the model parameters. It should also be intuitive that the non-contacting region of a tire under quasi-static conditions must adopt a smooth, unrippled shape. That is, the upper section of the tire remains nearly round. More specifically, changes in the radial deformation of the tire occur smoothly and in small increments in the circumferential direction. This implies that the contributions of higher spatial frequencies must remain small. This is in contrast to the deformed shape that a tire assumes when it is being harmonically excited, particularly in the free vibration mode. The remainder of this section proceeds as follows. First the physical restrictions on the tire's stiffness parameters are used to develop a set of constraints on the shape parameters. Next, a single point load is considered in order to develop the spatial frequency constraints. The low-frequency spatial constraint is formulated and the resulting constraints on the shape parameters are developed. This set of constraints forms the admissible region for the shape parameters.

Physical Constraints

The tire shape is constrained by its physical properties which must be enforced as constraints on the shape parameters α_1 and α_2 . Specifically, the tire must possess positive bending, shear and radial stiffness. The positive sense for the deflections and corresponding forces are defined such that the overall tire stiffness, k_0 must also be positive. Therefore, the relationships developed in Equation (1), Equation (2), Equation (3) are rearranged to develop constraints on the tire shape parameters, α_1 and α_2 , as given in Equation (11), Equation (12), Equation (13).

$$\alpha_2 = \frac{EIN^3}{k_0(2\pi R)^3} > 0 \quad (11)$$

$$\alpha_1 + 4\alpha_2 = \frac{GAN}{k_0(2\pi R)} > 0 \quad (12)$$

$$2\alpha_1 + 2\alpha_2 + 1 = \frac{\kappa(2\pi R)}{k_0 N} > 0 \quad (13)$$

Quasi-Static Point Loading

Next an additional constraint that enforces the low spatial frequency requirement (smoothness for the non-contacting region of the tire) is formulated by considering the effects of a single point load. Consider a single point load on the tire such that only one degree of freedom is active as shown in Figure 1. The tire segments are shown as solid lines, the virtual single point loading is shown as a dashed line, the tire model segments are indexed by n , and the indexing begins with the only active constraint.

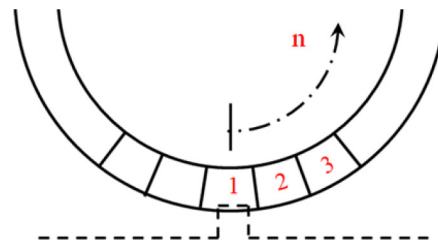


Figure 1. Single point loading and indexing scheme

Without loss of generality, the indices of the degrees of freedom are assigned such that this active point is the first degree of freedom. The deformed tire shape is simply computed as Equation (14).

$$\{\hat{u}\} = [\hat{K}]^{-1} \begin{Bmatrix} \hat{F}^a \\ \emptyset \end{Bmatrix} = [\alpha]^{-1} \begin{Bmatrix} k_0^{-1} \hat{F}^a \\ \emptyset \end{Bmatrix} \quad (14)$$

Since there is only one active segment, $\hat{u}^a$ and $k_0^{-1} \hat{F}^a$ are scalar values and the deformed shape of the tire, $\{\hat{u}\}$, is the product $\{a\}$, the first column of the matrix $[\alpha]^{-1}$, and the scalar $k_0^{-1} \hat{F}^a$. To compute the inverse matrix, $[\alpha]^{-1}$, an eigen-decomposition method is applied to the original shape matrix $[\alpha]$. The eigenvalues of the shape matrix $[\alpha]$