



AEROSPACE INFORMATION REPORT

AIR1168™/1**REV. B**

Issued	1990-03
Reaffirmed	2004-06
Revised	2019-04
Stabilized	2025-02

Superseding AIR1168/1A

Thermodynamics of Incompressible and Compressible Fluid Flow

RATIONALE

This document has been determined to contain basic and stable technology which is not dynamic in nature.

STABILIZED NOTICE

This document has been declared "STABILIZED" by SAE AC-9 Aircraft Environmental Systems Committee and will no longer be subjected to periodic reviews for currency. Users are responsible for verifying references and continued suitability of technical requirements. Newer technology may exist.

SAE Executive Standards Committee Rules provide that: "This report is published by SAE to advance the state of technical and engineering sciences. The use of this report is entirely voluntary, and its applicability and suitability for any particular use, including any patent infringement arising therefrom, is the sole responsibility of the user."

SAE reviews each technical report at least every five years at which time it may be revised, reaffirmed, stabilized, or cancelled. SAE invites your written comments and suggestions.

Copyright © 2025 SAE International

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, or used for text and data mining, AI training, or similar technologies, without the prior written permission of SAE.

TO PLACE A DOCUMENT ORDER: Tel: 877-606-7323 (inside USA and Canada)
Tel: +1 724-776-4970 (outside USA)
Fax: 724-776-0790
Email: CustomerService@sae.org
<http://www.sae.org>

SAE WEB ADDRESS:

Full document details: SAE AIR1168_1B

For more information on this standard, visit
<https://www.sae.org/standards/content/AIR1168/1B/>

**Documents**

Document Sharing Hub

PREFACE

This document is one of 14 Aerospace Information Reports (AIR) of the Third Edition of the SAE Aerospace Applied Thermodynamics Manual. The manual provides a reference source for thermodynamics, aerodynamics, fluid dynamics, heat transfer, and properties of materials for the aerospace industry. Procedures and equations commonly used for aerospace applications of these technologies are included.

In the Third Edition, no attempt was made to update material from the Second Edition nor were SI units added. However, all identified errata were corrected and incorporated and original figure numbering was retained, insofar as possible.

The SAE AC-9B Subcommittee originally created the SAE Aerospace Applied Thermodynamics Manual and, for the Third Edition, used a new format consisting of AIR1168/1 through AIR1168/10. AIR1168/11 through AIR1168/14 were created by the SAE SC-9 Committee.

The AIRs comprising the Third Edition are shown below. Applicable sections of the Second Edition are shown parenthetically in the third column.

AIR1168/1	Thermodynamics of Incompressible and Compressible Fluid Flow	(1A,1B)
AIR1168/2	Heat and Mass Transfer and Air-Water Mixtures	(1C,1D,1E)
AIR1168/3	Aerothermodynamic Systems Engineering and Design	(3A,3B,3C,3D)
AIR1168/4	Ice, Rain, Fog, and Frost Protection	(3F)

AIR1168/5	Aerothermodynamic Test Instrumentation and Measurement	(3G)
AIR1168/6	Characteristics of Equipment Components, Equipment Cooling System Design, and Temperature Control System Design	(3H,3J,3K)
AIR1168/7	Aerospace Pressurization System Design	(3E)
AIR1168/8	Aircraft Fuel Weight Penalty Due to Air Conditioning	(3I)
AIR1168/9	Thermophysical Properties of the Natural Environment, Gases, Liquids, and Solids	(2A,2B,2C,2D)
AIR1168/10	Thermophysical Characteristics of Working Fluids and Heat Transfer Fluids	(2E,2F)
AIR1168/11	Spacecraft Boost and Entry Heat Transfer	(4A,4B)
AIR1168/12	Spacecraft Thermal Balance	(4C)
AIR1168/13	Spacecraft Equipment Environmental Control	(4D)
AIR1168/14	Spacecraft Life Support Systems	(4E)

F.R. Weiner, formerly of Rockwell International and past chairman of the SAE AC-9B Subcommittee, is commended for his dedication and effort in preparing the errata lists that were used in creating the Third Edition.

Table of Contents

SECTION 1A - INCOMPRESSIBLE FLUID FLOW	1
1. INTRODUCTION	1
1.1 Scope	1
1.2 Nomenclature	1
2. BASIC FLUID MECHANICS	3
2.1 Continuity Equation	3
2.2 Momentum Equation	4
2.3 Energy Equation	4
2.4 Viscosity Concepts	4
2.5 Boundary Layer Flow	5
2.6 Total Pressure Loss Due to Change of Fluid Velocity Profile	8
3. SPECIFIC PRESSURE LOSS DATA:	9
3.1 Pressure Losses, Straight Ducts	9
3.2 Pressure Losses, Elbows	15
3.3 Pressure Losses, Duct Area Changes	31
3.4 Pressure Losses, Duct Branches	40
3.5 Pressure Losses, Internal Installations	46
3.6 Pressure Losses, Exits To Compartments	49
3.7 Example of Pressure Loss Calculations	51
4. REFERENCES	53
SECTION 1B - THERMODYNAMICS AND COMPRESSIBLE FLOW	59
1. INTRODUCTION	59
1.1 Definition and Scope	59
1.2 Nomenclature	59
2. FIRST LAW OF THERMODYNAMICS	61
3. IDEAL OR PERFECT GASES AS WORKING FLUIDS	63
3.1 Equation of State	63
3.2 Energy Equations of a Ideal Gas	63
3.3 Equations for Mixtures of Ideal Gases	70
4. THE SECOND LAW OF THERMODYNAMICS	71
4.1 Carnot Cycle and Available Energy	71
4.2 Methods of Graphical Representations	71
5. REAL GAS CHARACTERISTICS	73
5.1 Van der Waals Equation	74
5.2 Beattie-Bridgeman	74
5.3 Generalized Reduced Coordinate System	75
5.4 Changes of Phase	76
5.5 Triple Point and Triple Line	76
5.6 Latent Heat	76
5.7 Critical Point	76
6. THERMODYNAMICS OF HIGH-VELOCITY GAS FLOWS	76
6.1 Introduction	76
6.2 Steady-Flow Energy Equation	78
6.3 Sonic Velocity and Mach Number	79
6.4 One-Dimensional Energy Equation for Steady Flow Without Heat Transfer	80

6.5	Total Temperature, Static Temperature, Velocity, and Mach Number Relations	81
6.6	Pressure Ratio, Density Ratio, and Mach Number Relations	83
6.7	Continuity Relations	85
6.8	Illustrative Examples	86
6.9	Forces on Internal Flow Passages	89
6.10	General Duct Flow	90
6.11	Constant Area Duct Flow With Heat Transfer And No Friction	91
6.12	Constant Area Duct Flow with Friction and No Heat Transfer	95
6.13	Constant Area Duct Flow with Heat Transfer and Friction	99
6.14	Mach Waves and Prandtl-Meyer Flow	100
6.15	Normal, Oblique, and Conical Shock Waves	104
6.16	Subsonic Diffusers	108
6.17	Auxiliary Inlets and Diffusers	110
6.18	Auxiliary Outlets	123
6.19	Supersonic Nozzles and Sonic Venturis	127
6.20	Duct Component Losses in Compressible Flow	129
6.21	Real Gas Effects	136
7.	REFERENCES	142

List of Figures

Figure 1A-1 -	Incompressible Flow, Pressure Loss Error	1
Figure 1A-2 -	Boundary Layers and Velocity Distribution	6
Figure 1A-3 -	Pipe Velocity and Friction Factors	7
Figure 1A-4 -	Relative Roughness, Circular Ducts	7
Figure 1A-5 -	Friction Factor Versus Reynolds Number (Ref. 14)	10
Figure 1A-6 -	Reynolds Number in Gases	11
Figure 1A-7 -	Reynolds Number in Liquids	12
Figure 1A-8 -	Friction Factor Versus Weight Flow Per Unit Diameter, Air in Smooth Round Ducts	16
Figure 1A-9 -	Bend Loss	16
Figure 1A-10 -	Loss Coefficient Versus Reynolds Number for 90 deg Radius Bends, Circular Ducts. (Data for $N_{Re} > 10^5$ From Ref. 1, for $N_{Re} < 10^5$ From Ref. 28)	17
Figure 1A-11 -	Correction Factor for Bends Other Than 90 deg for Elliptical and Circular Ducts (From Ref. 12, Fig. 16)	17
Figure 1A-12 -	Loss Coefficient for 90 deg Radius Bends, Elliptical Ducts, $N_{Re} = 150,000$ (From Ref. 1, Fig. 3a)	18
Figure 1A-13 -	Loss Coefficient for Radius Bends, Elliptical Ducts, $N_{Re} = 300,000$ (From Ref. 1, Fig. 3b)	18
Figure 1A-14 -	Loss Coefficient for 90 deg Bends, Elliptical Ducts, $N_{Re} = 600,000$ (From Ref. 1, Fig. 3c)	19
Figure 1A-15 -	Loss Coefficient for Miter Bends, Circular Ducts, $N_{Re} < 1000$ (From Ref. 12)	19
Figure 1A-16 -	Loss Coefficient Versus Reynolds Number for 90 deg Radius Elbows, Square Ducts (From Ref. 1, Fig. 2; Ref. 9)	20
Figure 1A-17 -	Correction Factor for Bends Other Than 90 deg, Rectangular and Square Ducts (From Ref. 5)	20
Figure 1A-18 -	Loss Coefficient for 90 deg Bends, Rectangular Ducts, $N_{Re} = 100,000$ (From Ref. 1, Fig. 2a)	21
Figure 1A-19 -	Loss Coefficient for 90 deg Radius Bends, Rectangular Ducts, $N_{Re} = 300,000$ (From Ref. 1, Fig. 2b)	21
Figure 1A-20 -	Loss Coefficient for 90 deg Radius Bends, Rectangular Ducts, $N_{Re} = 600,000$ (From Ref. 1, Fig. 2c)	22
Figure 1A-21 -	Loss Coefficient for Square Mitered Elbow (From Ref. 13)	22
Figure 1A-22 -	Effect of Exit Duct on 90 deg Elbow (Data From Ref. 5)	23
Figure 1A-23 -	Correction Factor for Bends Other Than 90 deg for Bends Without Exit Ducts (Data From Refs. 5 and 12)	24
Figure 1A-24 -	Loss Coefficient for Compound U-bends, Circular Ducts (Reduced From Ref. 3, Figs. 27-32; Ref. 8, Fig. 30)	24
Figure 1A-25 -	Loss Coefficient for 90 deg Offset Bends, Circular Ducts (Reduced From Ref. 3, Figs. 39-44; Ref. 8, Fig. 32)	25
Figure 1A-26 -	Loss Coefficient for Z-bends, Circular Ducts (Reduced From Ref. 3, Figs. 33-38; Ref. 8, Fig. 31)	25
Figure 1A-27 -	Turning Vanes Geometry	26
Figure 1A-28 -	Number of Vanes for Thin Circular Arc Vanes, $c=2r \sin(\theta/2)$, $n=(5b/c) [\sin(\theta/2)] - 1$ (From Ref. 1)	27
Figure 1A-29 -	Angle of Attack for Thin Circular Arc Vanes (Calculated From Ref. 13)	27
Figure 1A-30 -	Thin Vanes, Noncircular Profile, Coordinates. (Data From Ref. 13)	28

Figure 1A-31 - Bend Angle and Gap to Chord Ratio. *The discrepancy in the angle of attack curve (shown by the broken diagonal line) is apparently a result of different methods used in developing the vane profiles. (From Ref. 1)	28
Figure 1A-32 - Thick Vanes (Calculated From Ref. 29)	29
Figure 1A-33 - Loss Coefficient for Thin Circular Arc Vanes	30
Figure 1A-34 - Effect of Gap to Chord Ratio on Thin Circular Arc Vanes (From Refs. 13 and 15)	30
Figure 1A-35 - Loss Coefficient for Thin Noncircular Arc Vanes (From Ref. 1)	30
Figure 1A-36 - Loss Coefficient for 90 deg Thick Vane Bends (From Ref. 29)	30
Figure 1A-37 - Sketch of Inside and Outside Radii with Splitters	31
Figure 1A-38 - Pressure Loss Correction Factor for Transitional Elbows. Note: Radius Ratio of Constant Area Bend = $r_i/b_1 + 1/2$, $r_o = r_i + b_2$, $a_1 = a_2$ (From Ref. 1)	32
Figure 1A-39 - Sketch of Typical Types of Expansion and Contraction Areas	32
Figure 1A-40 - Loss Coefficient of a Sudden Expansion (From Ref. 24)	33
Figure 1A-41 - Loss Coefficient of a Sudden Contraction	34
Figure 1A-42 - Sketch of a Typical Gradual Contraction	34
Figure 1A-43 - Total Loss Coefficient Versus Expansion Angle	34
Figure 1A-44 - Sketch of a Straight Wall Conical Diffuser	35
Figure 1A-45 - Sketch of a Straight Wall Rectangular Diffuser	35
Figure 1A-46 - Curved Wall Diffuser	35
Figure 1A-47 - Sketch of a Truncated Diffuser	36
Figure 1A-48 - Diffuser Expansion Factor (curve A-straight wall, conical and square; curve B-straight wall, two dimensional; curve C-curved wall, conical)	37
Figure 1A-49 - Sketch of a Diffuser Followed by a Resistance Unit	38
Figure 1A-50 - Equivalent Expansion Angle for a Diffuser Followed by a Resistance Unit (From Ref. 26)	38
Figure 1A-51 - Expansion Factor for a Diffuser Followed by a Resistance Unit (From Ref. 26)	39
Figure 1A-52 - Sketch of an Orifice in a Duct	39
Figure 1A-53 - Loss Coefficient for an Orifice in a Duct; $A_1 = A_3$ (From Ref. 45, Table 11)	40
Figure 1A-54 - Loss Coefficient for an Orifice in a Duct: $K_t = [(1/C_c) - (A_2/A_3)]^2$	41
Figure 1A-55 - Contraction Coefficient of an Orifice (From Ref. 45, Tables 8 and 13)	41
Figure 1A-56 - Approximate Pressure Loss Through Several ASME Flow Metering Devices P_{s1} = Upstream Static Pressure; P_{s2} = Throat Static Pressure (From Ref. 35, Fig. 5)	42
Figure 1A-57 - Diverging Branch, Functions for Computing Loss Coefficient (Ref. 7)	42
Figure 1A-58 - Converging Branch, Functions for Computing Loss Coefficient (From Ref. 7)	44
Figure 1A-59 - Sketch of a Typical Manifold	44
Figure 1A-60 - Loss Coefficients, Gate Valves (From Ref. 12)	46
Figure 1A-61 - Loss Coefficient, Butterfly Valves (From Ref. 65)	47
Figure 1A-62 - Loss Coefficients for Screens and Grids (From Ref. 56)	48
Figure 1A-63 - Sketch of Heat Exchanger Set at an Angle to the Entrance Duct Centerline	48
Figure 1A-64 - Effect of Angle on Heat Exchangers (From Ref. 6)	48
Figure 1A-65 - Example of a Duct System for Flow Loss Calculations	51
Figure 1B-1 - Nonflow Work, Compressor Cylinder and Piston	64
Figure 1B-2 - Steady-Flow Compression or Expansion	69
Figure 1B-3 - T-S Diagram	72
Figure 1B-4 - Carnot Cycle	72

Figure 1B-5 -	Brayton Cycle	72
Figure 1B-6 -	Rankine Vapor Cycle in Refrigeration	73
Figure 1B-7 -	Vapor Cycle in Refrigeration	73
Figure 1B-8 -	Compressibility Factor	77
Figure 1B-9 -	Solid and Liquid Phases with Changing P, V, T	78
Figure 1B-10 -	Temperature Ratio as a Function of Mach Number	82
Figure 1B-11 -	Velocity-Temperature Parameter as a Function of Mach Number	83
Figure 1B-12 -	Pressure Ratio as a Function of Mach Number	84
Figure 1B-13 -	Weight Flow Parameter, $w\sqrt{T_1}/P_1 A$, as a Function of Mach Number	86
Figure 1B-14 -	Weight Flow Parameter, $w\sqrt{T_1}/P_1 A$, as a Function of Mach Number	87
Figure 1B-15 -	Flow in a Duct	89
Figure 1B-16 -	Control Volume	91
Figure 1B-17 -	Mach Number Function in a Constant Area Duct	92
Figure 1B-18 -	Control Volume	95
Figure 1B-19 -	Friction Factor in Duct Flow	97
Figure 1B-20 -	Effect of Wall Friction on Duct Flow Properties (The asterisk denotes integration of differential forms of flow property equations.)	98
Figure 1B-21 -	Mach Wave Formation	101
Figure 1B-22 -	Infinitesimal Flow Deflection	102
Figure 1B-23 -	Expansion Fan	103
Figure 1B-24 -	Prandtl-Meyer Function	103
Figure 1B-25 -	Notation for Normal Shock Wave Analysis	104
Figure 1B-26 -	Oblique Shock Diagram	106
Figure 1B-27 -	Geometric Properties of Diffusers Reported in Ref. 14 ($A_1/A_2 = 0.510$ rake station; $r_1 = 1.50$ in. Diffuser identification system: (1) first number refers to maximum total divergence angle, (2) second number refers to throat extension in terms of inlet throat radii, (3) letter C refers to conical diffuser).	110
Figure 1B-28 -	Effects of Diffuser Geometry Variations and Boundary Layer Thickness on Performance	111
Figure 1B-29 -	Types and Locations of Auxiliary Inlets. (a) Ram Scoop; (b) Flush Inlet	112
Figure 1B-30 -	Characteristics of Auxiliary Inlets Operating in Freestream and in Boundary Layer at Subsonic Flight Speeds	113
Figure 1B-31 -	Pressure Recovery in a Turbulent Boundary Layer Including Normal Shock and Mixing Losses (Ref. 19)	114
Figure 1B-32 -	Mass Flow in Turbulent Boundary Layer Relative to That in Freestream Tube at Same Height; $N=7$, $u/U=(y/\delta)^{1/N}$ (Ref. 19)	115
Figure 1B-33 -	Comparison of NACA Diverging Wall Flush Inlet with Parallel Wall Flush Inlet (Refs. 20 and 34)	115
Figure 1B-34 -	Ram Recovery and Drag Coefficient for Flush Inlets	116
Figure 1B-35 -	Characteristics of Auxiliary Inlets Operating at Supersonic Flight Speeds	117
Figure 1B-36 -	Effect of Throat Mixing Sections on Diffuser Recovery (β = mixing section divergence angle).	119
Figure 1B-37 -	Discharge Coefficient as a Function of Discharge Flow Ratio for Inclined Square Duct Outlets (Ref. 35)	124
Figure 1B-38 -	Comparison of Discharge Coefficients of Square and Circular Inclined Outlets (Ref. 35)	124
Figure 1B-39 -	Discharge Coefficient as a Function of Discharge Flow Ratio for Square Outlets with Curved Axis (Ref. 35)	125

Figure 1B-40 - Discharge Coefficient as a Function of Discharge Flow Ratio for 60 deg Inclined Outlets With Various Radii at Trailing Edge (Ref. 35)	125
Figure 1B-41 - Discharge Coefficient as a Function of Discharge Flow Ratio for Recessed Outlets (Ref. 35)	126
Figure 1B-42 - Flow Characteristics of a Converging-Diverging Nozzle	128
Figure 1B-43 - Characteristics of a Particular Diverging Nozzle: $W = 31.9 C A P_1 / \sqrt{T_1} =$ lb/min, where C = flow factor; A = throat area, in. ² ; P_1 = inlet total pressure, psi; T_1 = inlet total temperature, °R (From General Electric Co. Bulletin D.F.81127)	130
Figure 1B-44 - Total Pressure Losses in Mitre Bends (circular constant area duct)	132
Figure 1B-45 - Total Pressure Losses in Radius Elbows (circular area ducts, entrance Mach No. = 0.5)	132
Figure 1B-46 - Total Pressure Losses in Radius Elbows (circular constant area ducts, $R/D=1.0$).	133
Figure 1B-47 - Total Pressure Losses in Radius Elbows (circular constant area ducts, $R/D=2.0$).	133
Figure 1B-48 - Total Pressure Losses in Radius Elbows (circular constant area ducts, $R/D=3.0$).	134
Figure 1B-49 - Total Pressure Losses in U-Bends	134
Figure 1B-50 - Total Pressure Losses in Z-Bends	135
Figure 1B-51 - Total Pressure Losses In Offset Bends	135
Figure 1B-52 - Weight-Flow Division for a 90 deg Flow Split	136
Figure 1B-53 - Real Gas Effects in Stagnation Regions of Flight Vehicles	137
Figure 1B-54 - Real Gas Effects: (a) isentropic total-to-static pressure ratio; (b) total pressure ratio across shock (conditions: $q_o = 1000$ lb/ft ² abs, $T_{so} = 400^\circ\text{R}$)	140
Figure 1B-55 - Station Nos.	140

List of Tables

Table 1A-1 - Table for Unit Conversion Factor C for ρg (lb/ft ³)	8
Table 1A-2 - Table for ρg of Air	9
Table 1A-3 - Coordinates for Reynolds Number Nomograph for Gases (Fig. 1A-6) ¹	13
Table 1A-4 - Coordinates for Reynolds Number Nomograph for Liquids (Fig. 1A-7) ¹	14
Table 1A-5 - Angular Displacement Versus Loss Coefficient ¹	33
Table 1A-6 - Values of $C' = \text{Constant}$	50
Table 1A-7 - Example of Flow Loss Calculation	52
Table 1B-1 - Conversion Factors	62
Table 1B-2 - Characteristic Constants for Gases	64
Table 1B-3 - Summary of Ideal Gas Equations for Nonflow Processes (for a unit weight of fluid)	68
Table 1B-4 - Constants a and b for Gases, Van der Waals	74
Table 1B-5 - Beattie-Bridgeman Constants	75
Table 1B-6 - P_{cr} and T_{cr} for Some Common Cases	75
Table 1B-7 - Triple Point Data	78
Table 1B-8 - Comparison of Perfect and Imperfect Gases	139

COMMON ABBREVIATIONS

abs	— Absolute
AFTR	— Air Force Technical Report
AIR	— Aerospace Information Report
ASTIA	— Armed Services Technical Information Agency
atm	— Atmosphere
av	— Average
Btu	— British Thermal Unit
cal	— Calorie
comp	— Compressible
cr	— Critical
<i>d</i>	— Differential notation
deg	— Degree (angle)
Dept.	— Department
dia	— Diameter
Eng	— Engineering
Eq./Eqs.	— Equation/Equations
°F	— Degrees Fahrenheit
Fig./Figs.	— Figure/Figures
fpm	— Feet per minute
fps	— Feet per second
ft	— Feet
ft/min	— Feet per minute
ft/sec	— Feet per second
Hg	— Mercury
hp	— Horsepower
hr	— Hour
in.	— Inch
in. ²	— Square inches
in. Hg	— Inches of mercury
in. Hg abs	— Inches of mercury absolute
in. H ₂ O	— Inches of water
in. H ₂ O abs	— Inches of water absolute
incomp	— Incompressible
isen	— Isentropic
K	— Degrees Kelvin
lb	— Pound
lb/in. ²	— Pounds per square inch
lb/in. ² abs	— Pounds per square inch absolute
lb/ft ²	— Pounds per square foot

lb/ft ² abs	— Pounds per square foot absolute
lb/min	— Pounds per minute
lb/hr	— Pounds per hour
lb/sec	— Pounds per second
ln	— Natural logarithm to the base <i>e</i>
max	— Maximum
min	— Minimum
NACA	— National Advisory Committee for Aeronautics
no.	— Number
%	— Percent
Par./Pars.	— Paragraph/Paragraphs
p.	— Page
pp.	— Pages
psf	— Pounds per square foot
psfa	— Pounds per square foot absolute
psi	— Pounds per square inch
psia	— Pounds per square inch absolute
psig	— Pounds per square inch gage
PTC	— Power Test Code
°R	— Degrees Rankine
R.A.E.	— Royal Aircraft Establishment (now Royal Aerospace Establishment)
Ref./Refs.	— Reference/References
RM	— Research Memorandum
rpm	— Revolutions per minute
SAE	— Society of Automotive Engineers
Sat	— Saturated
S.L.	— Sea level
sta.	— Station
sec	— Second
TN	— Technical Note
TR	— Technical Report
tr	— Triple
Trans. ASME	— Transactions of the American Society of Mechanical Engineers
W	— Watts
WADC	— Wright Air Development Center (now, Wright Research and Development Center)
Δ	— Increment
Σ	— Summation

SECTION 1A - INCOMPRESSIBLE FLUID FLOW

1. INTRODUCTION

1.1 Scope

The fluid flow treated in this section is isothermal, subsonic, and incompressible. The effects of heat addition, work on the fluid, variation in sonic velocity, and changes in elevation are neglected. An incompressible fluid is one in which a change in pressure causes no resulting change in fluid density. The assumption that liquids are incompressible introduces no appreciable error in calculations, but the assumption that a gas is incompressible introduces an error of a magnitude that is dependent on the fluid velocity and on the loss coefficient of the particular duct section or piece of equipment. Fig. 1A-1 shows the error in pressure drop resulting from assuming that air is incompressible.

With reasonably small loss coefficients and the accuracy that is usually required in most calculations, compressible fluids may be treated as incompressible for velocities less than Mach 0.2. At higher velocities and for large loss coefficients (K_t and $4fL/D$), compressible flow analysis should be used.

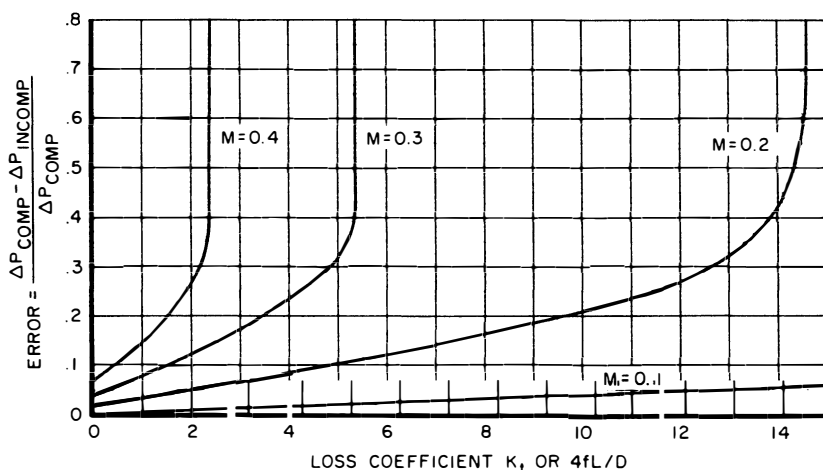


Figure 1A-1 - Incompressible Flow, Pressure Loss Error

1.2 Nomenclature

- a = Dimension of rectangle or ellipse, ft, in.
- A = Area, ft^2 , in^2
- b = Dimension of rectangle or ellipse, ft, in.
- b = Width, ft, in.
- c or C = Chord, ft, in.
- C = Correction factor, dimensionless

C	= Unit conversion factor (Table 1A-1), dimensionless
C	= Expansion factor in Fig. 1A-48, dimensionless
C'	= Constant (Table 1A-6), dimensionless
C_C	= Contraction coefficient (Eq. 1A-41), dimensionless
C_D	= Drag coefficient, dimensionless
D	= Diameter, ft, in.
D_e	= Equivalent diameter, ft, in.
f	= Fanning friction factor, dimensionless
f	= Friction coefficient, dimensionless
F	= Force, lb
g	= Gravitational acceleration, ft/sec ²
G	= Weight flow per unit area per unit time, lb/sec-ft ² , lb/min-in. ² , lb/hr-ft ²
K or K_f	= Pressure loss coefficient, dimensionless
L	= Length, ft, in.
m	= Mass (W/g), lb-sec ² /ft, lb
n	= Number of vanes or elbows
N_{Re}	= Reynolds number ($VD\rho g/\mu$, $Vx\rho g/\mu$), dimensionless
P	= Pressure, psi, lb/ft ² , in. Hg
P_s	= Static pressure, psi, lb/ft ²
P_t	= Total pressure, lb/ft ² , in. H ₂ O
q	= Dynamic pressure, $(1/2)\rho V^2$, lb/ft ² , in. Hg, in. H ₂ O
Q	= Volumetric flow rate, ft ³ /sec, ft ³ /min
r	= Radius, ft, in.
R_{fa}	= Ratio of free area to total area of exit, dimensionless
s	= Curvilinear distance, ft, in.
s or S	= Gap between vanes, ft, in.
S	= Area, ft ²
t	= Temperature, °F
T	= Temperature, absolute, °R
V	= Velocity, ft/sec, ft/min
w	= Mass flow rate, lb/sec, lb/min
x,y	= Distance, ft
X,Y	= Nomograph coordinates
α	= Angle of attack, deg
α	= Angle, deg
β	= Duct merge angle, deg
δ	= Duct merge angle, deg
ϵ	= Surface roughness, average, in.
η	= Efficiency, dimensionless
θ	= Angle, deg

λ	= Function of α , β or δ (See paragraphs 3.4.1 and 3.4.2)
μ	= Proportionality factor, dimensionless
μ	= Absolute viscosity, lb-sec/ft ²
ρ	= Mass density, lb-sec ² /ft ⁴
ρg	= Specific weight (density), lb/ft ³
τ	= Time, sec
ϕ	= Expansion angle, deg
ϕ	= Solidity, dimensionless

Subscripts:

<i>av</i>	= Average
<i>c</i>	= Compressible
<i>c</i>	= Inside chord
<i>COMP</i>	= Compressible
<i>D</i>	= Drag
<i>ef</i>	= Effective
<i>i</i>	= Inner
<i>INCOMP</i>	= Incompressible
<i>m</i>	= Mean
<i>max</i>	= Maximum
<i>o</i>	= Outer
<i>o</i>	= Freestream
<i>o</i>	= Outside
<i>p</i>	= Pressure
<i>r</i>	= Resistance
<i>t</i>	= Total
<i>t</i>	= Vena contracta
0.5	= Location where velocity is 1/2 of centerline velocity
90	= For 90° bend
1	= Point 1
2	= Point 2

Superscripts:

—	= Mean Value
---	--------------

2. BASIC FLUID MECHANICS

2.1 Continuity Equation

The continuity equation is generally expressed in the form

$$\rho AV = \text{constant} \quad (1A-1)$$

and reduces to

$$AV = \text{constant}$$

for ideal incompressible flow.

2.2 Momentum Equation

The form of the momentum equation most commonly used in fluid flow is

$$\sum F = \frac{d(mV)}{dt} \quad (1A-2)$$

where the summation of the forces is equal to the pressure, wall, and frictional forces on the fluid.

2.3 Energy Equation

The energy equation for incompressible frictionless flow can be expressed in a form called Bernoulli's equation:

$$P_s + (1/2)\rho V^2 = \text{constant} \quad (1A-3)$$

where the first term is the static pressure and the second term is the dynamic pressure of the fluid. The sum of the two is called the total pressure. To give physical meaning to the above pressures, they can be defined as follows:

- (a) Static Pressure: This is the pressure sensed by a probe moving with the same velocity as the fluid stream. Practically, it is measured by a pressure probe located normal to the fluid stream.
- (b) Dynamic Pressure: This is the static pressure rise that a fluid experiences when it is reversibly brought to rest, commonly noted as q .
- (c) Total Pressure: This is the pressure of a moving fluid if it is reversibly brought to rest. It is equal to the sum of the static and dynamic pressure.

2.4 Viscosity Concepts

In Bernoulli's equation, Eq. 1A-3, fluid frictional forces caused by viscosity have not been considered. In any real fluid these forces are caused by the shearing stresses that exist between the individual streamlines, where they are flowing at different velocities. For conditions of laminar flow, shearing stress is given by Newton's equation

$$\frac{F}{S} = \mu \frac{dV}{dx} \quad (1A-4)$$

where the proportionality factor μ is the property of the fluid called absolute viscosity and dV/dx is the velocity gradient normal to the flow. The shearing stress that exists in a fluid is therefore proportional to the viscosity and the velocity gradient that exists in the fluid normal to the direction of its flow. In most liquids and gases the absolute viscosity depends primarily on the temperature of the fluid. There is only a slight dependency of absolute viscosity on pressure, except when the fluid is near the critical point, or if the gases have dissociated.

2.5 Boundary Layer Flow

When a real fluid is in motion along a wall, the fluid particles adjacent to the wall are retarded by frictional forces. These retarding forces cause a velocity gradient near the wall, thereby defining a zone of flow called the "boundary layer."

The thickness of the boundary layer is usually defined by the two extremes of velocity: zero at the wall and 99% (or 99.5%) of local free stream velocity at the outer edge. Within this layer, the distribution of velocity is dependent on whether the flow is laminar or turbulent, and thereby it determines the velocity gradient at the wall and the shear force or "drag" on the wall. In a constant area duct, this drag force per unit surface area per unit length is balanced and thus is equal to a pressure loss or pressure drop per unit cross-sectional area per unit length.

2.5.1 Laminar-Turbulent

In laminar flow, the streamlines are orderly and undisturbed, and all shear developed is purely by molecular interaction. In turbulent flow, random eddies exist, causing momentum transfer across streamlines due to the irregular movement of finite masses of fluid.

2.5.2 Reynolds Number

In order to predict whether the boundary layer flow will be laminar or turbulent, the ratio of inertia forces to viscous forces must be considered. This ratio is, by definition, the Reynolds number (N_{Re}) and is equal to $Vx\rho g/\mu$ or $VD_e\rho g/\mu$. Flow conditions in a smooth closed channel will usually be laminar if the Reynolds number (based on hydraulic diameter) is less than 2000 and turbulent if above 4000. A "transition" type of flow exists between these values. For the case of a smooth flat plate submerged in an infinite flow field, the transition occurs at approximately $N_{Re} = 500,000$ to $1,000,000$ where the distance along the surface from the leading edge is the characteristic dimension used in the Reynolds number.

The growth in thickness of the boundary layer is illustrated in Fig. 1A-2. Over a flat plate (negligible pressure gradient), the laminar portion increases in thickness as the square root of the distance, and the turbulent region increases as the 0.8 power of the distance. Fig. 1A-2 also indicates the transition from laminar to turbulent flow as well as the limitation (beyond the entry region) of the growth of the boundary layer when the flow is confined to a duct or channel.

2.5.3 Friction Drag and Duct Pressure Loss Concepts

Eq. 1A-4 defined the shear stress that acts on the internal surface of a duct when a known velocity gradient exists at the wall. Downstream of the entrance region of a duct, the velocity gradient is independent of duct length, as indicated in the lower part of Fig. 1A-2. The shear stress causes a force along the wall proportional to the area in contact with the fluid, and this force is referred to as a drag or friction force.

In classical aerodynamic theory, the drag force acting on a surface A is expressed as

$$F_D = C_D q A, \text{ where } q = (1/2)\rho V^2$$

For a round duct, the surface area is

$$A = \pi DL$$

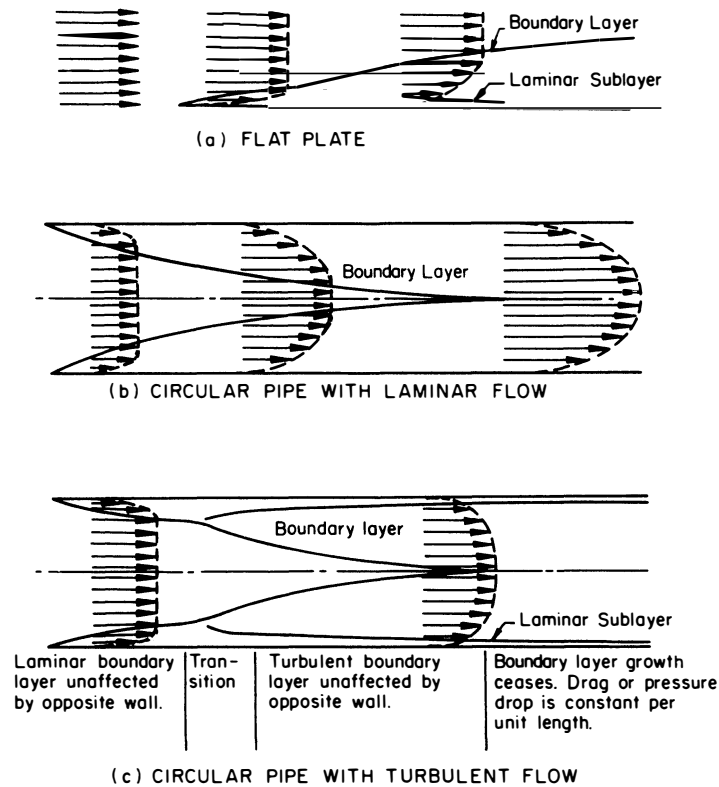


Figure 1A-2 - Boundary Layers and Velocity Distribution

The drag or friction force must be overcome by a pressure force corresponding to the duct pressure drop, so that

$$F_D = (P_{i1} - P_{i2}) \left(\frac{\pi D^2}{4} \right) = \Delta P_t \left(\frac{\pi D^2}{4} \right) \quad (1A-5)$$

Equating:

$$\Delta P_t = C_D \left(\frac{\pi D L}{\pi D^2 / 4} \right) q = 4 C_D \left(\frac{L}{D} \right) q = \frac{4 f L}{D} q \quad (1A-6)$$

In duct flow, C_D is replaced by the symbol of the Fanning friction factor, f . As mentioned above, the shear stress at the wall becomes a constant value for pipe flow in equilibrium, and therefore the friction factor is also a constant. Fig. 1A-3 shows the friction factor and the ratio of average to maximum velocity in pipe flow for these equilibrium conditions.

The roughness of a pipe changes the velocity profile at the wall. It also changes the shear stress and the friction factor. (See Fig. 1A-4.)

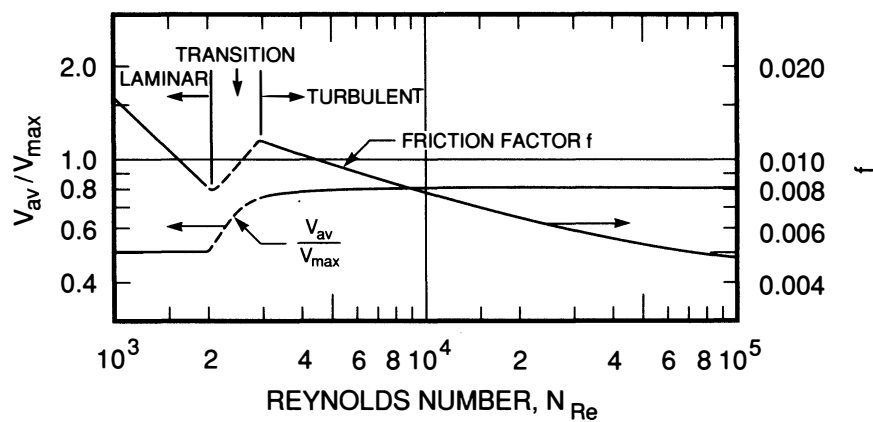


Figure 1A-3 - Pipe Velocity and Friction Factors



Figure 1A-4 - Relative Roughness, Circular Ducts

2.6 Total Pressure Loss Due to Change of Fluid Velocity Profile

In the discussion above, it was shown how the viscosity caused a velocity gradient at the fluid boundaries, with its associated friction drag and pressure loss. It may be logically reasoned from this that any change in the fluid velocity profile will cause a loss of energy and result in a corresponding pressure loss. Any change in direction, cross-sectional area, or shape will cause a loss in total pressure. In dealing with losses of this type, the following equation is used:

$$\Delta P_t = K_t (1/2) \rho V^2 = K_t q \quad (1A-7)$$

where K_t is the pressure loss coefficient for the change in direction, shape, or other type of loss. It should be emphasized that the loss coefficients are determined for one particular arrangement of ducting leading into and out of the device whose loss coefficient is being measured. Therefore, when combinations of devices are used, the loss coefficient of the system will not necessarily be the sum of the individual loss coefficients, but may be either larger or smaller. By careful selection of the location of bends, diffusers, and contractions to achieve the minimum change in velocity profile, the pressure loss can be reduced to a minimum. Examples of this would be the use of one 180 deg bend instead of two 90 deg bends separated by a straight section of pipe. Do not diffuse around a bend, and avoid large area changes at duct junctions.

In summary, the total losses in a system can be expressed by the equation

$$\Delta P_t = \sum K_t \left(\frac{1}{2} \rho V^2 \right) + \sum \frac{4fL}{D} \left(\frac{1}{2} \rho V^2 \right) \quad (1A-8)$$

where the dynamic or velocity pressure $(1/2)\rho V^2$ is evaluated at the reference area of the loss coefficient and the friction factor f . For convenience, the dynamic pressure $(1/2)\rho V^2$ can be evaluated from Tables 1A-1 and 1A-2 and the equation

$$q = \frac{1}{2} \rho V^2 = \frac{G^2}{\rho g C} \quad (1A-9)$$

where C = a unit conversion factor, dimensionless

Table 1A-1 - Table for Unit Conversion Factor C for ρg (lb/ft³)

Units of q	Units of G				
	lb/hr-ft ²	lb/min-ft ²	lb/sec-ft ²	lb/sec-in. ²	lb/min-in. ²
lb/ft ²	8.35×10^8	2.32×10^5	64.4	3.11×10^{-3}	11.20
lb/in. ²	1.202×10^{11}	3.34×10^7	9.28×10^3	0.447	1.61×10^3
in. H ₂ O	4.34×10^9	1.205×10^6	3.35×10^2	1.615×10^{-2}	58.14
in. Hg	5.90×10^{10}	1.64×10^7	4.56×10^3	0.2196	7.91×10^2

Table 1A-2 - Table for ρ_g of Air

	Units of P_s ($T = ^\circ R$)			
	in. Hg abs	psia lb/in. ² abs	psfa lb/ft ² abs	in. H ₂ O abs
lb/ft ³	$1.326 \frac{P_s}{T}$	$2.70 \frac{P_s}{T}$	$1.875 \times 10^{-2} \frac{P_s}{T}$	$9.76 \times 10^{-2} \frac{P_s}{T}$

A sample usage of Tables 1A-1 and 1A-2 is

$$\rho_g = 2.70 P_s / T, \text{ lb/ft}^3 \quad (\text{from Table 1A-2})$$

$$G = w/A, \text{ lb/min-in.}^2$$

$$C = 58.14 \quad (\text{from Table 1A-1})$$

$$q = \frac{TG^2}{2.70 P_s (58.14)}, \text{ in. H}_2\text{O}$$

where w is in lb/min, P_s in psia, T in $^\circ R$, and A in in.².

3. SPECIFIC PRESSURE LOSS DATA:

3.1 Pressure Losses, Straight Ducts

As noted in Par. 2.5.3, the pressure loss in straight ducts can be determined as follows:

$$\Delta P_t = \frac{4fL}{D} \left(\frac{1}{2} \right) \rho V^2 \quad (1A-10)$$

For most aircraft ducting, the ducts can be treated as smooth tubes, but to obtain a physical feel for the roughness where actual data are lacking, Fig. 1A-4 is presented for a few typical aircraft duct materials. The relative roughness presented in this figure is the mean roughness projection (ϵ) and is the arithmetic average surface deviation from the mean line of the surface. Friction factors are shown in Fig. 1A-5 for long pipe lengths, where entrance effects are small. Nomographs for determining Reynolds numbers are shown in Figs. 1A-6 and 1A-7 together with Tables 1A-3 and 1A-4 for gases and liquids. Usually pressure losses due to friction losses in most aircraft air duct systems are negligible compared to expansion and fitting losses. If the values using these approximate methods approach the values from the expansion and fitting losses, more specific friction loss data should be investigated. The data is available online and in such references as Reference 70.

In computing the pressure drop from Eq. 1A-10 and the Reynolds number for ducts of noncircular section, an equivalent diameter of the section must be used and is defined as

$$D_e = \frac{4 \times (\text{cross-sectional area})}{\text{wetted perimeter}} \quad (1A-11)$$

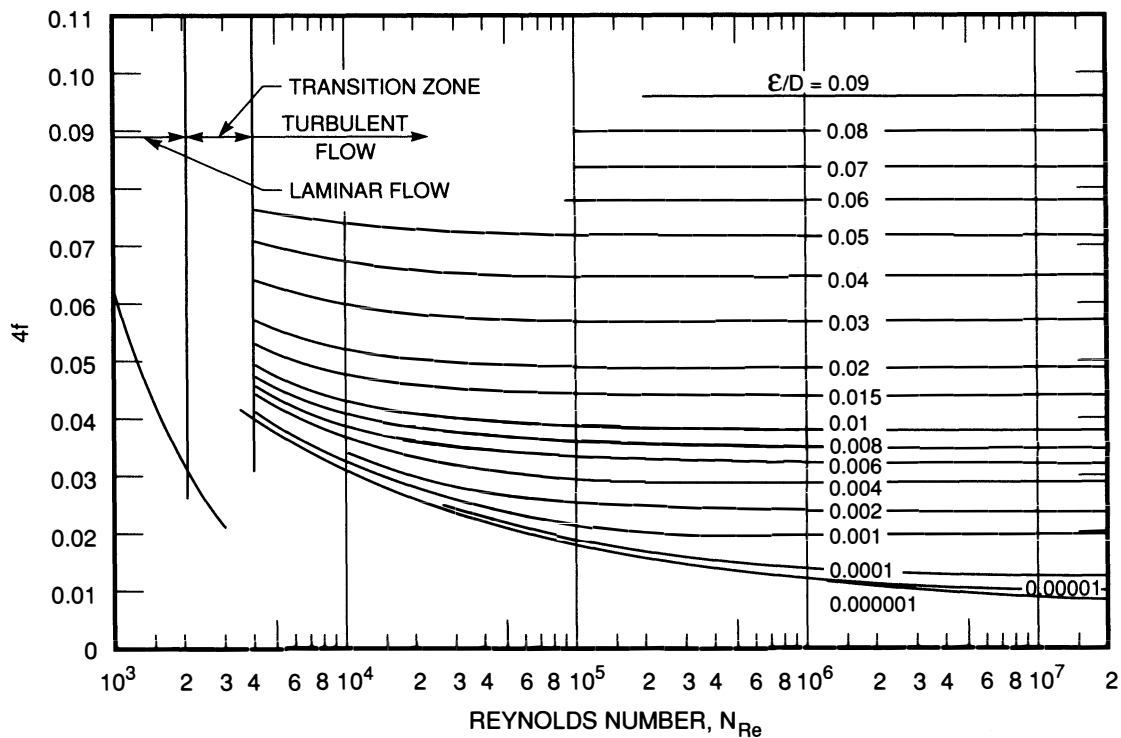


Figure 1A-5 - Friction Factor Versus Reynolds Number (Ref. 14)

For some of the more common shapes the equivalent diameters are:

Square duct: D_e = length of side

Rectangular duct: $D_e = 2ab/(a+b)$ (a and b are sides)

Elliptical duct:

$$D_e = 2ab \left(\frac{2}{a^2 + b^2} \right)^{1/2}$$

(a and b are semi-major and semi-minor axes, respectively)

Annular duct: $D_e = D_o - D_i$

where D_o and D_i are the outer and inner duct diameters, respectively.

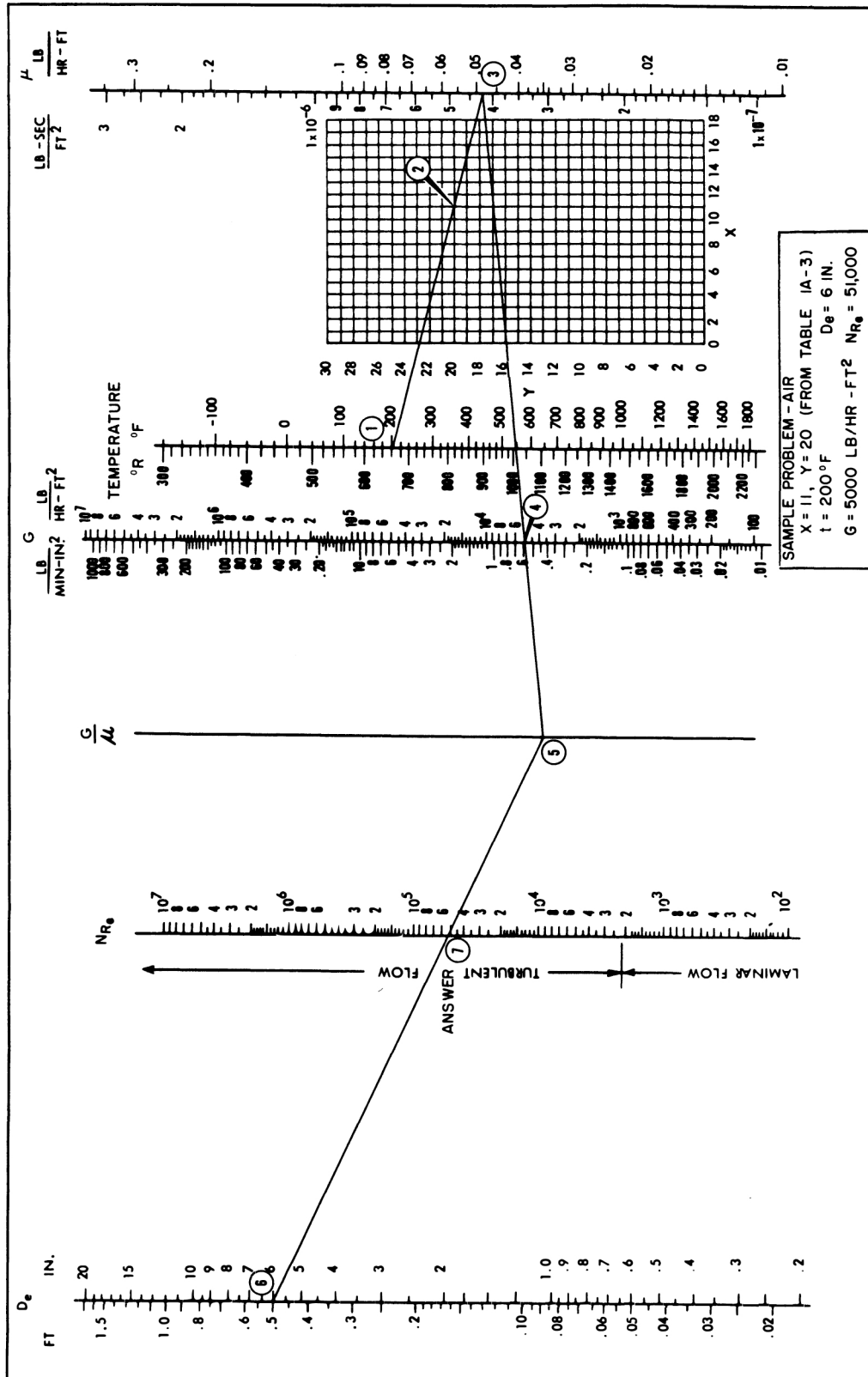


Figure 1A-6 - Reynolds Number in Gases

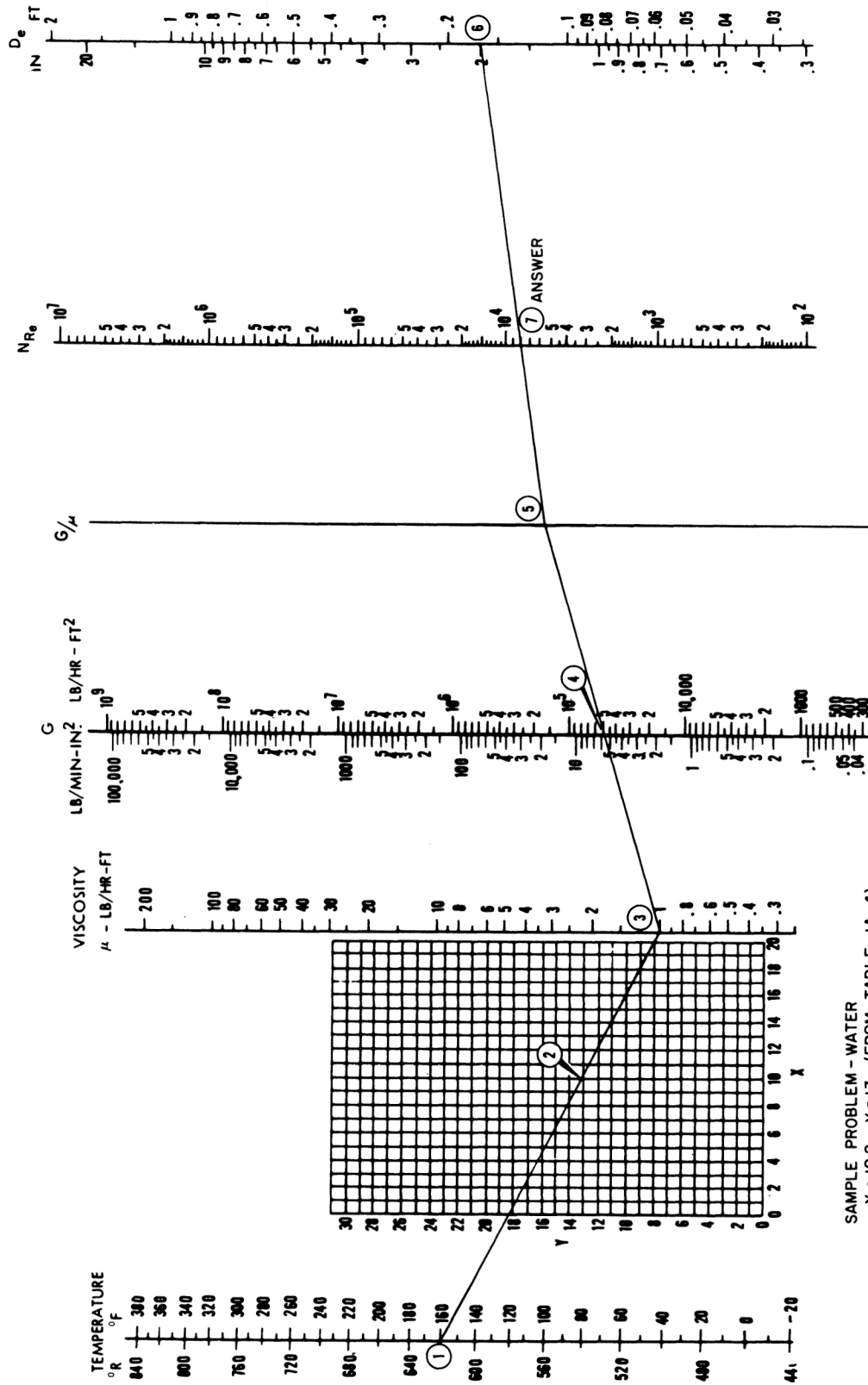


Figure 1A-7 - Reynolds Number in Liquids

Table 1A-3 - Coordinates for Reynolds Number Nomograph for Gases (Fig. 1A-6)¹

Gas	X	Y	Gas	X	Y
Acetic Acid	7.7	14.3	Freon-113	11.3	14.0
Acetone	8.9	13.0	Helium	10.9	20.5
Acetylene	9.8	14.9	Hexane	8.6	11.8
Air	11.0	20.0	Hydrogen	11.2	12.4
Ammonia	8.4	16.0	3 H ₂ N ₂	11.2	17.2
Argon	10.5	22.4	Hydrogen Bromide	8.8	20.9
Benzene	8.5	13.2	Hydrogen Chloride	8.8	18.7
Bromine	8.9	19.2	Hydrogen Cyanide	9.8	14.9
Butene	9.2	13.7	Hydrogen Iodide	9.0	21.3
Butylene	8.9	13.0	Hydrogen Sulfide	8.6	18.0
Carbon Dioxide	9.5	18.7	Iodine	9.0	18.4
Carbon Disulfide	8.0	16.0	Mercury	5.3	22.9
Carbon Monoxide	11.0	20.0	Methane	9.9	15.5
Chlorine	9.0	18.4	Methyl Alcohol	8.5	15.6
Chloroform	8.9	15.7	Nitric Oxide	10.9	20.5
Cyanogen	9.2	15.2	Nitrogen	10.6	20.0
Cyclohexane	9.2	12.0	Nitrosyl Chloride	8.0	17.6
Ethane	9.1	14.5	Nitrous Oxide	8.8	19.0
Ethyl Acetate	8.5	13.2	Oxygen	11.0	21.3
Ethyl Alcohol	9.2	14.2	Pentane	7.0	12.8
Ethyl Chloride	8.5	15.6	Propane	9.7	12.9
Ethyl Ether	8.9	13.0	Propyl Alcohol	8.4	13.4
Ethylene	9.5	15.1	Propylene	9.0	13.8
Fluorine	7.3	23.8	Sulfur Dioxide	9.6	17.0
Freon-11	10.6	15.1	Toluene	8.6	12.4
Freon-12	11.1	16.0	2,2,3-Trimethylbutane	9.5	10.5
Freon-21	10.8	15.3	Water Vapor	8.0	16.0
Freon-22	10.1	17.0	Xenon	9.3	23.0

¹ Ref. 11.

The following formulas are useful in determining the friction factors of smooth pipes over the ranges listed and approximate the data on Fig. 1A-5.

$$f = \frac{16}{N_{Re}} \quad \text{for } N_{Re} < 2100 \quad (1A-12)$$

$$f = \frac{0.0791}{N_{Re}^{0.25}} \quad \text{for } N_{Re} = 3000 \text{ to } 10,000 \quad (1A-12a)$$

$$f = \frac{0.046}{N_{Re}^{0.2}} \quad \text{for } N_{Re} = 10,000 \text{ to } 5 \times 10^6 \quad (1A-13)$$

**Table 1A-4 - Coordinates for Reynolds Number Nomograph for Liquids
(Fig. 1A-7)¹**

Liquid	X	Y	Liquid	X	Y
Acetaldehyde	15.2	4.8	Freon-22	17.2	4.7
Acetic Acid 100%	12.1	14.2	Freon-113	12.5	11.4
Acetic Acid 70%	9.5	17.0	Glycerol 100%	2.0	30.0
Acetic Anhydride	12.7	12.8	Glycerol 50%	6.9	19.6
Acetone 100%	14.5	7.2	Heptane	14.1	8.4
Acetone 35%	7.9	15.0	Hexane	14.7	7.0
Allyl Alcohol	10.2	14.3	Hydrochloric Acid 31.5%	13.0	16.6
Ammonia 100%	12.6	2.0	Isobutyl Alcohol	7.1	18.0
Ammonia 26%	10.1	13.9	Isobutyric Acid	12.2	14.4
Amyl Acetate	11.8	12.5	Isopropyl Alcohol	8.2	16.0
Allyl Alcohol	7.5	18.4	Kerosene	10.2	16.9
Aniline	8.1	18.7	Linseed Oil, Raw	7.5	27.2
Anisole	12.3	13.5	Mercury	18.4	16.4
Arsenic Trichloride	13.9	14.5	Methanol 100%	12.4	10.5
Benzene	12.5	10.9	Methanol 90%	12.3	11.8
Brine, CaCl ₂ , 25%	6.6	15.9	Methanol 40%	7.8	15.5
Brine, NaCl, 25%	10.2	16.6	Methyl Acetate	14.2	8.2
Bromine	14.2	13.2	Methyl Chloride	15.0	3.8
Bromotoluene	20.0	15.9	Methyl Ethyl Ketone	13.9	8.6
Butyl Acetate	12.3	11.0	Naphthalene	7.9	18.1
Butyl Alcohol	8.6	17.2	Nitric Acid 95%	12.8	13.8
Butyric Acid	12.1	15.3	Nitric Acid 60%	10.8	17.0
Carbon Dioxide	11.6	0.3	Nitrobenzene	10.6	16.2
Carbon Disulfide	16.1	7.5	Nitrotoluene	11.0	17.0
Carbon Tetrachloride	12.7	13.1	Octane	13.7	10.0
Chlorobenzene	12.3	12.4	Octyl Alcohol	6.6	21.1
Chloroform	14.4	10.2	Pentachloroethane	10.9	17.3
Chlorosulfonic Acid	11.2	18.1	Pentane	14.9	5.2
Chlorotoluene, ortho-	13.0	13.3	Phenol	6.9	20.8
Chlorotoluene, meta-	13.3	12.5	Phosphorus Tribromide	13.8	16.7
Chlorotoluene, para-	13.3	12.5	Phosphorus Trichloride	16.2	10.9
Cresol, meta-	2.5	20.8	Propionic Acid	12.8	13.8
Cyclohexanol	2.9	24.3	Propyl Alcohol	9.1	16.5
Dibromoethane	12.7	15.8	Propyl Bromide	14.5	9.6
Dichloroethane	13.2	12.2	Propyl Chloride	14.4	7.5
Dichloromethane	14.6	8.9	Propyl Iodide	14.1	11.6
Diethyl Oxalate	11.0	16.4	Sodium	16.4	13.9
Dimethyl Oxalate	12.3	15.8	Sodium Hydroxide 50%	3.2	25.8
Diphenyl	12.0	18.3	Stannic Chloride	13.5	12.8
Dipropyl Oxalate	10.3	17.7	Sulfur Dioxide	15.2	7.1
Ethyl Acetate	13.7	9.1	Sulfuric Acid 110%	7.2	27.4
Ethyl Alcohol 100%	10.5	13.8	Sulfuric Acid 98%	7.0	24.8
Ethyl Alcohol 95%	9.8	14.3	Sulfuric Acid 60%	10.2	21.3
Ethyl Alcohol 40%	6.5	16.6	Sulfuryl Chloride	16.2	12.4
Ethyl Benzene	13.2	11.5	Tetrachloroethane	11.9	15.7
Ethyl Bromide	14.5	8.1	Tetrachloroethylene	14.2	12.7
Ethyl Chloride	14.8	6.0	Titanium Tetrachloride	14.4	12.3
Ethyl Ether	14.5	5.3	Toluene	13.7	10.4
Ethyl Formate	14.2	8.4	Trichloroethylene	14.8	10.5
Ethyl Iodine	14.7	10.3	Turpentine	11.5	14.9
Ethylene Glycol	6.0	23.6	Vinyl Acetate	14.0	8.8
Formic Acid	10.7	15.8	Water	10.2	13.0
Freon-11	14.4	9.0	Xylene, ortho-	13.5	12.1
Freon-12	16.8	5.6	Xylene, meta-	13.9	10.6
Freon-21	15.7	7.5	Xylene, para-	13.9	10.9

¹ Ref. 11.

The following equation approximates data over the entire range of experiments:

$$\frac{1}{\sqrt{f}} = 40 \log_{10} (N_{Re} \sqrt{f}) - 0.40 \quad (1A-14)$$

It is recommended for extrapolating to higher Reynolds numbers.

For air, the factor $4f$ versus w/D is presented in Fig. 1A-8 to simplify the calculation of ΔP_t using Eq. 1A-10.

3.2 Pressure Losses, Elbows

The flow loss in a bend or an elbow is

$$\Delta P_t = \left(4f \frac{L}{D} + CK_{90} \right) \left(\frac{1}{2} \rho V^2 \right) \quad (1A-15)$$

where C = Correction factor for bends other than 90 deg, dimensionless
 L = Length along centerline of duct

The loss coefficients for all bends and elbows given in the succeeding sections give total pressure loss due to turning only. The wall or straight duct friction loss must be added to this loss to determine the overall loss of the elbow. Fig. 1A-9 illustrates the combined pressure loss, turning plus duct friction, in a circular duct for various radius ratios. It is seen from this figure that the minimum loss occurs at an r/D ratio of 3. While this particular figure is for a Reynolds number equal to 10^6 , the principle holds true for other Reynolds numbers. Therefore, considering both pressure drop and fabrication, a bend with a radius ratio between 2 and 3 should be used wherever possible.

Note: The loss coefficients for bends and elbows in Figs. 1A-10 to 1A-21, unless otherwise specified, apply to elbows followed by a length of duct equal to at least four duct diameters. If no duct follows the elbow, the loss coefficient should be corrected as in Par. 3.2.1.

3.2.1 Effect of Discharge Duct

When the discharge duct at the exit of an elbow is removed, the flow pattern through the elbow is changed, and this changes the loss coefficient. For bend angles less than 90 deg, the flow losses are increased by removing the exit duct; for angles greater than 90 deg, the flow losses are decreased by removing the exit duct. This can be explained by the fact that for angles greater than 90 deg, the removal of the exit duct reduces the net turning angle, and this is the predominant loss effect. The effect of the exit duct is plotted in Fig. 1A-22 for 90 deg bends and presents the ratio of the loss factor with and without the exit duct. The loss without the exit duct is for the loss just up to the exit end of the elbow. Therefore, an additional loss for the sudden expansion should be added to the values obtained to determine the total loss of an elbow without an exit duct (3.3). For angles other than 90 deg, Fig. 1A-23 is used to correct the loss factors obtained from Fig. 1A-22 for bend angle. Although the data used to obtain Fig. 1A-22 were obtained from rectangular ducts, they can be applied to elliptical and circular ducts in lieu of specific data for these ducts.

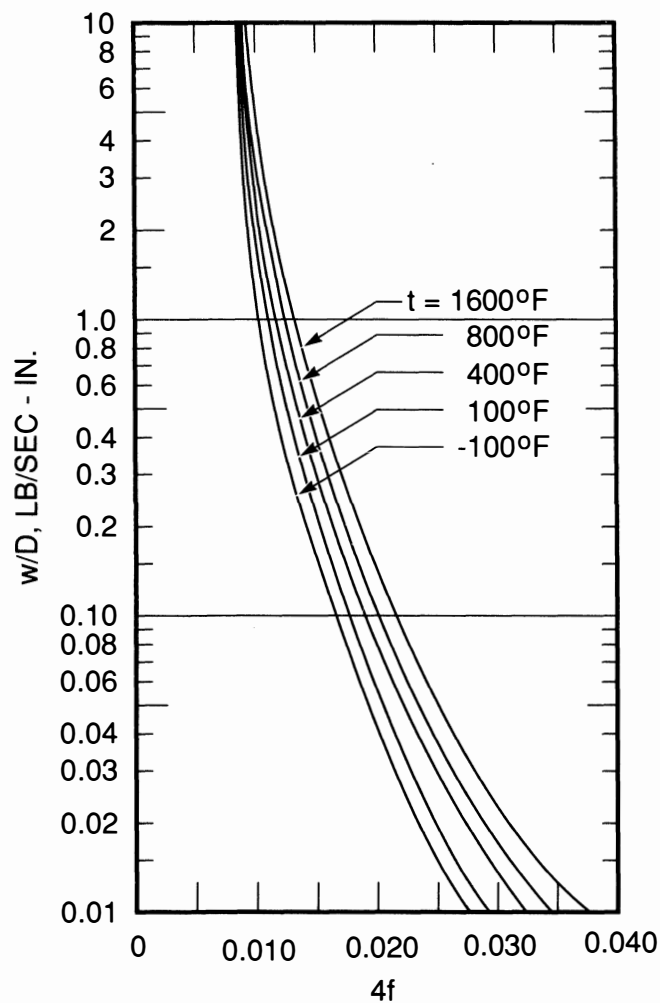


Figure 1A-8 - Friction Factor Versus Weight Flow Per Unit Diameter, Air in Smooth Round Ducts

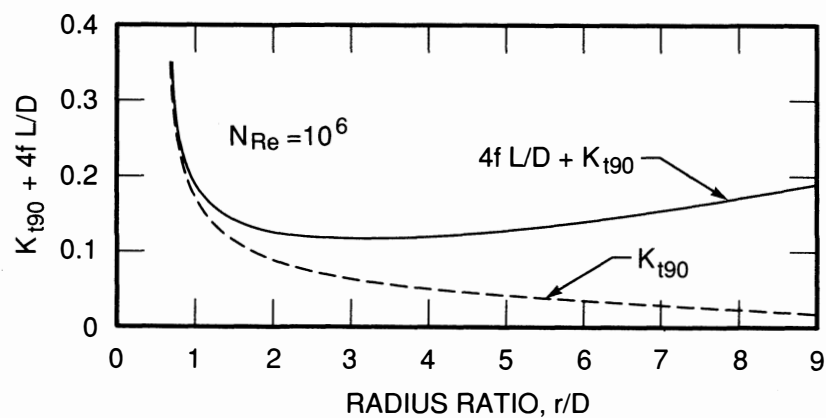


Figure 1A-9 - Bend Loss

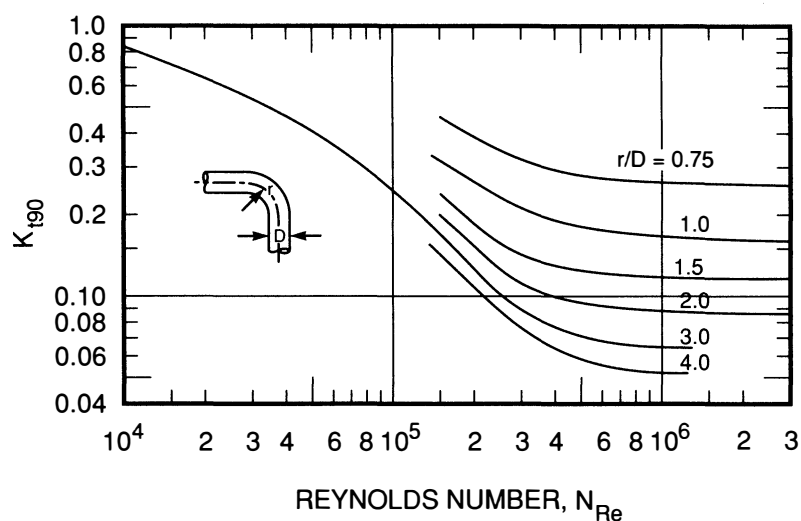


Figure 1A-10 - Loss Coefficient Versus Reynolds Number for 90 deg Radius Bends, Circular Ducts. (Data for $N_{Re} > 10^5$ From Ref. 1, for $N_{Re} < 10^5$ From Ref. 28)

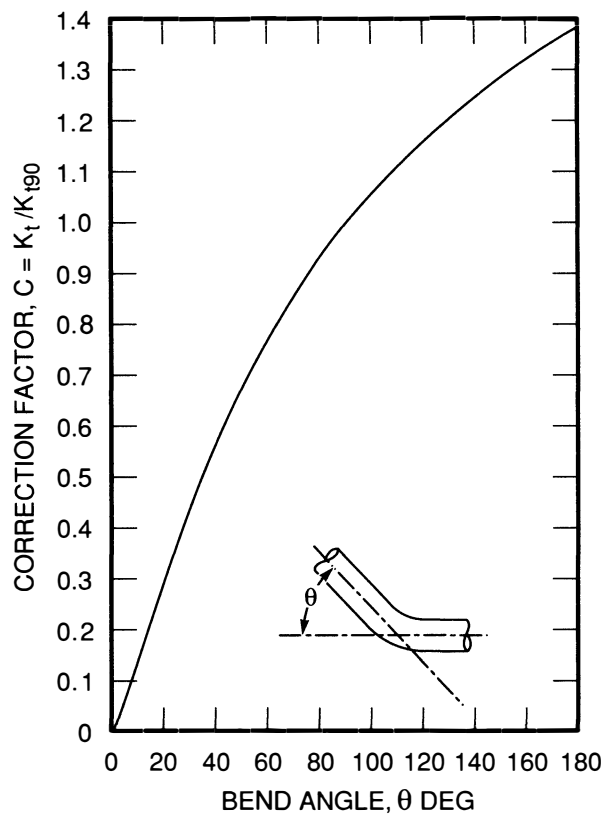


Figure 1A-11 - Correction Factor for Bends Other Than 90 deg for Elliptical and Circular Ducts (From Ref. 12, Fig. 16)

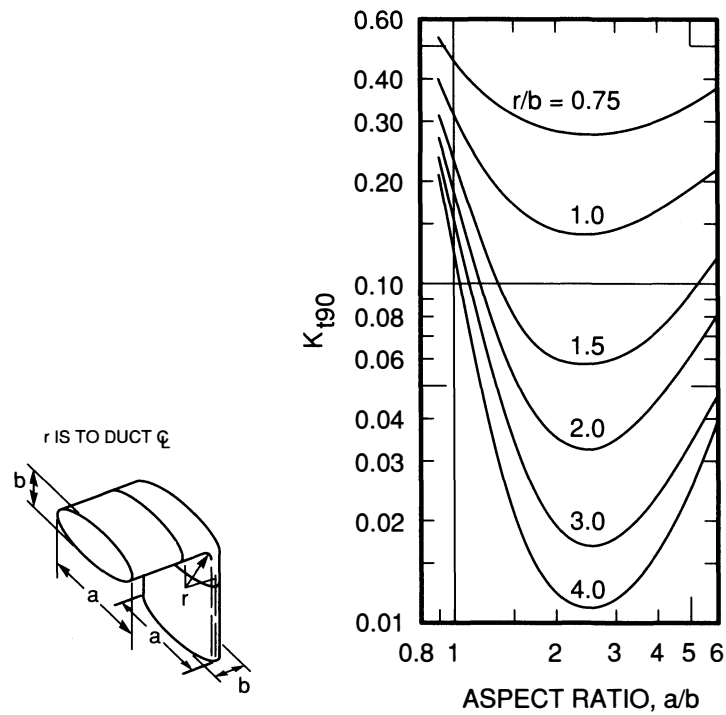


Figure 1A-12 - Loss Coefficient for 90 deg Radius Bends, Elliptical Ducts, $N_{Re} = 150,000$ (From Ref. 1, Fig. 3a)

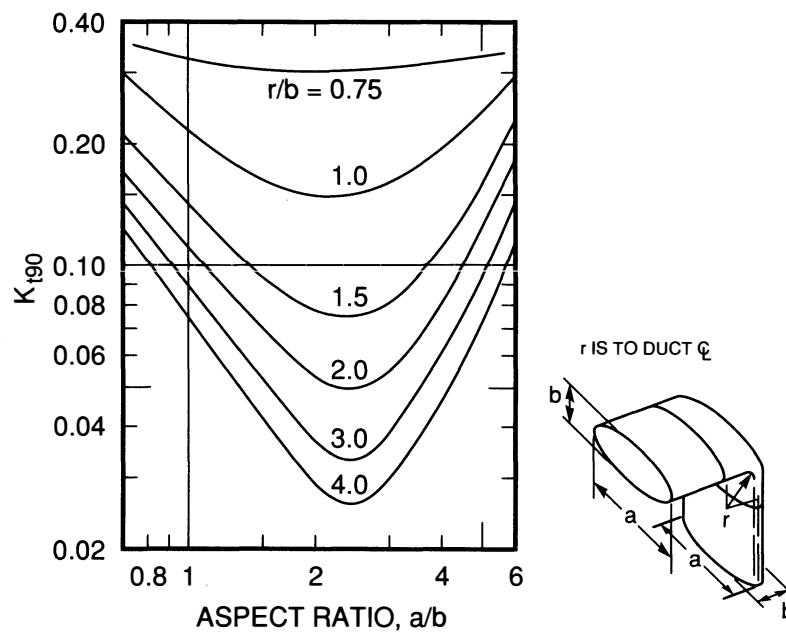


Figure 1A-13 - Loss Coefficient for Radius Bends, Elliptical Ducts, $N_{Re} = 300,000$ (From Ref. 1, Fig. 3b)

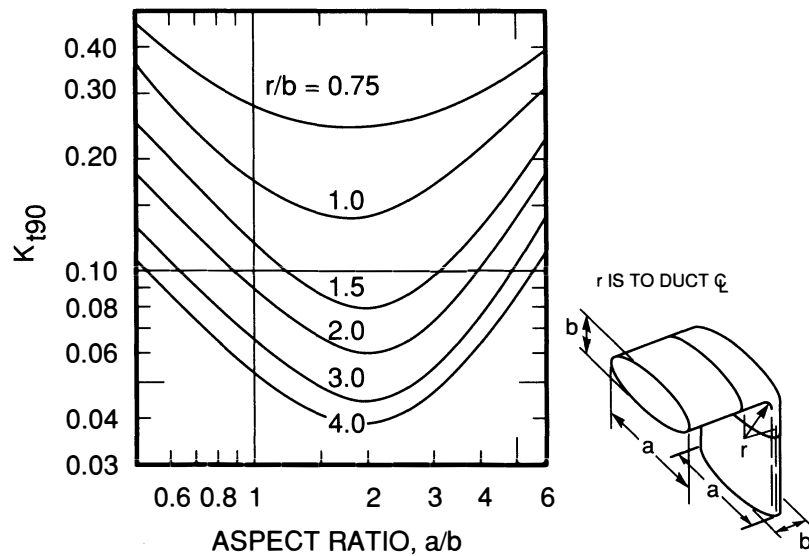


Figure 1A-14 - Loss Coefficient for 90 deg Bends, Elliptical Ducts, $N_{Re} = 600,000$ (From Ref. 1, Fig. 3c)

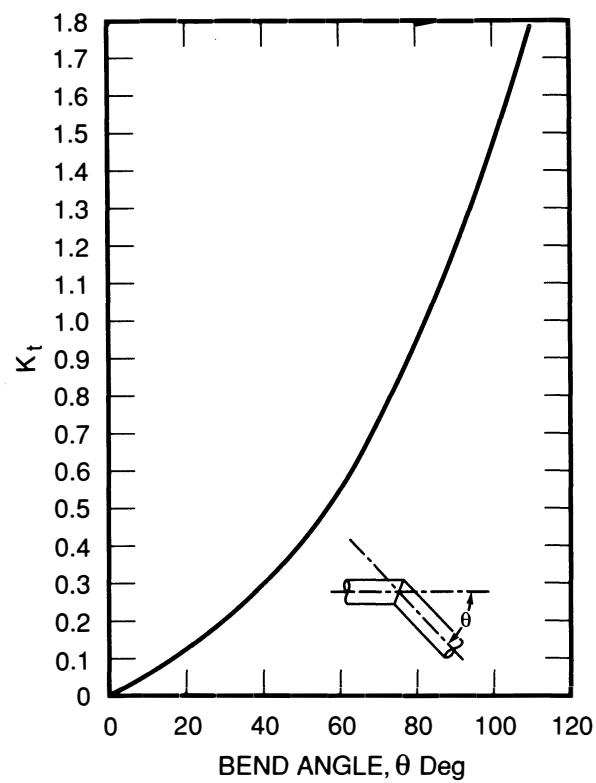


Figure 1A-15 - Loss Coefficient for Miter Bends, Circular Ducts, $N_{Re} < 1000$ (From Ref. 12)

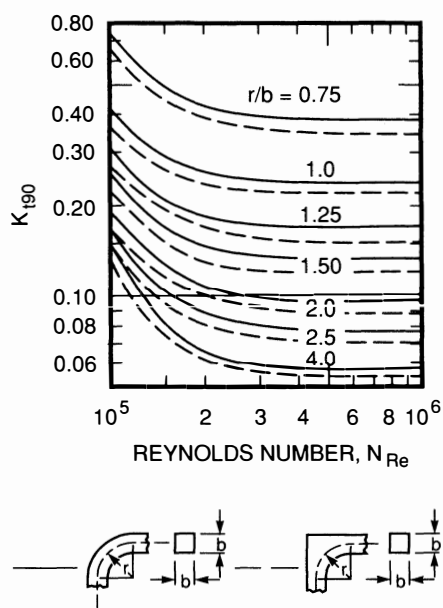


Figure 1A-16 - Loss Coefficient Versus Reynolds Number for 90 deg Radius Elbows, Square Ducts (From Ref. 1, Fig. 2; Ref. 9)

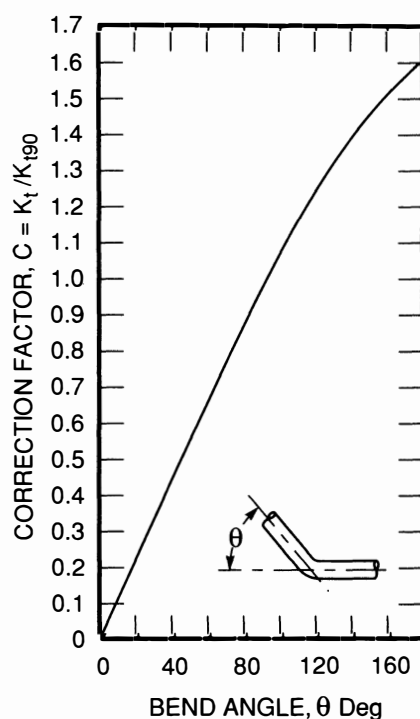


Figure 1A-17 - Correction Factor for Bends Other Than 90 deg, Rectangular and Square Ducts (From Ref. 5)

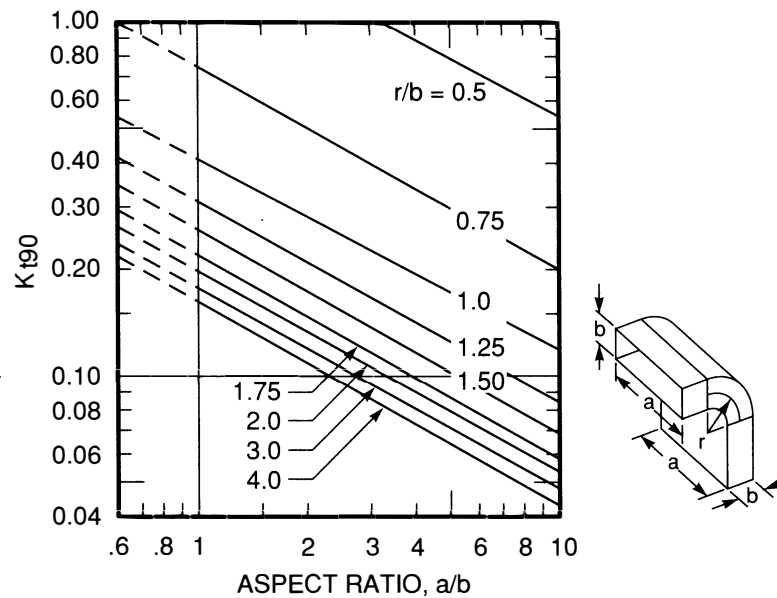


Figure 1A-18 - Loss Coefficient for 90 deg Bends, Rectangular Ducts, $N_{Re} = 100,000$ (From Ref. 1, Fig. 2a)

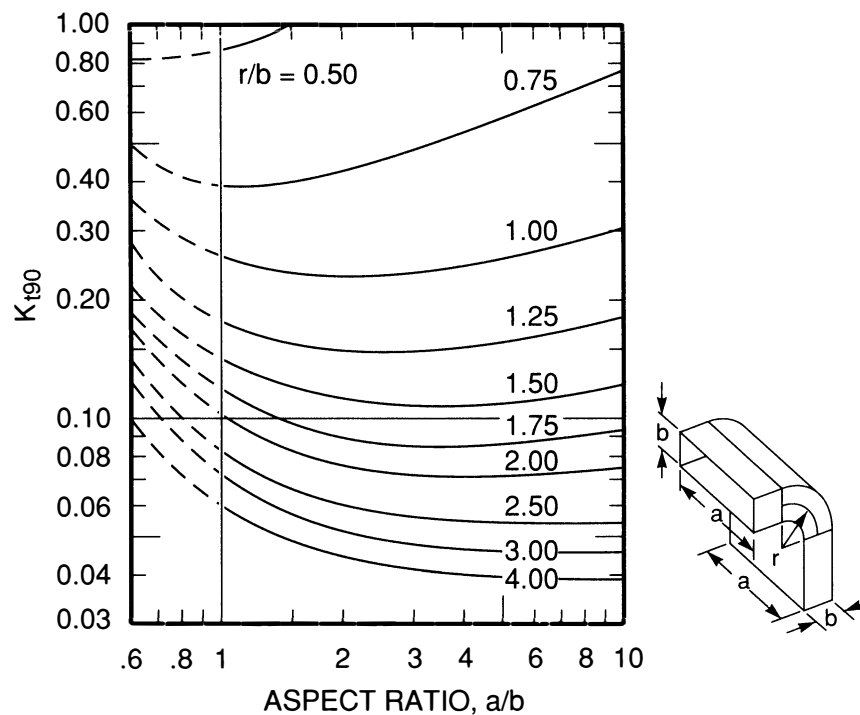


Figure 1A-19 - Loss Coefficient for 90 deg Radius Bends, Rectangular Ducts, $N_{Re} = 300,000$ (From Ref. 1, Fig. 2b)

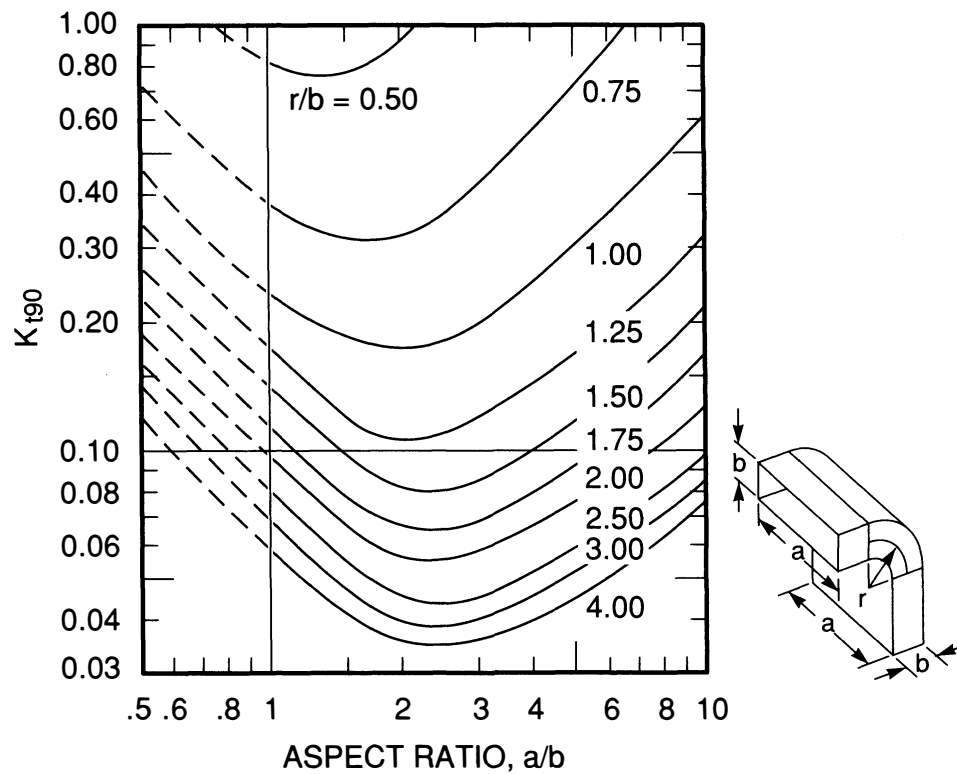


Figure 1A-20 - Loss Coefficient for 90 deg Radius Bends, Rectangular Ducts, $N_{Re} = 600,000$
(From Ref. 1, Fig. 2c)

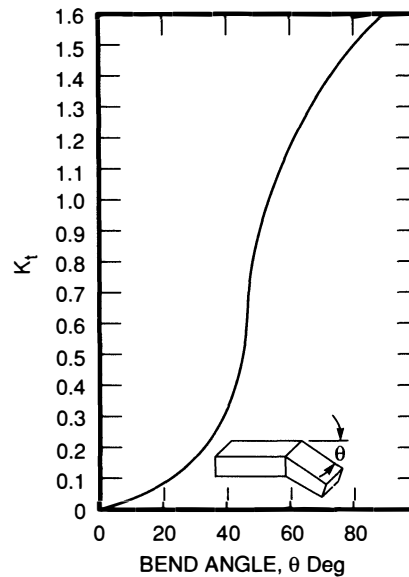


Figure 1A-21 - Loss Coefficient for Square Mitered Elbow (From Ref. 13)

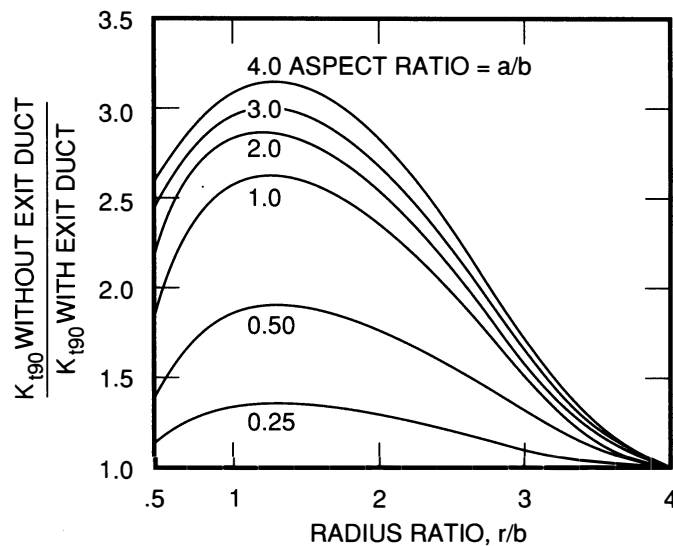


Figure 1A-22 - Effect of Exit Duct on 90 deg Elbow (Data From Ref. 5)

3.2.2 Compound Elbows

Figs. 1A-24 and 1A-25 for circular ducts plot K_f versus r/D for compound U-bends and 90 deg offset bends, respectively. Fig. 1A-26 plots the same values for Z-bends.

3.2.3 Improving Bend Efficiency

3.2.3.1 Turning Vanes

Turning vanes are used in sharp bends to improve the exit velocity distribution and reduce the pressure loss. All the turning vanes in a particular elbow are of the same size and shape as opposed to splitter elbows (3.2.3.2). Eqs. 1A-19 and 1A-20 were obtained from Ref. 1, pages 10 to 13, and the performance data for thin vanes were obtained from Refs. 1, 9, 13, and 15; data for thick vanes are from Ref. 29. The geometry referred to in the following paragraphs is shown in Fig. 1A-27.

$$\Delta \bar{V} = \bar{V}_1 \left[1 + \left(\frac{A_1}{A_2} \right)^2 - \left(\frac{2A_1}{A_2} \right) \cos \theta \right]^{1/2} \quad (1A-16)$$

The actual gap, s , between vanes for all types of vanes is determined from the equation

$$s = \frac{b_1}{(n+1) \cos(\theta/2)} \quad (1A-17)$$

(A) Design

(1) Thin Vanes, Circular Profile: Due to the ease of installation, a thin circular arc vane is recommended for general use. These vanes are equally spaced within the bend. A vane is considered thin when its thickness is less than $1/40$ of the inside bend radius (r_i).

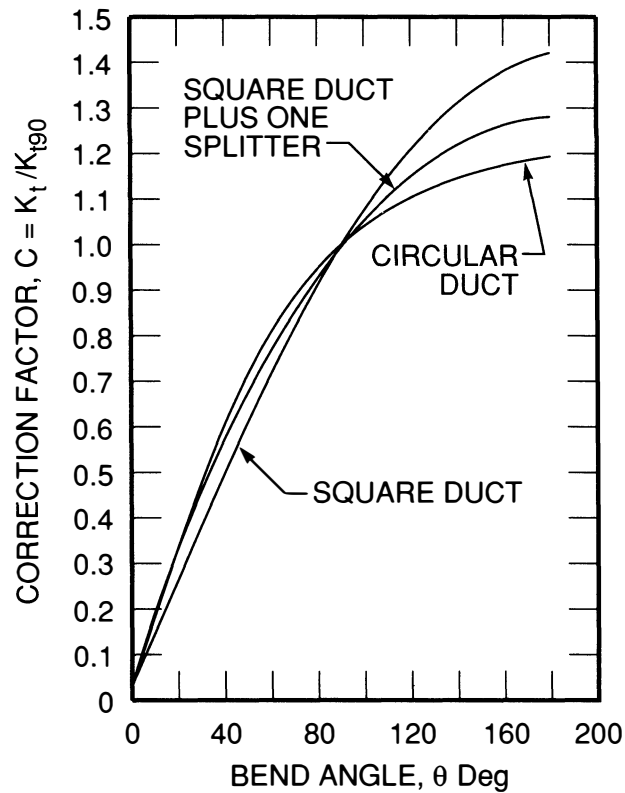


Figure 1A-23 - Correction Factor for Bends Other Than 90 deg for Bends Without Exit Ducts (Data From Refs. 5 and 12)

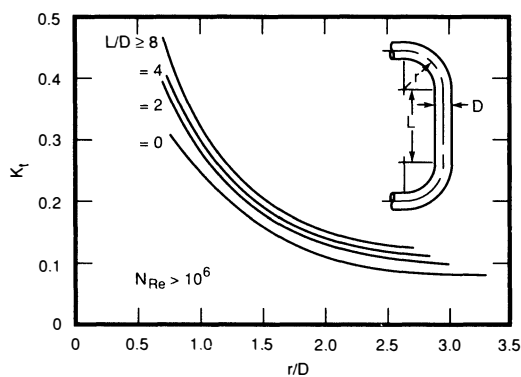


Figure 1A-24 - Loss Coefficient for Compound U-bends, Circular Ducts (Reduced From Ref. 3, Figs. 27-32; Ref. 8, Fig. 30)

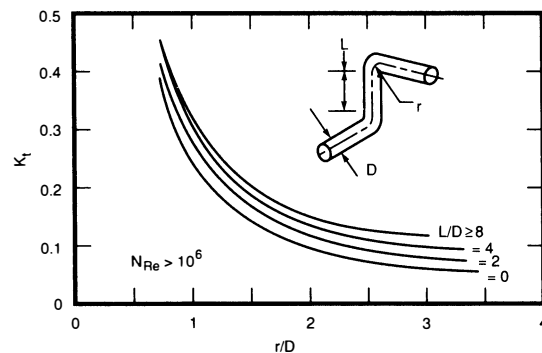


Figure 1A-25 - Loss Coefficient for 90 deg Offset Bends, Circular Ducts (Reduced From Ref. 3, Figs. 39-44; Ref. 8, Fig. 32)

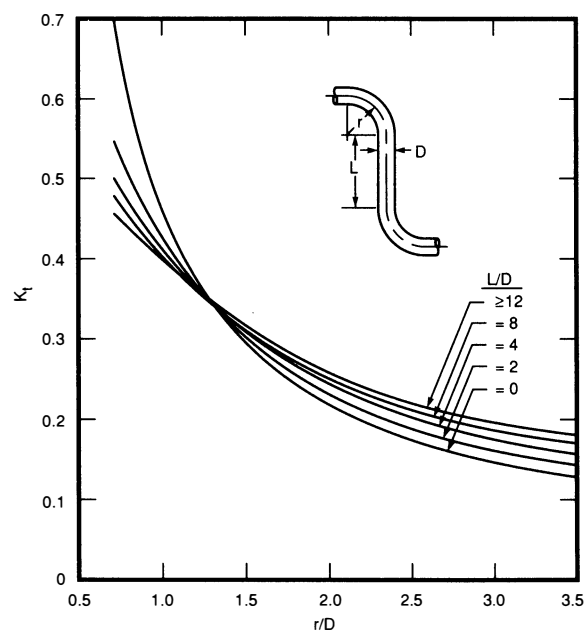


Figure 1A-26 - Loss Coefficient for Z-bends, Circular Ducts (Reduced From Ref. 3, Figs. 33-38; Ref. 8, Fig. 31)

The vane geometry is determined from the duct width, b , the inside bend radius, r_i , the bend angle, θ , and the area ratio, A_1/A_2 . The inside bend radius, r_i , should be made as large as space will permit.

Vane bend radius: $r = r_i$

Chord length: $c = 2r \sin(\theta/2)$

Number of vanes $= n$

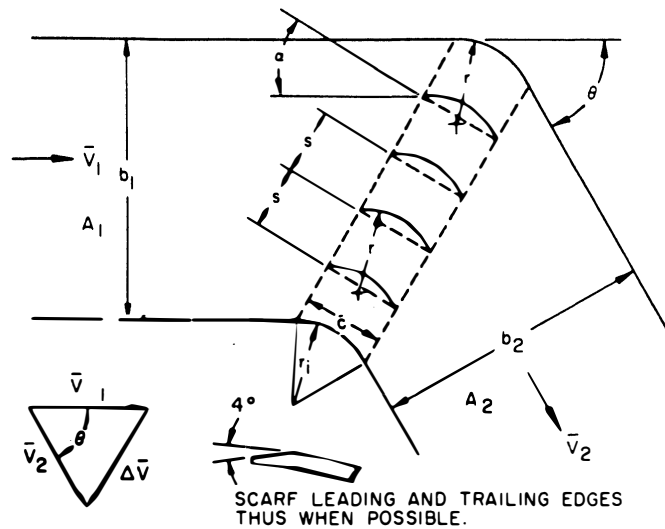


Figure 1A-27 - Turning Vanes Geometry

For all bends (Fig. 1A-27):

$$n = 2.5 \left[\frac{\Delta \bar{V}}{\bar{V}_1} \frac{b_1}{c} \right] - 1 \quad (1A-18)$$

For bends of equal inlet and outlet dimensions, this equation reduces to

$$n = 5 \frac{b}{c} \left(\sin \frac{\theta}{2} \right) - 1 \quad (1A-19)$$

In Fig. 1A-28, n is plotted as a function of r/b for equal inlet and outlet dimensions. The angle of attack is presented in Fig. 1A-29.

(2) Thin Vanes, Noncircular Profile: For use in bends with equal inlet and outlet dimensions, a noncircular thin vane, which is more efficient than the circular arc vane, has been developed. However, since the noncircular arc is more difficult to fabricate, it should be used only where the decreased pressure drop is necessary. The coordinates for this type of vane are shown in Fig. 1A-30, and the angle of attack is presented in Fig. 1A-31.

At the upstream end of the vane, $x=0$ and $y=0$. For this type of vane, the shape of the inside and outside corners should be the same shape as the blade instead of having a radius r_i . Therefore, a chord length c should be selected as large as the bend inside-corner clearance will permit. The blade and corner blade locations in the duct are determined from c and the angle of attack, α (Fig. 1A-31). The number of blades is determined from c , the s/c value from Fig. 1A-31, and the following equation:

$$n = \frac{b}{c(s/c) \cos(\theta/2)} - 1 \quad (1A-20)$$

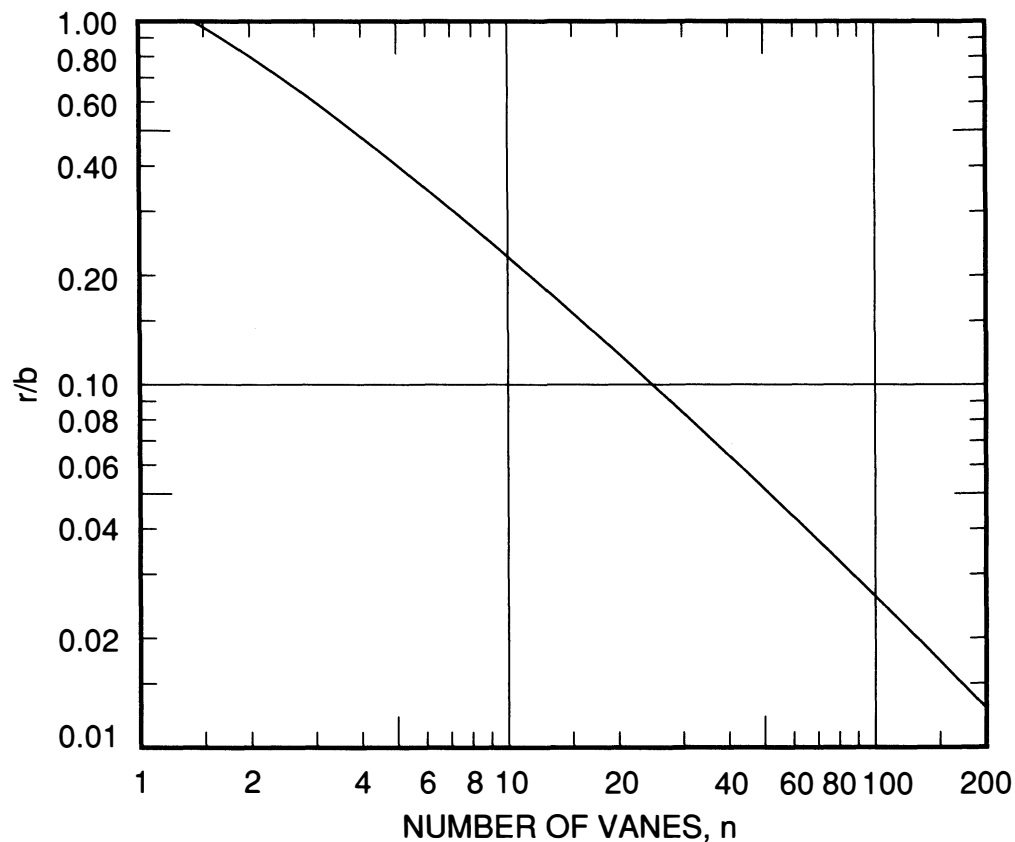


Figure 1A-28 - Number of Vanes for Thin Circular Arc Vanes, $c=2r \sin(\theta/2)$, $n=(5b/c) [\sin(\theta/2)] - 1$ (From Ref. 1)

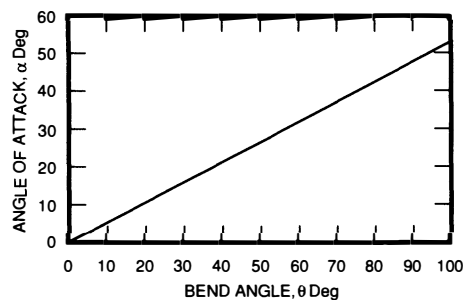


Figure 1A-29 - Angle of Attack for Thin Circular Arc Vanes (Calculated From Ref. 13)

(3) **Thick Vanes:** Two types of thick vanes are presented and are designated as type A and type B in Fig. 1A-32. Both types are very similar and vary only in the contour of the leading edges.

Number of vanes for both types of vanes

$$n = \left[4 \left(\frac{b}{c} \right) \sin \frac{\theta}{2} \right] - 1 \quad (1A-21)$$

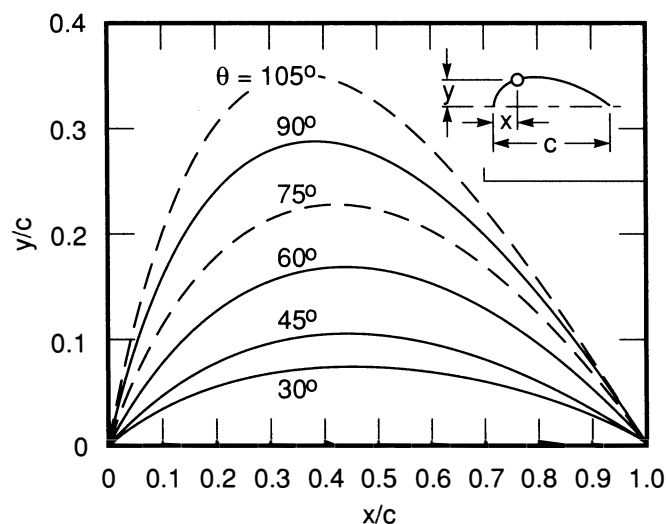


Figure 1A-30 - Thin Vanes, Noncircular Profile, Coordinates. (Data From Ref. 13)

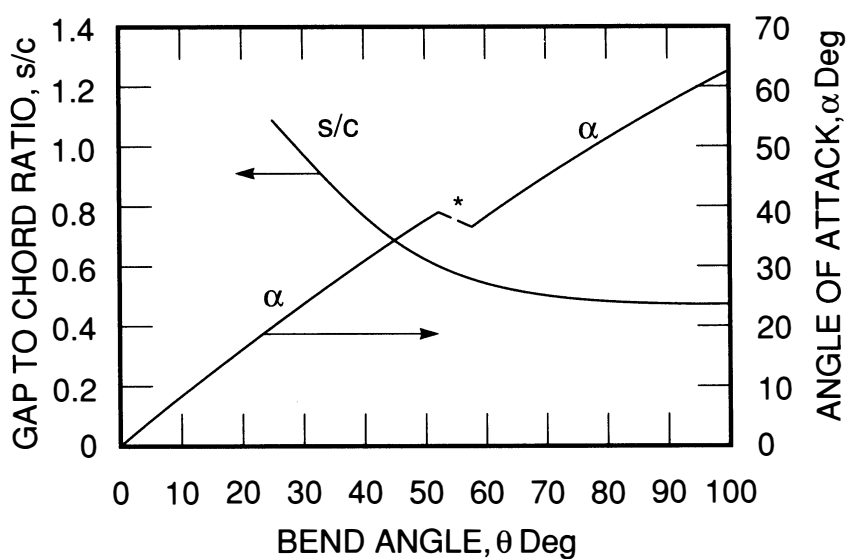


Figure 1A-31 - Bend Angle and Gap to Chord Ratio. *The discrepancy in the angle of attack curve (shown by the broken diagonal line) is apparently a result of different methods used in developing the vane profiles. (From Ref. 1)

Chord length: Type A has the following chord length; type B starts with a type A vane and is shortened by virtue of the leading-edge contour change:

$$c = 2 r_c \sin \frac{\theta}{2} \quad (1A-22)$$

where r_c = Inside chord radius.

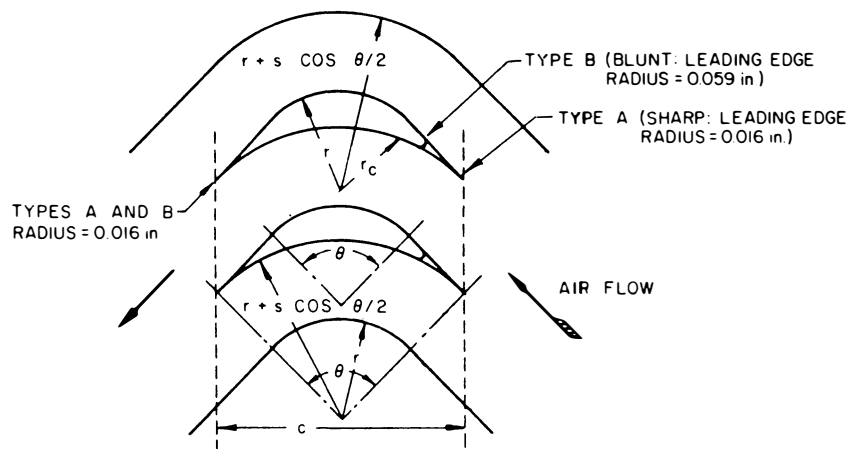


Figure 1A-32 - Thick Vanes (Calculated From Ref. 29)

(B) Pressure Loss: The following pressure loss coefficients are based on the upstream q immediately preceding the vaned bend. The Reynolds number is based on duct velocity and chord length.

(1) Thin Vanes, Circular Profile: Figs. 1A-33 and 1A-34 plot loss coefficients and gap-to-chord ratio versus K_t , respectively.

(2) Thin Vanes, Noncircular Arc: Fig. 1A-35 plots K_t versus bend angle.

(3) Thick Vanes: The pressure loss versus Reynolds number for thick vanes is presented in Fig. 1A-36. The pressure drop data were determined for 90 deg bends with a spacing s equal to 1 in. and a radius, r , equal to 0.743 in. Some of the data were determined with an open duct downstream; therefore, the pressure loss data may be a little low under some flow conditions. The recommended number of vanes as determined in 3.2.3.1 (a) was determined from good turning vane practice and is believed to be good for extrapolating the data to other angles and vane radii. For other angles, the loss coefficient can be determined with reasonable accuracy by applying the bend angle correction factor of Fig. 1A-17 for rectangular elbows.

3.2.3.2 Splitters

The pressure loss in a rectangular elbow may be reduced by dividing the elbow with a number of flow dividers called "splitters." These splitters divide the larger elbow into a number of smaller elbows, each having a larger radius-to-duct width ratio and therefore a smaller loss coefficient. When the splitters are positioned in a manner such that all smaller elbows have the same radius-to-duct width ratio, the loss will be a minimum for a particular number of splitters. This means that the ratio of the inside duct radius, r_i , to the outside duct radius, r_o , of all smaller elbows will be the same and is equal to

$$\left(\frac{r_i}{r_o}\right)_A = (r_i/r_o)^{1/n} \quad (1A-23)$$

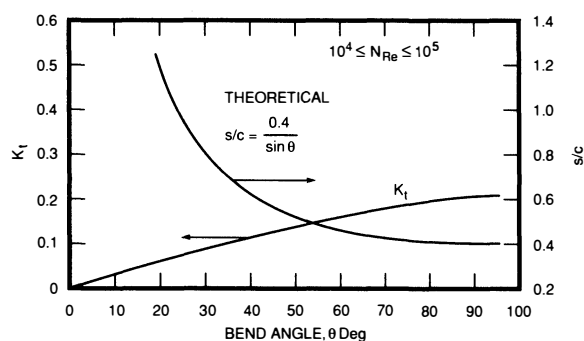


Figure 1A-33 - Loss Coefficient for Thin Circular Arc Vanes

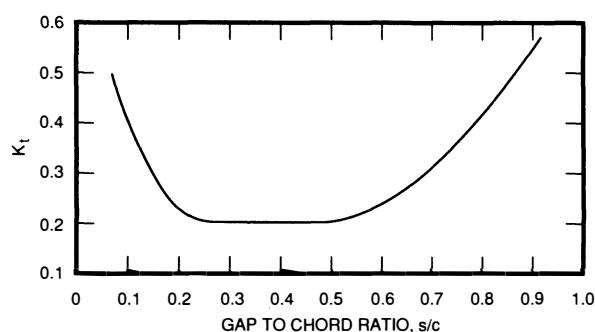


Figure 1A-34 - Effect of Gap to Chord Ratio on Thin Circular Arc Vanes (From Refs. 13 and 15)

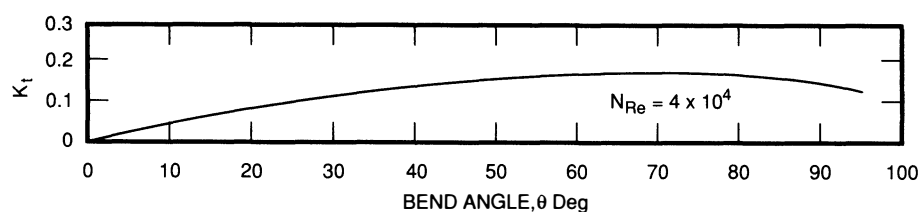


Figure 1A-35 - Loss Coefficient for Thin Noncircular Arc Vanes (From Ref. 1)

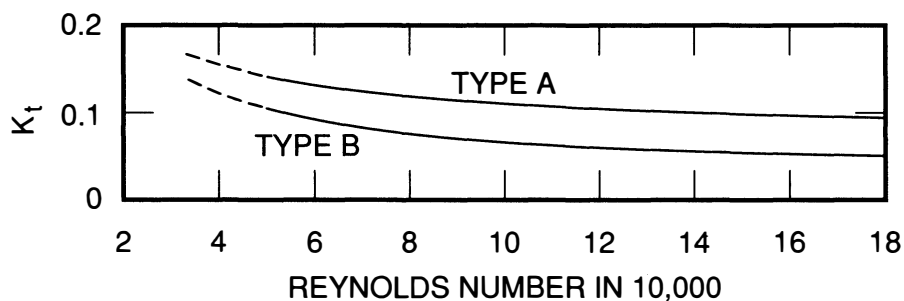


Figure 1A-36 - Loss Coefficient for 90 deg Thick Vane Bends (From Ref. 29)

where $(r_i/r_o)_A$ = Ratio of inside radius to outside radius of any subdivided elbows

r_i/r_o = Ratio of inside to outside radius of original duct before splitters are added
(see Fig. 1A-37)

n = Number of splitters plus 1, that is, for one splitter, $n=2$

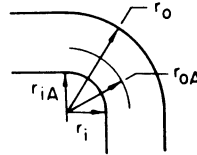


Figure 1A-37 - Sketch of Inside and Outside Radii with Splitters

The pressure loss of the new elbow with splitters is calculated by using a loss coefficient based on the radius-to-duct width ratio of the subdivided elbows. See Figs. 1A-18 through 1A-20 for rectangular elbows. The Reynolds number should also be based on the subdivided elbow. It should be mentioned that turning vanes may be lighter and show an advantage in Reynolds number if a large number of duct divisions is used. Ref. 27 contains additional information on losses in elbows of this type.

3.2.4 Transition Elbows

The loss coefficient for 90 deg rectangular transitional elbows may be obtained by using the correction factor of Fig. 1A-38 and the loss factor for 90 deg constant area rectangular elbows (Figs. 1A-18 through 1A-20).

3.3 Pressure Losses, Duct Area Changes

3.3.1 Sudden Expansions or Contractions

The accompanying figure (Fig. 1A-39) represents expansion (a) and contraction (b) in direction of the flow, for which

$$\Delta P = K_t q \quad (1A-24)$$

where q is the dynamic pressure or velocity head in the *smaller* area section.

The loss coefficient K_t is given in Figs. 1A-40 and 1A-41.

3.3.2 Gradual Contractions

Table 1A-5 shows the angular displacement versus pressure loss coefficient for a gradual contraction as diagrammed in Fig. 1A-42 and expressed by

$$\Delta P = \left(K_t + 4f \frac{L}{D_m} \right) q_2 \quad (1A-25)$$

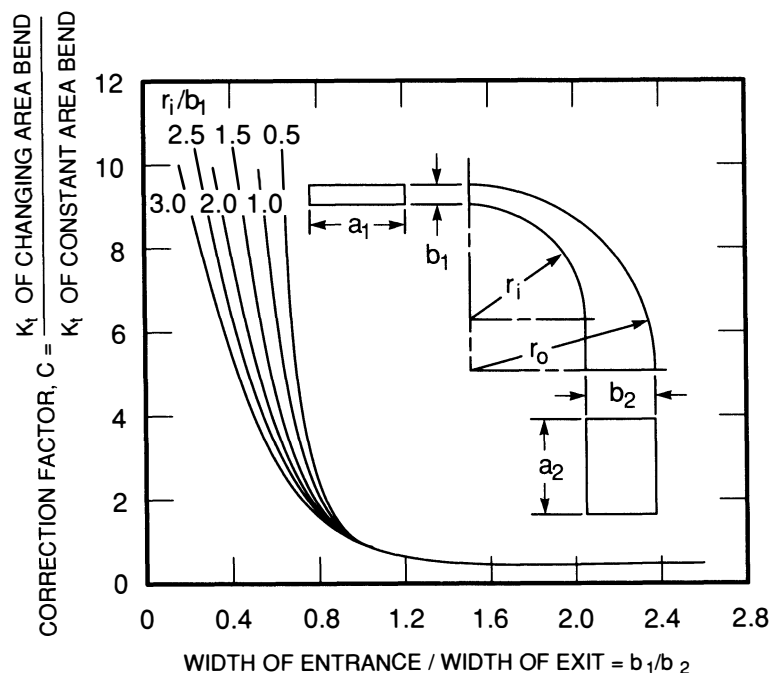


Figure 1A-38 - Pressure Loss Correction Factor for Transitional Elbows. Note: Radius Ratio of Constant Area Bend = $r_i/b_1 + 1/2$, $r_o = r_i + b_2$, $a_1 = a_2$ (From Ref.1)

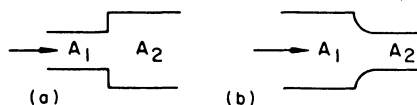


Figure 1A-39 - Sketch of Typical Types of Expansion and Contraction Areas

$$\text{where } D_m = \frac{D_1 + D_2}{2}$$

= mean equivalent diameter

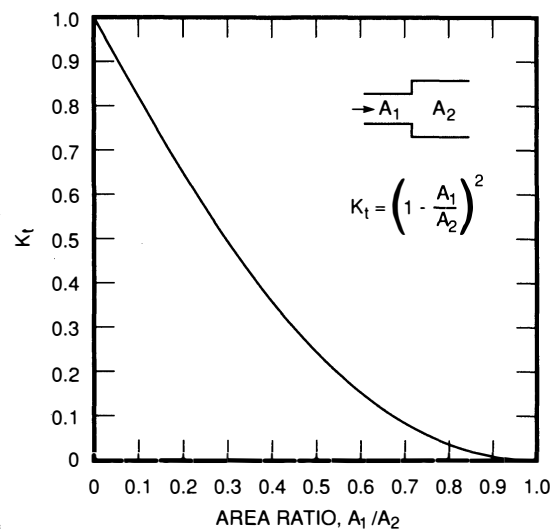
3.3.3 Diffusers (Subsonic)

3.3.3.1 Geometry

Whenever possible, a diffuser should be designed symmetrically about its axis, with an expansion angle (2θ) between 7 and 12 deg. Nonsymmetrical flow or too large an expansion angle may cause the flow to separate from the diffuser walls and thereby cause an increase in pressure loss. When space limitations necessitate a large expansion angle ($2\theta > 15$ deg), separation may be prolonged by use of a curved wall or truncated diffuser, by the addition of guide vanes (Refs. 19, 22, and 24), or in extreme cases by the use of boundary layer control (Ref. 23).

Table 1A-5 - Angular Displacement Versus Loss Coefficient¹

2θ, deg	K_t
0-30	0
30-45	0.05
45-180	Sudden contraction from A_1 to A_2 , as shown in Par. 3.3.1

¹ Ref. 27, p. 115.**Figure 1A-40 - Loss Coefficient of a Sudden Expansion (From Ref. 24)**

The type of intersection (sharp or rounded corner) between inlet duct and diffuser walls appears to have little effect on pressure loss characteristics except at high entrance Mach numbers ($M_1 > 0.7$). (Ref. 21.)

The optimum expansion angle is a function of area ratio, surface roughness, and Reynolds number. For example, for a given area ratio and surface roughness, the expansion angle with minimum pressure loss coefficient (as determined from Table 1A-5 and Eq. 1A-25) varies with inlet Reynolds number as shown in Fig. 1A-43. These curves are based on a smooth, straight-walled conical diffuser with an inlet-to-exit area ratio of 0.36 and a length-to-mean diameter ratio of 0.25 ($\cot \theta$).

Some types of diffusers are listed here.

(a) Straight Wall, Conical: For the diffuser shown in the following drawing (Fig. 1A-44)

$$D_2 = 2L \tan \theta + D_1 \quad (1A-26)$$

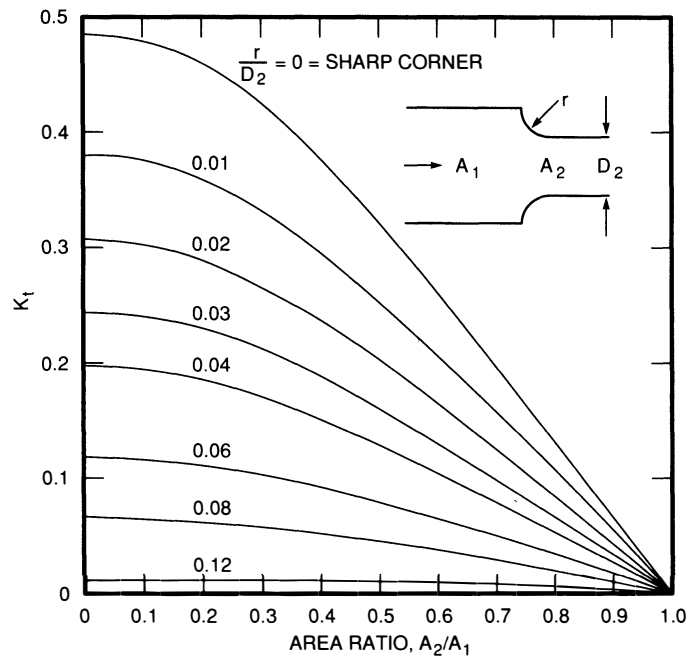


Figure 1A-41 - Loss Coefficient of a Sudden Contraction

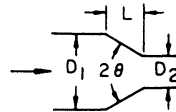


Figure 1A-42 - Sketch of a Typical Gradual Contraction

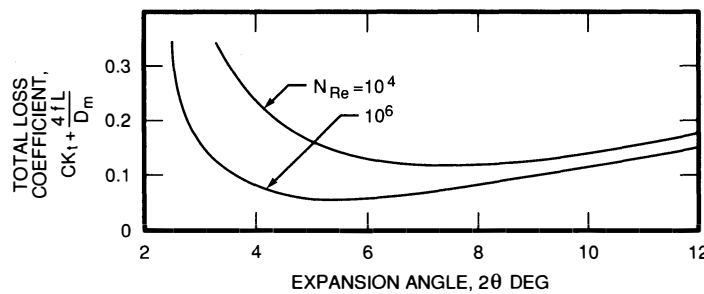


Figure 1A-43 - Total Loss Coefficient Versus Expansion Angle

(b) Straight Wall, Rectangular: Referring to Fig. 1A-45, values are:

Two Dimensions

$$a_2 = a_1 \quad (1A-27)$$

$$b_2 = b_1 + 2L \tan \theta \quad (1A-28)$$

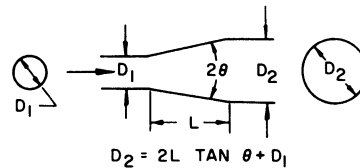


Figure 1A-44 - Sketch of a Straight Wall Conical Diffuser

Square

$$a_2 = a_1 + 2L \tan \theta \quad (1A-29)$$

$$b_2 = b_1 + 2L \tan \theta \quad (1A-30)$$

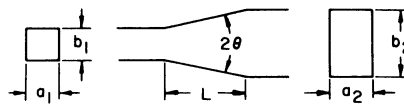


Figure 1A-45 - Sketch of a Straight Wall Rectangular Diffuser

(c) Curved Wall: At large expansion angles, diffusers of the shape shown in Fig. 1A-46 (Ref. 1, Fig. 10) appear to have lower pressure loss than do straight wall diffusers. The equation for the curved section is

$$y = \frac{D_1}{2} \left[\frac{1}{1 + (x/L) \sqrt{(A_1/A_2) - 1}} \right] \quad (1A-31)$$

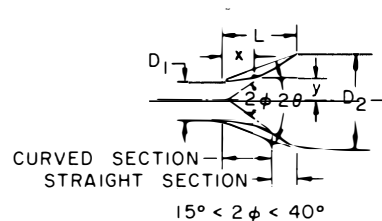


Figure 1A-46 - Curved Wall Diffuser

(d) Truncated: This type is identical to a straight wall diffuser except that there is a sudden enlargement into the downstream pipe (see Fig. 1A-47). This is useful where space is prohibitive and a curved wall diffuser is too expensive to fabricate.