

Model-Based Control of BMEP and NOx Emissions in a Euro VI 3.0L Diesel Engine

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ABSTRACT

A model-based approach to control BMEP (Brake Mean Effective Pressure) and NOx emissions has been developed and assessed on a FPT F1C 3.0L Euro VI diesel engine for heavy-duty applications. The controller is based on a zero-dimensional real-time combustion model, which is capable of simulating the HRR (heat release rate), in-cylinder pressure, BMEP and NOx engine-out levels. The real-time combustion model has been realized by integrating and improving previously developed simulation tools. A new discretization scheme has been developed for the model equations, in order to reduce the accuracy loss when the computational step is increased. This has allowed the required computational time to be reduced to a great extent. The real-time combustion model has been first calibrated and assessed at both steady-state and transient conditions, on the basis of experimental data acquired at the highly dynamic test bench of ICEAL-PT (Internal Combustion Engines Advanced Laboratory - Politecnico di Torino), in the frame of a research activity in collaboration with FPT Industrial. The model has then been used to realize a model-based control of BMEP and NOx emissions. In particular, the controller provides the injected fuel quantity and the injection timing of the main pulse, for given targets of BMEP and engine-out NOx levels. Finally, the developed controller has been tested on a rapid prototyping device (ETAS ES910) through HiL (Hardware-in-the-Loop) techniques, and demonstrated to have real-time capability.

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INTRODUCTION

Interest in model-based combustion control has increased in the last few years [1]. A model-based approach for engine control can in fact lead to several advantages compared to the traditional map-based one. For example, the development of a model-based approach requires in general lower experimental effort compared to that required for the calibration of the engine maps, also considering that a large number of engine maps are implemented in modern ECUs (Engine Control Units) [1], which are even difficult to manage. Moreover, a model-based control offers the possibility of optimizing the combustion process onboard, and to adjust the main calibration parameters during the engine operation in real time. For example, this would allow the management of the after-treatment system to be optimized, by setting variable targets of exhaust temperatures and/or pollutant engine-out emissions over a vehicle mission, and these targets may be achieved by adapting the main ECU variables in real-time. In addition, a model-based approach may be capable of taking into account the effects, on the combustion process, of the variability of the environmental conditions (humidity, temperature, pressure) or the effects related to the engine transient operation (turbo-lag, EGR delays, ...), without the need of implementing correction maps.

The implementation of model-based controls has been made possible, in the last few years, by the increasing computational performance of modern ECUs.

Therefore, the development of combustion models has become of great interest.

In general, engine simulation can be carried out with different degrees of detail. The main simulation approaches include multidimensional, one-dimensional or zero-dimensional methods, that are characterized by a different degree of detail and computational effort.

3D-CFD (Computer Fluid-Dynamics) methods [2, 3, 4, 5, 6, 7, 8, 9, 10] and 1D-CFD approaches [11, 12, 13, 14, 15, 16] require a computational time that is not currently suitable to develop model-based combustion controls.

Mean-value engine and combustion models [17-18] are capable of simulating the combustion and emission formation processes with a good level of detail [18]. These methods offer the opportunity of

further decreasing the computational time in comparison to 1D-CFD approaches, while guaranteeing at the same time a good predictive capability at steady-state and transient engine operating conditions. Moreover, they are physically consistent, so that they do not require a high calibration effort, and at the same time their accuracy is still acceptable outside the calibration range [18]. Therefore, they can be considered as good candidates for the development of model-based control algorithms. A real-time combustion model, which belongs to this category, has been developed by the authors in [18].

Finally, artificial intelligence systems [19, 20, 21, 22, 23, 24], such as support vector machines (SVM), genetic algorithms (GA) and artificial neural networks (ANNs) constitute a last category of models which are often used in the field of engine design and control. These methods do not require the detailed physical knowledge of the investigated process and are able to capture complex nonlinear system behavior with relatively simple mathematical operations. Moreover, they are characterized by a very small computational time, so that they are good candidates for the development of model-based control algorithms to be implemented in ECUs. However, their training usually requires a high number of experimental tests, and their performance is usually not reliable outside the calibration range.

Contribution of the Present Study

It should be noted, from the previous background, that mean-value models are among the best candidates for the development of model-based combustion control algorithms, as they are physically consistent, and generally require a low calibration effort and a low computational time.

In this study, a previously developed real-time combustion model [18] has been calibrated and validated for a FPT F1C 3.0L Euro VI diesel engine, and has been used to develop a model-based controller of BMEP and NOx emissions. The activity was carried out in the frame of a research project in collaboration with FPT Industrial.

The real-time combustion model is capable of simulating the HRR (heat release rate) and in-cylinder pressure, along with the related metrics, such as MFB50, PFP (Peak Firing Pressure), IMEP (Indicated Mean Effective Pressure) and BMEP. In particular, the chemical energy release has been simulated using an improved version [18, 25] of the accumulated fuel mass approach, previously presented in [26, 27, 28, 29, 30]. A 3-zone thermodynamic model is also included to simulate the in-cylinder burned gas temperatures. The latter model has been enhanced in this paper with respect to previous versions, in order to be able to account for the effect of intake air humidity. Engine-out NOx emissions were simulated on the basis of a semi-empirical correlation, that takes into account the in-cylinder burned gas temperature, as well as MFB50 and additional metrics.

The real-time combustion model used in this study has been improved with respect to previous versions in terms of computational efficiency. In particular, a new discretization scheme has been developed for the in-cylinder pressure model, in order to reduce the accuracy loss when the computational step is increased. This has allowed the required computational time to be reduced to a great extent.

The model has been first calibrated and assessed at both steady-state and transient conditions over several speed/load ramps, on the basis of experimental data acquired at the highly dynamic test bench of ICEAL-PT (Internal Combustion Engines Advanced Laboratory - Politecnico di Torino), in the frame of a research activity in collaboration with FPT Industrial. Then, it has been inverted in order to realize a model-based controller of BMEP and NOx emissions. In particular, the controller is capable of providing the injected fuel quantity and the injection timing of the main pulse, for given targets of BMEP and NOx engine-out emissions.

Finally, the developed controller has been tested on a rapid prototyping device (ETAS ES910) through HiL (Hardware-in-the-Loop), and demonstrated to have real-time capability.

ENGINE SETUP AND EXPERIMENTAL ACTIVITY

The experimental tests used for model calibration and validation were conducted on a FPT F1C 3.0L Euro VI diesel engine. The main technical specifications of the engine are reported in [Table 1](#).

Table 1. Main technical specifications of the engine.

Engine type	FPT F1C Euro VI diesel engine
Displacement	2998 cm ³
Bore x stroke	95.8 mm x 104 mm
Rod length	160 mm
Compression ratio	17.5
Valves per cylinder	4
Turbocharger	VGT type
Fuel injection system	High pressure Common Rail

The engine ([Fig. 1](#)) is equipped with a short-route cooled EGR system, in which the EGR valve is located upstream from the cooler. A flap is installed in the exhaust pipe downstream the turbine, to control the temperature of the exhaust gas flowing to the aftertreatment system and to allow high EGR rates to be obtained when the pressure drop between the exhaust and intake manifolds is not sufficiently high.

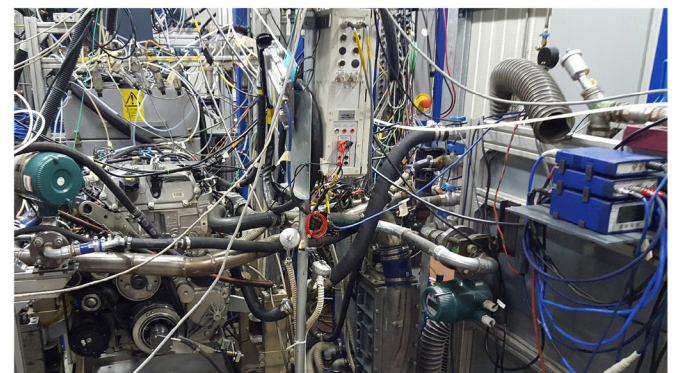


Figure 1. FPT F1C 3.0L Euro VI diesel engine installed on the highly dynamic test bench at the Politecnico di Torino. The rapid prototyping device can be observed on the right side.

The test engine was instrumented with piezoresistive pressure transducers and thermocouples to measure the pressure and temperature at different locations, such as upstream and downstream from the compressor, from the turbine and intercooler, in the intake manifold and in the EGR circuit. Thermocouples were also used to measure the temperatures in each intake and exhaust runners. KISTLER 6058A high-frequency piezoelectric transducers were fitted to the glow-plug seat to measure the in-cylinder pressure time-histories, which were used to realize a pressure-based MFB50 control [1]. The in-cylinder pressure traces were corrected on the basis of the intake pressure that was measured by means of high-frequency KISTLER 4007C piezoresistive transducers, which were located at the inlet runners of the cylinders.

All the experimental tests were carried out on the highly dynamic test bed at ICEAL at the Politecnico di Torino. The test rig is equipped with an 'ELIN AVL APA 100' cradle mounted AC dynamometer and an 'AVL KMA 4000', with a reading accuracy of 0.1% over a 0.28-110 kg/h range, to continuously measure the fuel consumption. An 'AVL AMAi60' system, consisting of three analyzer trains, was used to measure the engine-out gaseous raw emissions. Two analyzer trains were equipped with complete devices for the analysis of THC, CH₄, NO_x, and low as well as high CO, CO₂ and O₂, and were used to measure the intake and exhaust gas composition. All of the abovementioned measurement devices were controlled by a PUMA OPEN 1.3.2 automation system. In order to minimize the testing effort, the test bed environment was interfaced with AVL CAMEO software to run intelligent engine calibration procedures on the basis of the DoE (Design of Experiment) approach.

An ETAS ES910 rapid prototyping device was used to realize pressure-based and model-based controls of the combustion phasing (see [1]), and to test the real-time capability of the model-based controller of BMEP and NO_x which has been developed in this study. The main specifications of the ETAS ES910 device are reported in Tab. 2.

Table 2. Main specifications of the ETAS ES910 rapid prototyping device.

Main processor	Freescall PowerQUICC™ III MPC8548 with 800 MHz clock Double precision floating point unit
Memory	512 MByte DDR2-RAM (400 MHz clock) 64 MByte Flash 128 kByte NVRAM

Experimental Activity

The experimental tests that have been considered in the present paper include steady-state tests and transient tests. The steady-state tests were mainly used to calibrate the real-time combustion model. To this aim, the following tests were considered (Fig. 2):

- A full engine map with baseline operating parameters, including 123 points.
- EGR-sweep tests at fixed key-points, including 162 points. EGR rate was varied from 0 to 50% by setting different levels of trapped air mass with steps of 50 mg/cycle.
- sweep tests of main injection timing (SOI_{main})/injection pressure (p_i) at fixed key-points, including 125 points. A SOI_{main}

variation of ± 6 deg around the nominal values and a p_i variation of $\pm 20\%$ around the nominal values were set.

The developed control technique was tested on the engine over different load/speed ramps. Details on these ramps are reported in the "Results and discussion" section.

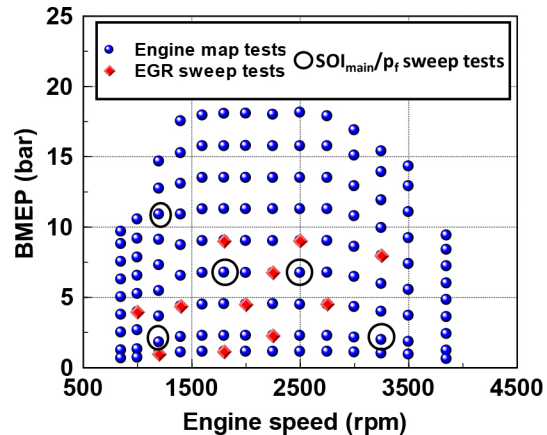


Figure 2. Experimental tests used for the calibration of the real-time combustion model.

REAL-TIME COMBUSTION MODEL

The real-time combustion model that has been used for the development of the BMEP/NO_x controller includes the simulation of:

1. Chemical energy release: the approach is based on an enhanced version [25] of the model previously presented by the authors, which was based on the accumulated fuel mass approach [30]. The input data of the model are the injection parameters, as well as the main thermodynamic conditions in the intake manifold and the engine operating parameters.
2. In-cylinder pressure: the approach is based on the inversion of a single-zone heat release model [31] which requires the net energy release as input; the latter is derived starting from the predicted chemical energy release and estimating the heat transfer between the charge and the walls. Polytropic evolutions are assumed during the compression and expansion phases. Several metrics, such as PFP and IMEP, can be extracted from the simulated in-cylinder pressure.
3. Friction losses: the Chen-Flynn approach has been used to predict FMEP on the basis of the engine speed and peak firing pressure; the simulation of friction losses allows BMEP to be evaluated starting from IMEP.
4. Pumping losses: the pumping losses (PMEP) were simulated on the basis of a semi-empirical correlation which takes into account the intake and exhaust manifold pressure levels, as well as engine speed.
5. NO_x emission levels: an improved version of the semi-empirical correlation previously developed by the authors for a 2.0L Euro 5 diesel engine and reported in [32] has been tuned and validated for the 3.0L F1C Euro VI engine considered in the present study.

The detailed description of the combustion model and the calibration methodology is reported in [18]. However, a summary is provided hereafter for the sake of clarity. Moreover, the correlations of the model parameters which have been obtained from the model recalibration on the FIC engine have also been reported.

Figure 3 reports the scheme of the real-time combustion model.

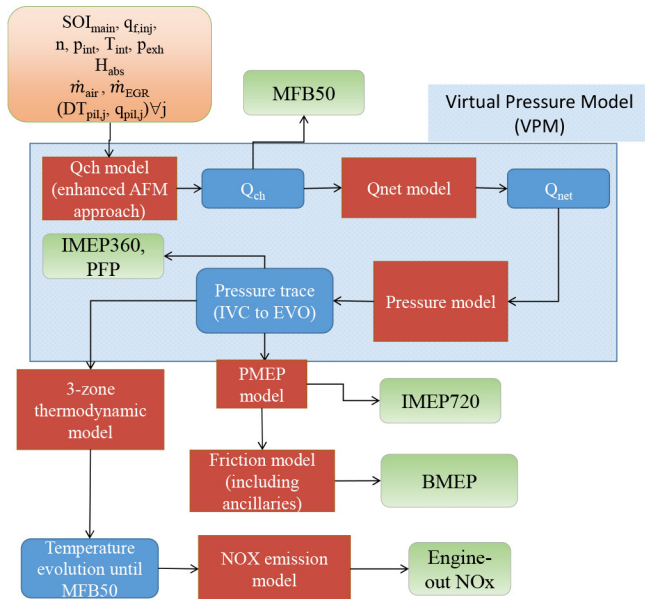


Figure 3. Scheme of the real-time combustion model.

Estimation of the Chemical Energy Release Q_{ch} and of the Net Energy Release Q_{net}

The equations of the Q_{ch} and Q_{net} sub-models are reported in [Table 3](#). The reader may refer to [\[18\]](#) for further details related to these equations.

The Q_{ch} and Q_{net} models were assessed for the steady-state conditions reported in Fig. 2. Physically-consistent correlations were identified for the model calibration parameters, as a function of the in-chamber thermodynamic quantities at SOI/SOC (start of injection/start of combustion) and other engine variables, as follows:

$$K_{pil}[\frac{1}{\text{deg}}] = 2.73E-04 p_f^{-0.7887} O_2^{1.90} n^{0.452} q_{pil,tot}^{-0.239} \quad (I)$$

$$K_{l,main}[\frac{1}{\text{deg}}] = 7.78E-8 \cdot SOI_{main}^{2.16} p_{\text{int}}^{0.55} O_2^{0.937} n^{-0.159} q_{f,inj}^{-0.274} \quad (2)$$

$$K_{2,main}[-] = 0.567 p_f^{0.215} \rho_{SOC,main}^{-1.13} O_2^{0.0926} n^{-0.0484} q_{main}^{0.401} \quad (3)$$

$$\tau_{pil}[\text{deg}] = 1.31 p_f^{0.0496} \rho_{SOIP}^{-1.16} n^{0.606} O_2^{-0.0298} \quad (4)$$

$$\tau_{main}[\text{deg}] = 7.71 p_f^{-0.789} \rho_{SOI,main}^{-2.46} n^{1.41} q_{f,inj}^{0.259} \quad (5)$$

$$Q_{f,evap}[kJ]=3.75E-6 T_{int}^{1.17} n^{0.227} q_{f,inj}^{-0.162} \quad (6)$$

$$\mathcal{Q}_{ht, glob}[kJ] = 1.701E3 \cdot SOI_{main}^{-2.06} p_{int}^{-0.466} O_2^{0.465} n^{-0.351} q_{f, inj}^{1.29} \quad (7)$$

Table 3. Main equations of the Q_{ch} and Q_{net} models.

Q_{ch} model	$\frac{dQ_{ch,pil,j}}{dt}(t) = K_{pil,j} [Q_{fuel,pil,j}(t - \tau_{pil,j}) - Q_{ch,pil,j}(t)]$ $\frac{dQ_{ch,main}}{dt}(t) = K_{1,main} [Q_{fuel,main}(t - \tau_{main}) - Q_{ch,main}(t)] +$ $+ K_{2,main} \frac{dQ_{fuel,main}(t - \tau_{main})}{dt}$ $Q_{fuel,j}(t) = \int_{t_{SOI,j}}^t \dot{m}_{f,inj}(t) H_L dt \quad t \leq t_{EOI,j}$ $Q_{fuel,j}(t) = \int_{t_{SOI,j}}^{t_{EOI,j}} \dot{m}_{f,inj}(t) H_L dt \quad t > t_{EOI,j}$ $Q_{ch} = \sum_{j=1}^n Q_{ch,j}$ where j indicates the generic injection pulse, H_L indicates the fuel lower heating value, K and τ are the combustion rate coefficient and ignition delay coefficient of the Q_{ch} model.
Q_{net} model	$Q_{net,ht} \cong Q_{ch} \frac{m_{f,inj} H_L - Q_{ht,glob}}{m_{f,inj} H_L}$ $Q_{net} \cong Q_{net,ht} - Q_{f,evap}$ where Q_{f,evap} and Q_{ht,glob} indicate the fuel evaporation heat from SOI to SOC (J) and the heat exchanged by the charge with the walls over the combustion cycle (J), and m_{f,inj} is the total injected fuel mass per cyc/cyl

In equations (1, 2, 3, 4, 5, 6, 7), ρ_{SOF} , ρ_{SOC} indicate the in-chamber densities evaluated at the start of injection or combustion, respectively, and are expressed in kg/m³. The injection pressure p_f is expressed in bar, the engine speed n in rpm, the total injected fuel quantity $q_{f,inj}$ (used as a load parameter) in mm³/cyc/cyl, the total injected fuel quantity of the pilot shots $q_{pil,tot}$ in mm³/cyc/cyl and finally the intake oxygen concentration O_2 in %. T_{int} and p_{int} indicate the intake manifold temperature and pressure, respectively.

Estimation of the In-Cylinder Pressure

The equations of the in-cylinder pressure sub-model are reported in [Table 4](#). The reader may refer to [\[18\]](#) for further details related to these equations.

Table 4. Main equations of the in-cylinder pressure model.

Pressure model	Starting condition (pIVC): $p_{IVC} = p_{int} + \Delta p_{int}$
	Compression phase (IVC to SOC): $pV^m = const$
	Combustion phase (SOC to EOC): $p^i = \frac{\Delta Q_{net} - \frac{p^{i-1}}{2}(V^i - V^{i-1}) + \frac{1}{\gamma-1} p^{i-1} V^{i-1}}{\frac{V^i - V^{i-1}}{2} + \frac{V^i}{\gamma-1}}$
	Expansion phase (EOC to EVO): $pV^{m'} = const$ where p_{int} indicates the intake manifold pressure, pIVC the in-cylinder pressure at IVC, m and m' the compression and expansion polytropic exponents.

With reference to [table 4](#), the in-chamber pressure was evaluated, during the combustion interval, on the basis of the inversion of a single-zone heat release model, which can be summarized as follows [\[31\]](#):

$$dQ_{ch} - dQ_{ht} = dQ_{net} = p dV + m C_v dT \quad (8)$$

where Q_{ht} represents heat transfer and C_v the specific heat at constant volume. In [Eq. \(8\)](#) the last term is usually rewritten using the ideal-gas law and assuming a constant mass m, so that:

$$dQ_{net} = p dV + \frac{1}{\gamma-1} d(pV) \quad (9)$$

where $\gamma = c_p / c_v$. The last differential in [Eq. \(9\)](#) is typically rewritten in terms of pressure and volume differentials, and the following differential equation is commonly proposed in the literature [\[31\]](#):

$$dQ_{net} = \frac{\gamma}{\gamma-1} p dV + \frac{1}{\gamma-1} V dp \quad (10)$$

In the previous versions of the combustion model proposed by the authors [\[25, 30\]](#), the pressure differential was evaluated on the basis of [Eq. \(10\)](#), as follows:

$$dp = \left(\frac{\gamma-1}{V} \right) \left(dQ_{net} - \frac{\gamma}{\gamma-1} p dV \right) \quad (11)$$

which had been then discretized according to the following scheme:

$$p^i = p^{i-1} + \frac{\left[Q_{net}^i - Q_{net}^{i-1} - \frac{\gamma}{\gamma-1} p^{i-1} (V^i - V^{i-1}) \right] (\gamma-1)}{V^i} \quad (12)$$

The new approach proposed in this study starts directly from [Eq. \(9\)](#) and is based on the following discretization scheme:

$$Q_{net}^i - Q_{net}^{i-1} = \Delta Q_{net} = \frac{p^i + p^{i-1}}{2} (V^i - V^{i-1}) + \frac{1}{\gamma-1} (p^i V^i - p^{i-1} V^{i-1}) \quad (13)$$

so that the pressure value at time instant t^i is evaluated as follows:

$$p^i = \frac{\Delta Q_{net} - \frac{p^{i-1}}{2} (V^i - V^{i-1}) + \frac{1}{\gamma-1} p^{i-1} V^{i-1}}{\frac{V^i - V^{i-1}}{2} + \frac{V^i}{\gamma-1}} \quad (14)$$

The new approach for pressure discretization has allowed the accuracy loss to be reduced to a great extent when the computational step is increased.

A comparison of the model accuracy (quantified by the RMSE, i.e., root mean squared error) when using the new scheme (i.e., [Eq. \(14\)](#)) and the old one (i.e., [Eq. \(12\)](#)) is reported in [Table 5](#), for different values of the computational step. It can be seen that the new discretization scheme leads to a virtually negligible loss of accuracy in terms of IMEP and BMEP prediction, when the crank angle integration step (CAstep) is increased to 0.5 or 1 deg, starting from a nominal value of 0.1 deg.

Table 5. Comparison between the values of RMSE for the main quantities of the real-time combustion model, when using the old (a) and new (b) pressure discretization schemes. Engine map tests are considered.

CAstep	0.1	0.2	0.5	1
RMSE MFB50 (deg)	0.85	0.85	0.87	0.90
RMSE PFP (bar)	2.94	2.96	3.04	3.71
RMSE IMEP360 (bar)	0.21	0.23	0.42	0.83
RMSE IMEP720 (bar)	0.20	0.23	0.42	0.83
RMSE BMEP (bar)	0.17	0.22	0.44	0.85
RMSE NOx (ppm)	37.0	36.9	37.1	37.9

(a) Old discretization scheme for pressure

CAstep	0.1	0.2	0.5	1
RMSE MFB50 (deg)	0.85	0.85	0.87	0.90
RMSE PFP (bar)	2.92	2.93	2.93	2.94
RMSE IMEP360 (bar)	0.21	0.21	0.21	0.22
RMSE IMEP720 (bar)	0.21	0.21	0.21	0.21
RMSE BMEP (bar)	0.17	0.17	0.17	0.17
RMSE NOx (ppm)	36.9	36.7	36.7	36.3

(b) New discretization scheme for pressure

In [Eq. \(14\)](#) the isentropic coefficient $\gamma = c_p / c_v$ was set constant and equal to 1.37.

The following correlations were identified for the pressure model parameters, which are functions of the intake manifold thermodynamic conditions and of the engine load and speed:

$$m = 2.136 T_{int}^{-0.0696} n^{0.0074} q_{f, inj}^{0.0022} \quad (15)$$

$$m' = 1.321 \cdot T_{int}^{-0.0532} n^{-0.0272} q_{f, inj}^{-0.0312}$$



$$\Delta p_{int} [\text{bar}] = 0.0929 p_{int}^{1.15} n^{0.032} q_{f, inj}^{-0.0319} \quad (17)$$

The in-cylinder pressure trace has been simulated over the compression and combustion phases only, and this allows the gross IMEP (i.e., IMEP360) to be estimated. Pumping losses were evaluated by means of a dedicated correlation, which is reported in the next section.

The simulation of the in-cylinder pressure traces also allows peak-firing pressure (PFP) to be evaluated.

Estimation of PMEP and FMEP

The following correlation was identified to evaluate PMEP for the engine considered in this study:

$$PMEP [\text{bar}] = 0.01248 \cdot p_{exh}^{1.068} n^{0.557} - 0.0337 \cdot p_{int}^{1.18} n^{0.402} \quad (18)$$

The Chen-Flynn approach [33] was adopted to estimate FMEP.

The experimental values of FMEP were evaluated as the difference between the experimental values of the net IMEP and the measured values of BMEP, as follows:

$$FMEP = IMEP - BMEP \quad (19)$$

The values of IMEP of one of the four cylinders were taken as being representative of all the cylinders, due to the low cylinder-to-cylinder dispersion.

The following correlation was identified to evaluate FMEP for the engine considered in this study:

$$FMEP [\text{bar}] = -0.2679 + 6.177E - 4 \cdot n - 5.164E - 8 \cdot n^2 + 0.00584 \cdot PFP \quad (20)$$

where the engine speed is expressed in rpm and PFP is expressed in bars.

The squared correlation coefficient R^2 between the predicted and experimental values of FMEP is the order of 0.9.

Estimation of the NOx Emissions

The engine-out NOx emissions were evaluated starting from the semi-empirical model developed by the authors in [32]. That correlation has been re-tuned for the engine considered in the present study, on the basis of the experimental tests reported in Fig. 2.

The following correlations were obtained from the tuning of the NOx emission model:

$$NO_x [\text{ppm}] = 1135 \exp^{\frac{6.78e+03}{MFB50}} \exp^{\frac{-4.69e+04}{T_{b,MFB50}}} O_2^{-1.99} n^{-0.430} q_{f, inj}^{0.568} \quad (q_{f, inj} \leq 45 \text{ mm}^3, n \leq 1900 \text{ rpm}) \quad (21)$$

$$NO_x [\text{ppm}] = 5698 \exp^{\frac{3.40e+03}{MFB50}} \exp^{\frac{-4.37e+04}{T_{b,MFB50}}} O_2^{1.55} n^{-0.138} q_{f, inj}^{0.306} \quad (q_{f, inj} > 45 \text{ mm}^3, n \leq 1900 \text{ rpm}) \quad (22)$$

$$NO_x [\text{ppm}] = 4042 \exp^{\frac{5.20e+03}{MFB50}} \exp^{\frac{-4.82e+04}{T_{b,MFB50}}} O_2^{0.724} n^{-0.334} q_{f, inj}^{0.583} \quad (q_{f, inj} \leq 45 \text{ mm}^3, n > 1900 \text{ rpm}) \quad (23)$$

$$NO_x [\text{ppm}] = 64.74 \exp^{\frac{2.92e+03}{MFB50}} \exp^{\frac{-3.44e+04}{T_{b,MFB50}}} O_2^{2.80} n^{-0.353} q_{f, inj}^{0.372} \quad (q_{f, inj} > 45 \text{ mm}^3, n > 1900 \text{ rpm}) \quad (24)$$

Equations (21, 22, 23, 24) were derived considering all the data related to the engine map, EGR-sweep and SOI_{main}/p_f sweep tests, as it was verified that this led to satisfactory results for all datasets. Moreover, at the beginning, all the main engine variables related to NOx formation were included in the correlations, and a sensitivity analysis was carried out in order to exclude the least influential ones.

In order to avoid discontinuities when applying the NOx model, Eqs. (21, 22, 23, 24) have been calibrated using overlapping calibration datasets. In other words, each of the four calibration datasets also included tests characterized by injected quantities and speed levels exceeding the breakpoints (i.e., $q_{f, inj} = 45 \text{ mm}^3$, $n = 1900 \text{ rpm}$).

It can be seen that the temperature of the burned gases evaluated at MFB50 (i.e., $T_{b,MFB50}$) has been used. The temperature of the burned gases was evaluated by means of the real-time thermodynamic three-zone model that is described in the next section. The evaluation of the $T_{b,MFB50}$ term, in general, requires the evaluation of the in-cylinder pressure, which is an input of the thermodynamic model.

Moreover, it can be seen that the MFB50 parameter has also been used in the correlation. It was verified that the use of MFB50 leads to a significant increase in the NOx prediction accuracy for all the experimental datasets.

Three-Zone Thermodynamic Model

A refined version of the real-time three-zone thermodynamic model presented in [34] has been used in order to evaluate the temperature of the burned gases. In particular, the model has been refined by taking into account the effect of the intake air humidity. The 3-zone thermodynamic model is based on the splitting of the in-chamber content into a vapor-fuel zone (f), an unburned gas zone (u), made up