

Design and Analysis of a Plug-Muffler Modified for Low Back-Pressure and Improved Acoustic Performance

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ABSTRACT

High insertion loss is desirable and can be achieved by using plug-muffler elements which consist of two cross-flow perforated sections. However, the plug-mufflers have an inherent disadvantage of high back-pressure which may affect the engine performance adversely. In this paper, a novel structural modifications has been introduced to the plug-muffler to obtain better acoustic performance as well as low back-pressure. Three configurations have been analyzed here including the classical plug-muffler configuration. Back-pressure has been calculated using the lumped flow-resistance network theory for all three configurations and compared. To evaluate the transmission loss, the 1-D (plane wave) analysis has been carried out using the Integrated Transfer Matrix (ITM) method and the results so obtained are validated against 3-D FEM using a commercial software. For the 1-D ITM method as well as the 3-D FEM, an existing perforate impedance model has been used for evaluating and comparing the acoustic performance of the three configurations. Incidentally, however, a simplified expression for the acoustic impedance of the perforate has been developed, based primarily on the existing models for the acoustic perforate impedance. The proposed perforate impedance expression is shown to predict the acoustic performance with reasonable accuracy as compared to the existing complicated models. Thus it provides a simple expression for perforate impedance which includes the bias flow and grazing flow effects as well as the viscous dissipation and inertance effects. Finally, the modified plug-muffler configuration (configuration 3) proposed here is shown to have superior acoustic performance, particularly at low frequencies where it is crucial, and lower back-pressure when compared to the other two configurations studied here. Physical explanation of the improvement in acoustic performance has been provided in terms of phase cancellation that is typical of the multiply-connected elements.

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INTRODUCTION

The plug muffler with a perforated baffle plate in-between as shown in Fig. 1, consists of the cross-flow perforated inlet and outlet pipe. The plug forces the mean flow to move out of the inlet-pipe and reenter the outlet-pipe through the perforations in segments 2-3 and 6-7, respectively as shown in Fig. 1. In this process, it introduces a pair of in-line flow-acoustic resistances in the system that dissipate the acoustic energy in the free shear layer of the jets at the orifices or holes. Thus a pair of cross-flow perforated elements constitute a reactive-cum-dissipative muffler [1, 2].

To obtain adequate insertion loss over the entire frequency range, particularly at low frequencies of the order of the engine firing frequency, cross-flow perforated elements are used which unfortunately have the disadvantage of high back-pressure which would raise the specific fuel consumption of the engine. In order to reduce the back-pressure whilst using the cross-flow elements, one can reduce the mean flow velocity through the muffler by increasing the diameter of the exhaust pipe, which in turn calls for a larger diameter muffler shell in order to maintain sufficient area expansion ratio. Thus, there is a tradeoff between the size of the muffler and the back pressure [1].

A transfer-matrix-based muffler program was used by Munjal *et al.* [2] to evaluate the acoustic performance of a plug muffler (without the perforated baffle plate) where open area ratio (OAR) was used as a design parameter. A lower value of OAR (1 or below) results in the increase of the mean flow velocity through the perforate, thereby increasing the flow-acoustic resistance and hence acoustic dissipation. This raises the potential troughs of the transmission loss (*TL*) curve. However, due to lower value of OAR, the plug muffler exerts high back-pressure. On the other hand, if OAR is increased to higher value (2 or more), the performance of the plug muffler degenerates to that of a simple expansion chamber characterized by periodic domes [1].

To make use of the advantage of cross-flow perforated elements along with low back-pressure, structural modifications have been introduced here to the existing plug muffler (Fig. 1) as shown in Figs. 2 and 3. These muffler configurations are referred as configurations 1, 2 and 3, corresponding to Figs. 1, 2 and 3, respectively. To evaluate the acoustic performance of a muffler configuration, knowledge of flow distribution is required to take into account the dissipative effect of the bias flow through the perforations. Moreover, knowing the flow distribution, one can estimate the back-pressure offered by the muffler. It is to be noted that the convective effect of the mean flow is

known to be relatively insignificant for the perforated element [1] and hence neglected. For all the configurations analyzed here, the flow distribution has been evaluated using the lumped flow-resistance network theory [3].

To evaluate the acoustic performance (TL) of configurations 1, 2 and 3, 1-D (plane wave) analysis has been carried out using the Integrated Transfer Matrix (ITM) method [4, 1], and the results obtained are validated against the 3-D Finite Element Method (FEM) using a commercial software. The acoustic performance of all the configurations evaluated by means of ITM method and are validated against the 3-D FEM analysis.

The following assumptions have been made while analyzing all the configurations by means of the 1-D ITM method as well as the 3-D FEM:

- The medium is considered to be inviscid.
- The muffler shell and the end plates are assumed to be rigid.
- The convective effects of the mean flow have been neglected in the governing equations, however the dissipative effect of the bias flow as well as grazing flow has been considered in the analysis by means of appropriate expressions for the perforate impedance.

DESCRIPTION OF THE MUFFLER SYSTEMS / CONFIGURATIONS

The general arrangement of the configuration-1 is shown in Fig. 1, where the inlet-pipe and the outlet-pipe are aligned on a common axis, with a plug in-between. Section 4-5 in Fig. 1 represents the plug as well as a baffle plate with two or more holes.

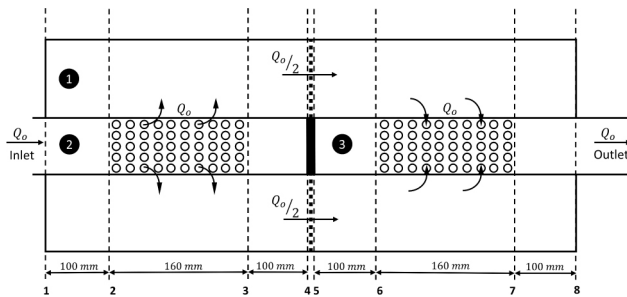


Figure 1. Schematic diagram of the configuration-1: plug muffler with perforated baffle plate in-between.

The geometrical details of the configuration-1 are listed in Table 1.

The general arrangement of the configuration-2 is shown in Fig. 2. Similar to the configuration-1, in configuration-2, the inlet-pipe and the outlet-pipe are plugged at one end with perforated section but they run parallel throughout the length of the muffler, being offset to the axis of the shell in opposite direction to each other. In Fig. 2, section 2-3 represents the perforation on the outlet-pipe, whereas section 6-7 represents perforation on the inlet-pipe. The geometrical details of the configuration-2 are same as those of configuration-1 and are listed in Table 1.

Table 1. Geometric details of the configurations 1 and 2

S. No.	Parameter		Value
1	Shell Diameter (<i>mm</i>)		288
2	Total Length of the muffler (<i>mm</i>)		720
3	Baffle plate	Hole Diameter (<i>mm</i>)	76
		No. of holes	2
		Thickness of plate (<i>mm</i>)	1.5
4	Inlet-pipe and Outlet-pipe	Diameter (<i>mm</i>)	100
		OAR	0.94
		Porosity	14.6 %
		No. of Holes	26×10
		Hole Diameter (<i>mm</i>)	6
		Perforated Length (<i>mm</i>)	160

The schematic of the configuration-3 is shown in Fig. 3. In configurations 1 and 2 each intermediate pipe (inlet-pipe and outlet-pipe) has only one type of perforation, on the other hand in configuration-3, each intermediate pipe has two different types of perforation (of widely different porosities). The geometrical details of the configuration-3 are listed in Table 2.

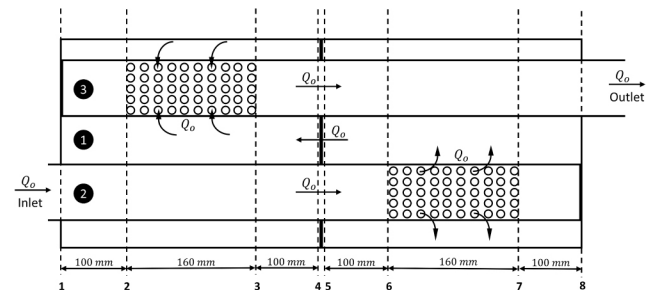


Figure 2. Schematic diagram of the configuration-2.

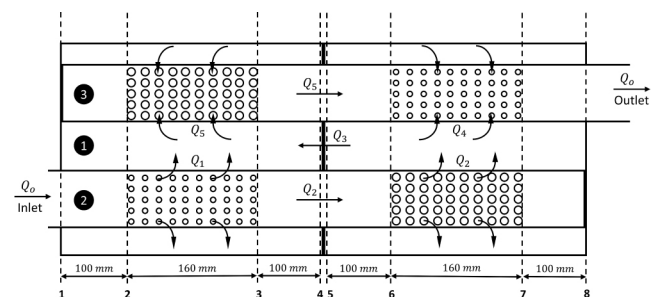


Figure 3. Schematic diagram of the configuration-3.

MEAN FLOW DISTRIBUTION AND BACK-PRESSURE CALCULATION

To account for the effect of the mean flow on the perforate impedance, evaluation of bias flow Mach number and grazing flow Mach number in perforations has been calculated here by means of the lumped flow-resistance network theory [3, 1]. The flow resistance R is defined as [5],

$$R = \frac{\Delta P}{Q | Q |} \quad (1)$$

where ΔP is the stagnation pressure drop across the element and Q is the volume flow rate passing through the element.

Configuration-1

In Configuration-1 shown in Fig. 1, mean flow Q_o enters through the inlet-pipe. As it encounters a plug, the whole flow passes through the perforation of the inlet-pipe and enters the first cavity. From the first cavity the flow Q_o passes through the two holes in the baffle plate and reaches the second cavity. The flow Q_o then enters the outlet-pipe through the perforation on the outlet-pipe from the second cavity. The exhaust that comes out from the outlet-pipe enters the tail pipe directly and vents out to the atmosphere.

One can observe that the flow through each element of the Configuration-1 is Q_o as there is no alternative path for the flow. This suggests that all the flow resistance elements are in series. The flow resistance network diagram for the Configuration-1 (Fig. 1) is shown in Fig. 4, where R_1 and R_3 are the flow resistances due to the perforations on the inlet-pipe and outlet-pipe, respectively, and R_2 is the flow resistance offered by the holes in the baffle plate.

Table 2. Geometric details of the configuration-3

S. No.	Parameter	Value
1	Shell Diameter (mm)	288
2	Total Length of the muffler (mm)	720
3	Inlet and Outlet pipe diameter (mm)	100
4	Length of each perforation (mm)	160
5	Baffle plate	Hole Diameter (mm)
		No. of holes
		Thickness of plate (mm)
6	Inlet-pipe left perforation and outlet-pipe right perforation	OAR
		Porosity
		Diameter of holes (mm)
		No. of Holes
7	Inlet-pipe right perforation and outlet-pipe left perforation	OAR
		Porosity
		Diameter of holes (mm)
		No. of Holes

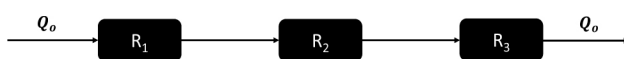


Figure 4. Flow network diagram for configuration-1 and 2.

The perforate flow resistance (R_{CE}) for cross flow is given by [5, 3, 1]

$$R_{CE} = \frac{\rho_o}{2(C_D S_P)^2} \quad (2)$$

where C_D is the coefficient of discharge of the orifices or holes, S_P is the perforate open area and ρ_o is the density of the medium. Eq. (2) is used to calculate all three resistances, R_1 , R_2 and R_3 making use of respective dimensions for the perforated area S_P from Table 2.

The total flow resistance (R_{total}) offered by the muffler is the series equivalent of the flow resistances shown in Fig. 4. Thus,

$$R_{total} = R_1 + R_2 + R_3 \quad (3)$$

and the total pressure drop (back-pressure, ΔP) is given by,

$$\Delta P = R_{total} \times Q_o^2 \quad (4)$$

For the dimensions listed in Table 1 and $Q_o = 0.355 \text{ m}^3/\text{s}$, ΔP works out to be

$$\Delta P = \begin{cases} 1.659 \text{ kPa} \\ 169.1 \text{ mm of water column} \\ 12.441 \text{ mm of Hg} \end{cases} \quad (5)$$

Configuration-2

Configuration-2, shown in Fig. 2, is similar to configuration-1 except for the flow reversal. In Configuration-2, the entire mean flow Q_o enters through the inlet-pipe and expands into the second cavity by passing through the inlet-pipe perforations. From the second cavity, the whole of mean flow Q_o passes through the two holes in the baffle plate and enters the first cavity. After reaching the first cavity the total mean flow enters the outlet-pipe through the perforations.

Similar to the Configuration-1, R_1 and R_3 are the flow resistances due to perforation on the inlet-pipe and the outlet-pipe, respectively and R_2 is the flow resistance offered by the two holes in the baffle plate. Total flow resistance offered by the Configuration-2 is given by Eq. (3), and the total back-pressure by Eq. (4). It can be observed that both the Configuration-1 and 2 offer the same resistance and exert the same back-pressure, but in reality Configuration-2 will have a slightly higher back-pressure than the Configuration-1, as the pressure drops due to the frictional drop across the duct length of inlet-pipe and outlet-pipe have been neglected here.

Configuration-3

The distinguishing feature of Configuration-3 is that the inlet-pipe and the outlet-pipe have two different kinds of perforations with widely different porosities as listed in Table 2. The flow resistance network diagram for Configuration-3 is shown in Fig. 5.

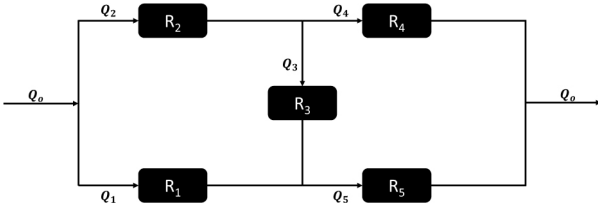


Figure 5. Flow network diagram for configuration-3.

As shown in Fig. 3, mean flow Q_o enters through the inlet-pipe and divides into two parts, Q_1 and Q_2 . The first part Q_1 passes through the left side perforation of the inlet-pipe and enters the first cavity, whereas the second part Q_2 passes through the right side perforations of the inlet-pipe and expands into the second cavity. The volume flow Q_2 divides and a part of it, Q_4 , enters the outlet-pipe through the right side perforation of the outlet-pipe. The other part of Q_2 , i.e. Q_3 , enters the first cavity through the two holes in the baffle plate and joins the flow Q_1 to form flow Q_5 . The flow Q_5 enters the outlet-pipe through the left side perforation of the outlet-pipe and joins the flow Q_4 to form the total flow Q_o .

For the circuit given in Fig. 5, applying Kirchhoff's first law and second law [5, 3, 1] we have:

$$Q_o = Q_1 + Q_2 \quad (6)$$

$$Q_2 = Q_3 + Q_4 \quad (7)$$

$$Q_5 = Q_1 + Q_3 \quad (8)$$

$$R_2 Q_2 |Q_2| + R_3 Q_3 |Q_3| - R_1 Q_1 |Q_1| = 0 \quad (9)$$

$$R_3 Q_3 |Q_3| + R_5 Q_5 |Q_5| - R_4 Q_4 |Q_4| = 0 \quad (10)$$

Thus we have five equations (Eqs. (6) to (10)) and five variables (Q_1, Q_2, Q_3, Q_4 and Q_5). This system of equations can be solved in terms of Q_o using the 'f solve' built-in function in MATLAB.

Considering the $R_2 - R_4$ path for the mean flow in Fig. 5 yields the following expression for back-pressure (ΔP):

$$\Delta P = R_2 Q_2 |Q_2| + R_4 Q_4 |Q_4| \quad (11)$$

Solving for all the five variables (Q_1, Q_2, Q_3, Q_4 and Q_5) and substituting the value of Q_2 and Q_4 in Eq. (11), we get

$$\Delta P = \begin{cases} 1.223 \text{ kPa} \\ 124.7 \text{ mm of water column} \\ 9.174 \text{ mm of Hg} \end{cases} \quad (12)$$

MODELING OF THE PERFORATE IMPEDANCE

Acoustic impedance of perforates is the most important parameter in the analysis of perforated-element mufflers. It depends upon many factors like porosity, bias flow and grazing flow Mach number, diameter and thickness of the tube and diameter of the holes. Adopting the impedance analogy, the non-dimensional specific acoustic impedance ζ is given by [1],

$$\zeta = \frac{p_o - p_i}{u \rho_o c_o} \quad (13)$$

where p_i and p_o are the complex acoustic pressure inside and outside of the perforate, u is the radial acoustic velocity through the perforate, and ρ_o and c_o are the density of the medium and the velocity of sound in the medium, respectively. Fig. 6 shows the state variables in a perforated duct where the arrow shows the direction in which acoustic velocity is considered to be positive.

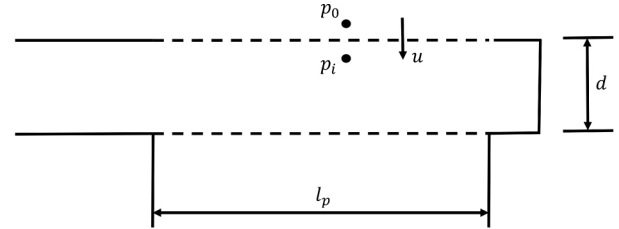


Figure 6. Acoustic state variables in a perforated duct.

Elnady *et al.* [5] provide an expression for the acoustic impedance of a perforate which is modified in Appendix A. The bias flow Mach number term was originally given by Zinn [6] and the modification was pointed out by Lahiri [7]. In addition to this, the effect of grazing flow Mach number suggested by Boden and Kabral is also included in the reactance term [8].

Following the admittance analogy, Rayleigh Conductivity (or acoustic conductivity, K_R) of an orifice is defined as [9, 7],

$$K_R = i \omega \rho_o \frac{Q}{p_o - p_i} \quad (14)$$

where $i = \sqrt{-1}$, ω is the angular frequency of the acoustic waves, and Q is the acoustic volume velocity through the orifice defined as $Q = Au$, where A is the area of the orifice.

Rayleigh conductivity for an orifice with bias flow, derived by Howe [9], is given by

$$K_R = 2r(\gamma + i\delta) \quad (15)$$

Here, $e^{i\omega t}$ convention is assumed for the time dependence, r is the radius of the orifice, and $\gamma + i\delta$ is defined as,

$$\gamma + i\delta = 1 + \frac{\frac{\pi}{2}I_1(St)e^{-St} + iK_1(St)\sinh(St)}{St\left[\frac{\pi}{2}I_1(St)e^{-St} - iK_1(St)\cosh(St)\right]} \quad (16)$$

where St is the Strouhal number, and I_n and K_n are the modified Bessel functions of order n of first and second kind, respectively. Strouhal number is given by $St = \omega r / U_B$, with U_B as the mean velocity through the orifice. This model assumes the orifice thickness to be negligible.

Jing and Sun [10], added the thickness term to the Howe's model, so the Rayleigh conductivity for a circular orifice of finite thickness is given by,

$$K_R = 2r \left(\frac{1}{\gamma + i\delta} + \frac{2l}{\pi r} \right)^{-1} \quad (17)$$

where l is the thickness of the orifice.

Luong *et al.* [11] arrived at a simplified expression for the Rayleigh conductivity (for both infinitesimal wall thickness and finite wall thickness). For the finite wall thickness, they obtained

$$K_R = \frac{A}{l_{eff}} \left(\frac{\omega l_{eff} / U_B}{\omega l_{eff} / U_B - \frac{i}{C_c^2}} \right) \quad (18)$$

where $l_{eff} = l + 2l'$, l' is the end correction generally given by $l' = 0.85r$, and $C_c = 0.75$, is the contraction coefficient of the orifice. For infinitesimal thin wall, replace l_{eff} by $\pi r / 2$ in Eq. (18).

Using Eq. (14) and substituting $\omega = kc_o$ and $Q = Au$, results in the following expression for Rayleigh Conductivity:

$$K_R = ikc_o \rho_o \frac{Au}{p_o - p_i} \quad (19)$$

Making use of Eq. (13) for the non-dimensionalized specific perforate impedance ζ and expressing it in terms of Rayleigh conductivity, we get ζ for a single hole, which when divided by porosity gives the following expression for the multiple-hole perforate:

$$\zeta = \frac{ikA}{\sigma K_R} \quad (20)$$

Substituting K_R from Luong *et al.*'s Eq. (18) into the above equation, we get

$$\zeta = \frac{ikl_{eff}}{\sigma} \left[\frac{\frac{\omega l_{eff}}{U_B} - \frac{i}{C_c^2}}{\frac{\omega l_{eff}}{U_B}} \right] \quad (21)$$

which on simplification becomes

$$\zeta = \frac{ik}{\sigma} \left[l_{eff} - \frac{iU_B}{\omega C_c^2} \right], \quad (22)$$

Using $k = \omega / c_o$ and $M_b = U_B / c_o$, Eq. (22) yields

$$\zeta = \frac{M_b}{\sigma C_c^2} + i \frac{kl_{eff}}{\sigma} \quad (23)$$

Substituting $C_c = 0.75$ [11,7], and incorporating viscous term given by Ingard [12] and grazing flow effect recommended by Dean [13], we get,

$$\zeta = \left\{ \frac{\sqrt{8\mu\rho_o\omega}}{\rho_o c_o \sigma} \left(1 + \frac{t}{d_h} \right) + \frac{1.78M_b}{\sigma} + \frac{0.3M_g}{\sigma} \right\} + \dots + \frac{ikl_{eff}}{\sigma} \quad (24a)$$

where, μ is the dynamic viscosity of the medium, σ is the porosity, t is the thickness of the perforated pipe, d_h is the diameter of the holes and M_b and M_g are the bias flow Mach number and grazing flow Mach number, respectively.

Expression (24a) may further be improved upon by incorporating the effect of grazing flow on inertance from Ref. [8] and the radiation resistance term [1]. Thus, we obtain a simplified, yet all inclusive, expression for the non-dimensionalized specific acoustic impedance of a perforate:

$$\zeta = \left\{ \frac{\sqrt{8\mu\rho_o\omega}}{\rho_o c_o \sigma} \left(1 + \frac{t}{d_h} \right) + \frac{k^2 r^2}{2\sigma} + \frac{1.78M_b}{\sigma} + \dots + \frac{0.3M_g}{\sigma} \right\} + i \left\{ \frac{kl_{eff}}{\sigma} - \frac{0.3M_g}{\sigma} \right\} \quad (24b)$$

This simplified expression for the acoustic impedance of a perforate is henceforth referred as the proposed perforate impedance model. The Luong *et al.*'s model [11] considers only the effect of bias flow, whereas the proposed perforate impedance model incorporates grazing flow term as per Dean [13] and viscous dissipation term as per Ingard [12], as well as the radiation resistance and reactance of the orifice.

1-D (PLANE WAVE) ANALYSIS

The Integrated Transfer Matrix (ITM) method [4] can be used to obtain the required transfer matrix between the inlet state variables (acoustic pressure and acoustic mass velocity) and the outlet state variables. The given muffler configurations are divided into a number of sections based upon perforated surfaces and baffle. For each section an ' $n \times n$ ' transfer matrix can be obtained relating the upstream and downstream state variables where n depends on the number of domains a segment has. For example, if a segment has two domains (as in configuration-1) then a 4×4 matrix will relate the