



AEROSPACE INFORMATION REPORT

AIR1657™**REV. C**Issued 1981-01
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Superseding AIR1657B

(R) Handbook of Hydraulic Metric Calculations

RATIONALE

AIR1657 has been updated to Revision C for the following reasons:

- To incorporate technical updates and corrections.
- To include editorial and pictorial improvements.

FOREWORD

Hydraulic calculations may be conducted directly in the International System (SI) metric units. Calculations in SI metric units could not be simpler. Data, however, is not always in these units and conversions are necessary. It is generally preferable to convert the data and then proceed with calculations in the SI metric units.

The terminology is defined in each section where it is required to avoid any misunderstanding of dual uses of symbols such as N = Newton or N = rpm.

There can be confusion where symbols for a physical quantity have the same letters, (for example, mN (milli newton) and Newton meter (Nm)). To prevent this confusion, where the first letter is a multiple of an SI unit, there should be no spaces between the two letters (mN). If the two letters are combinations of separate units, then there should be a space or a dot between the two letters. In this document, a space is employed (for example N m).

Care should be taken when converting from one unit to a higher or lower power of the same measurement, for example, 2.0 mm is 2.0×10^{-3} m, not 2.0×10^3 m.

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1. SCOPE

This SAE Aerospace Information Report (AIR) provides details of how to perform hydraulic system calculations using equations that incorporate the metric International System of Units (SI).

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or +1 724-776-4970 (outside USA), www.sae.org.

AIR1362 Aerospace Hydraulic Fluids Physical Properties

ARP4386 Terminology and Definitions for Aerospace Fluid Power, Actuation and Control Technologies

AS1241 Fire Resistant Phosphate Ester Fluid for Aircraft

2.1.2 ASTM Publications

Available from ASTM International, 100 Barr Harbor Drive, P.O. Box C700, West Conshohocken, PA 19428-2959, Tel: 610-832-9585, www.astm.org.

ASTM SI 10 American National Standard for Use of the International System of Units (SI): The Modern Metric System

2.1.3 ICAO Publications

Available from International Civil Aviation Organization, 999 University Street, Montreal, Quebec H3C 5H7, Canada, Tel: +1 514-954-8219, <http://www.icao.int/>.

Doc 7488-CD Manual of the ICAO Standard Atmosphere: extended to 80 kilometres (262 500 feet)

2.1.4 NAS Publications

Available from Aerospace Industries Association, 1000 Wilson Boulevard, Suite 1700, Arlington, VA 22209-3928, Tel: 703-358-1000, www.aia-aerospace.org.

NAS 10001 Preferred Metric Units for Aerospace

2.1.5 U.S. Government Publications

Copies of these documents are available online at <https://quicksearch.dla.mil>.

MIL-PRF-5606 Hydraulic Fluid, Petroleum Base; Aircraft, Missile and Ordinance, NATO Code 515 (Inactive for new design)

MIL-PRF-83282 Hydraulic Fluid, Fire Resistant, Synthetic Hydrocarbon Base, Aircraft, NATO Code H537

MIL-PRF-87257 Hydraulic Fluid, Fire Resistant, Low Temperature, Synthetic Hydrocarbon Base, Aircraft and Missile, NATO Code 538

2.2 Related Publications

The following publications are provided for information purposes only and are not a required part of this SAE Technical Report.

2.2.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

TSB 003 Rules for SAE Use of SI (Metric) Units

2.2.2 ISO Publications

Copies of these documents are available online at <http://webstore.ansi.org/>.

ISO 80000-1 Quantities and units — Part 1: General

2.3 Definitions

FLUID SHEAR STRESS: This is the shear stress within a fluid while it is flowing which varies with the velocity gradient across a given sheared section.

NOTE: Hydraulic fluids can be considered as Newtonian fluids, in which the shear stress is proportional to the rate of shear.

HYDROSTATIC PRESSURE: This is the pressure that is statically generated by the weight of a column of fluid.

RATED SYSTEM PRESSURE: This is the nominal steady-state pressure that is generated by the hydraulic power generation system.

NOTE: Rated System Pressure is equivalent to the Design Operating Pressure (DOP) for commercial aircraft and System Pressure for military aircraft.

SPECIFIC GRAVITY: This is the ratio of the density of a substance to the density of a given reference material.

NOTE: For the purpose of this document, the substance is a hydraulic fluid and water at 15 °C and at sea level is the given reference material.

Refer to ARP4386 for the other definitions and terms.

3. DEFINITION OF METRIC UNITS

The SI system is coherent because force, pressure, and power are expressed in terms of basic quantities of mass, length, and time without empirical constants. SI engineering units have been devised to be scaled to each domain to facilitate calculations, handling and criticizing of numerical results because having a size 0.1 to 1000 suits the way the human brain processes them.

Kilogram (kg), meter (m), and second (s) are the basic units. Unity (1) is the coefficient of the basic units in their definition. Multiples of the basic units are defined by powers of 10^3 as shown in Table 1.

NOTE: kg is now only a unit of mass (not weight), weight is a force in N.

Table 1 - Recommended multiples of SI units

10^9	1000000000	giga	G
10^6	1000000	mega	M
10^3	1000	kilo	k
1	1	basic unit	basic unit
10^{-3}	0.001	milli	m
10^{-6}	0.000001	micro	μ
10^{-9}	0.000000001	nano	n

These multiples provide usefulness and convenience for practical purposes. Multiples other than powers of 10^3 are obsolete as shown in Table 2.

Table 2 - Not recommended multiples of units

10^2	100	hecto	h
10^1	10	deca	da
10^{-1}	0.1	deci	d
10^{-2}	0.01	centi	c

The SI metric system was established by the International Organization for Standardization (ISO) and supersedes the CGS metric system, where centimeter (cm), and gram (g) and second (s) were the basic units and the earlier metric system where kilogram force was the basic unit.

NOTE: If it is not mentioned in this document, to conform to SAE policy:

- Pressure is in kPa, not Mpa.
- Gallons are U.S. gallons.

Table 3 provides the conversion of metric units. NAS 10001 and ASTM SI 10 are available for conversion of metric and English units.

Table 3 - Metric units

Physical Quantity	Symbol	Dimension	SI (Metric) Unit	Other SI Unit	Conversion to SI Units
Mass	M	M	kg, kilogram	g, gram	10^{-3} kg
Length	l	L	m, meter	cm, centimeter mm, millimeter	10^{-2} m 10^{-3} m
Time	t	T	s, second	min, minute h, hour	60 s 3600 s
Angle	α	-	rad, radian	rev, revolution degree	2π rad $\pi/180$ rad
Absolute Temperature	T	-	K, Kelvin		
Temperature	T	-	°C, degree Celsius (centigrade)		K - 273.2
Frequency	f	T^{-1}	Hz, Hertz	cps, cycle/s	1 Hz
Angular frequency	ω	T^{-1}	Hz, Hertz	Rad/s, radian/s	1 Hz
Area	A	L^2	m^2	mm^2 cm^2	$10^{-6} m^2$ $10^{-4} m^2$
Volume	V	L^3	m^3	L, liter mL cm^3 , cc mm^3 , μL	$10^{-3} m^3$ $10^{-6} m^3$ $10^{-6} m^3$ $10^{-9} m^3$
Mass Density	ρ	ML^{-3}	kg/m^3	kg/L g/cm^3	$10^{-3} kg/m^3$ $10^{-3} kg/m^3$
Moment of Inertia	J	ML^2	$kg m^2$	$g cm^2$	
Velocity	v	LT^{-1}	m/s	km/s km/h	103 m/s 103/3600 m/s
Acceleration	a	LT^{-2}	m/s^2	g	$9.81 m/s^2$
Force	F	MLT^{-2}	N, Newton	kgf (force) kN, kilo N daN	9.81 N 103 N 10 N
Work, Energy, Heat	F x l	ML^2T^{-2}	J, Joule	W·h kW·h N·m	3600 J 3600 kJ 1 J
Pressure	P	$ML^{-1}T^{-2}$	Pa, Pascal	N/m^2 kPa Mpa bar mm Hg	1 Pa 103 Pa 106 Pa 105 Pa 133.3 Pa
Flow, Volumetric	Qv	L^3T^{-1}	m^3/s	L/s L/min	$10^{-3} m^3/s$ $10^{-3}/60 m^3/s$
Torque	T	ML^2T^{-2}	N·m, Newton-meter	kN m	10^3 N m
Speed of Rotation	ω	T^{-1}	rad/s	rpm, rev/minute	$2\pi/60$ rad/s
Power, Heat Flow	W Qh	ML^2T^{-3}	W, Watt	kW mW	10^3 W 10^{-3} W
Spring Rate	F/l	MT^{-2}	N/m	kN/m kN/mm daN/mm	10^3 N/m 10^6 N/m 10^4 N/m
Surface Tension	γ	MT^{-2}	N/m	mN/m	10^{-3} N/m
Bulk Modulus	B	$ML^{-1}T^{-2}$	Pa, Pascal	kPa Mpa Gpa	10^3 Pa 10^6 Pa 10^9 Pa
Dynamic Viscosity	μ	$ML^{-1}T^{-1}$	Pa s	mPa s centipoise Poiseuille	10^{-3} Pa s 10^{-3} Pa s 1 Pa s



Table 3 - Metric units (continued)

Physical Quantity	Symbol	Dimension	SI (Metric) Unit	Other SI Unit	Conversion to SI Units
Kinematic Viscosity	ν	L^2T^{-1}	m^2/s	mm^2/s centistoke, cSt	$10^{-6} m^2/s$ $10^{-6} m^2/s$
Specific Heat	C_p	L^2T^{-2}	J/(kg K)	kJ/(kg K)	$10^3 J/(kg K)$
Thermal Conductivity	k	MLT^{-3}	W/(m K)		
Surface Thermal Conductance	h_c	MT^{-3}	W/(m ² K)	kW/(m ² K)	$10^3 W/(m^2 K)$
Heat Capacity Rate	C	ML^2T^{-3}	W/K watt/K	kW/K mW/K	$10^3 W/K$ $10^{-3} W/K$

Table 4 provides factors for conversion from English units to SI Metric units.

Table 4 - Factors for converting from English units to SI Metric units

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Mass	M	M	lb _m slug	kg kg	0.4536 kg 14.594 kg
Length	l	L	nautical mile mile ft, foot in, inch μin, microinch	km km m mm μm	1.852 km 1.609 km 0.3048 m 25.4 mm 0.0254 μm
Time	t	T	s, second min, minute h, hour	s	1 s 60 s 3600 s
Angle	α	-	rev, revolution degree	rad, radian	2π rad π/180 rad
Absolute Temperature	T	-	°R, degree Rankine	K, Kelvin	K/1.8
Temperature	T	-	°F, degree Fahrenheit	C, Celsius	(F-32)/1.8
Frequency	f	T^{-1}	cps, cycle/s	Hz, hertz	1 Hz
Area	A	L^2	ft ² ft ² in ²	m ² cm ² mm ²	0.0929 m ² 929 cm ² 645 mm ²
Volume	V	L^3	ft ³ U.S. gallon Imperial gallon in ³ drop	L, liter L L mL mm ³	28.32 L 3.785 L 4.546 L 16.39 mL 50 mm ³
Mass Density	ρ	ML^{-3}	lbm/gal lbm/ft ³	kg/L kg/m ³	0.1198 kg/L 16.02 kg/m ³
Moment of Inertia	J	ML^2	lbm·ft ² lbm·in ² lbf·in·s ²	Kg m ² Kg mm ² kg m ²	0.04214 kg m ² 292.6 kg mm ² 0.113 kg m ²
Velocity	v	LT^{-1}	ft/s in/s knot mile/hour	m/s mm/s km/h km/h	0.3048 m/s 25.4 m/s 1.852 km/h 1.609 km/h
Acceleration	a	LT^{-2}	ft/s ²	m/s ²	0.3048 m/s ¹
Force	F	MLT^{-2}	lbf kip, kilopound	N, Newton kN	4.448 N 4.448 kN

Table 4 - Factors for converting from English units to SI Metric units (continued)

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Work, Energy, Heat	F x l	ML ² T ⁻²	ft·lbf BTU, British Thermal Unit	J kJ	1.356 J 1.055 kJ
Pressure	P	ML ⁻¹ T ⁻²	lbf/in ² , psi ksi, kilopound per in ² in Hg	kPa kpa kPa	6.895 kPa 6895 kPa 3.386 kPa
Flow, Volumetric	Q _v	L ³ T ⁻¹	gal/min, gpm in ³ /s, cis	L/min L/min	3.785 L/min 983.4 mL/min
Torque	T	ML ² T ⁻²	lbf·ft lbf·in oz·in	N·m N·m N·m	1.356 N·m 0.1130 N·m 0.0071 N·m
Speed of Rotation	ω	T ⁻¹	rpm, rev/minute	rad/s	2π/60 rad/s
Power, Heat Flow	W Q _h	ML ² T ⁻³	ft·lbf/s hp kBTU/h	W kW kW	1.356 W 0.7457 kW 0.293 kW
Spring Rate	F/l	MT ⁻²	lbf/in kip/in	N/m kN/m	175.1 N/m 175.1 kN/m
Surface Tension	γ	MT ⁻²	lbf/in grainf/in	N/m mN/m	175.1 N/m 25 mN/m
Bulk Modulus	B	ML ⁻¹ T ⁻²	kip/in ² , ksi	kPa	6895 kPa
Dynamic Viscosity	μ	ML ⁻¹ T ⁻¹	lbf·s/ft ² 10-3 lb·s/ft ²	Pa·s mPa·s centipoise	47.88 Pa·s 47.88 mPa·s 47.88 cP
Kinematic Viscosity	ν	L ² T ⁻¹	ft ² /s ft ² /s	m ² /s mm ² /s	0.09259 m ² /s 926 mm ² /s
Specific Heat	C _p	L ² T ⁻²	BTU/(lbm·F)	kJ/(kg·K)	4.187 kJ/(kg·K)
Thermal Conductivity	k	MLT ⁻³	BTU/(h·ft·F)	W/(m·K)	1.731 W/(m·K)
Surface Thermal Conductance	h _c	MT ⁻³	BTU/(h·ft ² ·F)	kW/(m ² ·K)	5.679 W/(m ² ·K)
Heat Capacity Rate	C	ML ² T ⁻³	BTU/(h·F)	W/K	0.527 W/K

4. USE OF METRIC UNITS IN HYDRAULIC SYSTEM ANALYSIS - HYDROSTATICS

4.1.1 Primary Units

The hydrostatic pressure (P) is by definition:

$$P = \rho gh \quad (\text{Eq. 1A})$$

$$(ML^{-1}T^{-2} = ML^{-3} \times LT^{-2} \times L) \quad (\text{Eq. 1B})$$

where:

P = pressure, (Pa) - (ML⁻¹T⁻²)

ρ = mass density, (kg/m³) - (ML⁻³)

g = gravitational acceleration, (m/s²) - (LT⁻²)

$g = 9.81 \text{ m/s}^2$ at sea level, 45 degree latitude

h = static pressure head, (m) - (L)

4.1.1.1 Practical Units

By the definition of liter (L):

$$1 \text{ L} = 1 \text{ m}^3/1000 \quad (\text{Eq. 2})$$

Therefore:

$$\rho = \text{mass density (kg/L)} \quad (\text{Eq. 3A})$$

$$1 \text{ kg/L} = 1000 \text{ kg/m}^3 \quad (\text{Eq. 3B})$$

The hydrostatic pressure in practical units is:

$$P = \rho gh \quad (\text{Eq. 4})$$

where:

P = pressure, (kPa) - (1 kPa = 1000 Pa)(10^3 Pa , $\text{ML}^{-1}\text{T}^{-2}$)

ρ = mass density, (kg/L) - (10^3 kg/m^3 , ML^{-3})(1 kg/L = 1000 kg/m³)

g = gravitational acceleration, (m/s^2) - (LT^{-2}) = 9.81 m/s^2 at sea level, 45 degree latitude

h = head, (m) - (L)

4.1.1.2 Application - Suction Pressure

Given:

- MIL-PRF-5606 hydraulic fluid at 20 °C.
- The hydraulic reservoir is at 2.27 m above the baseline.
- An actuator is at 5.32 m above the baseline and is directly connected to the reservoir via the return connection of the controlling valve.

The hydraulic reservoir is open to the sea-level atmosphere during maintenance.

Calculate:

- The suction pressure (P) that the actuator seals should be able to withstand without air ingestion.
- The absolute pressure (P) at the actuator.

The density of MIL-PRF-5606 at 20 °C and 0 kPa per Figure 1 is:

$$\rho = 0.859 \text{ kg/L} \quad (\text{Eq. 5})$$

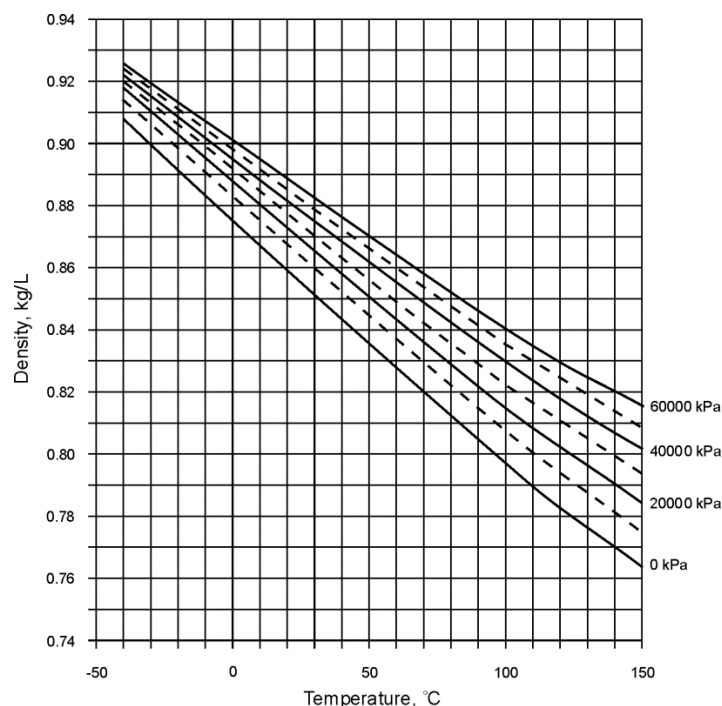


Figure 1 - Mass density of MIL-PRF-5606

The difference in elevation is:

$$h = 5.32 - 2.27 = 3.05 \text{ m} \quad (\text{Eq. 6})$$

The suction pressure (below atmospheric) in kilopascals (kPa):

$$P = 0.859 \times 9.81 \times 3.05 \quad (\text{Eq. 7A})$$

$$P = 25.7 \text{ kPa} \quad (\text{Eq. 7B})$$

The standard sea-level atmospheric pressure is 101.3 kPa per ICAO Doc 7488-CD.

The absolute pressure at the actuator is:

$$101.3 - 25.7 = 75.6 \text{ kPa} \quad (\text{Eq. 8})$$

4.2 Hydrostatic Axial Force

4.2.1 Primary Units

The axial force is:

$$F = A \times P \quad (\text{Eq. 9A})$$

$$(\text{MLT}^{-2} = \text{L}^2 \times \text{ML}^{-1}\text{T}^{-2}) \quad (\text{Eq. 9B})$$

where:

F = force, (Newton, N) - (MLT^{-2})

A = area, (m^2) - (L^2)

P = pressure, (Pascal, Pa) - ($\text{ML}^{-1}\text{T}^{-2}$)

4.2.2 Practical Units

F = force, (Newton, N) - (MLT⁻²)

A = area, (mm²) - (10⁻⁶ m², L²)

P = pressure, (kPa) - (10³ Pa, ML⁻¹T⁻²)

4.2.3 Application - Hydrostatic Test

A single-acting hydraulic cylinder of 50.8 mm inside diameter is proof pressure tested to 1.5 factor of the rated system pressure, 20690 kPa.

Calculate the axial force (F) on the cylinder cap on the full area side.

The test pressure is:

$$P = 1.5 \times \text{rated system pressure} \quad (\text{Eq. 10A})$$

$$P = 1.5 \times 20690 \text{ kPa} \quad (\text{Eq. 10B})$$

$$P = 31035 \text{ kPa} \quad (\text{Eq. 10C})$$

The cap area is:

$$A = \pi \times (50.8)^2 / 4 \quad (\text{Eq. 11A})$$

$$A = 2027 \text{ mm}^2 \quad (\text{Eq. 11B})$$

The force is:

$$F = 31035 \times 10^3 \text{ Pa} \times 2027 \times 10^{-6} \text{ m}^2 \quad (\text{Eq. 12A})$$

$$F = 62908 \text{ N} \quad (\text{Eq. 12B})$$

$$F = 62.9 \text{ kN} \quad (\text{Eq. 12C})$$

5. USE OF METRIC UNITS IN HYDRAULIC SYSTEM ANALYSIS - HYDRODYNAMICS

5.1 Bernoulli's Theorem

5.1.1 Total Head

The total head in steady state flow of an incompressible fluid is:

$$H = h + v^2 / 2g + z \quad (\text{Eq. 13A})$$

$$(L = L + (LT^{-1})^2 / LT^{-2} + L) \quad (\text{Eq. 13B})$$

where:

h = static head, (m) - (L)

NOTE: Head "loss" can be either negative or positive if energy is introduced from outside the considered fluid domain.

v = velocity of flow, (m/s) - (LT^{-1})

g = gravitational acceleration, (m/s^2) - (LT^{-2})

z = elevation of section above datum, (m) - (L)

Bernoulli's theorem states that total head (H) along a streamline is constant, provided that the head losses (DH) between sections are accounted for:

$$H_1 = H_2 + DH \quad (\text{Eq. 14A})$$

$$h_1 + v_1^2 / 2g + z_1 = h_2 + v_2^2 / 2g + z_2 + DH \quad (\text{Eq. 14B})$$

where:

Subscripts 1 and 2 refer to sections

DH = loss in total head, (m) - (L)

5.1.2 Velocity Head

By definition, the velocity head (h_v) is:

$$h_v = v^2 / 2g \quad (\text{Eq. 15A})$$

$$(L = (LT^{-1})^2 / LT^{-2}) \quad (\text{Eq. 15B})$$

5.1.3 Application - Liquid Column Head

MIL-PRF-5606 fluid at 50 °C and 20690 kPa pressure is flowing in a tube at a velocity of 3 m/s. What is the velocity head in terms of:

a. Liquid column height?

b. Pressure?

1. In terms of liquid column height:

$$h_v = v^2 / 2g \quad (\text{Eq. 16})$$

where:

$v = 3.0 \text{ m/s}$ - (LT^{-1})

$g = 9.81 \text{ m/s}^2$ - (LT^{-2}) at sea level, 45 degree latitude

Then:

$$h_v = 3.0^2 / (2 \times 9.81) \quad (\text{Eq. 17A})$$

$$h_v = 0.46 \text{ m} \quad (\text{Eq. 17B})$$

2. In terms of pressure head (P_v) (dynamic pressure):

$$P_v = \rho g h_v \quad (\text{Eq. 18A})$$

$$(ML^{-1}T^{-2} = ML^{-3}LT^{-2}L)$$

$$P_v = (\rho g) \times (v^2 / 2g) = \rho v^2 / 2 \quad (\text{Eq. 18C})$$

$$(ML^{-1}T^{-2} = ML^{-3}(LT^{-1})^2) \quad (\text{Eq. 18D})$$

where:

$$\rho = 0.85 \text{ kg/L} - (ML^{-3}) \text{ per Figure 1}$$

$$v = 3 \text{ m/s} - (LT^{-1})$$

Then:

$$P_v = 0.85 \times 3^2 / 2 \quad (\text{Eq. 19A})$$

$$P_v = 3.83 \text{ kPa} \quad (\text{Eq. 19B})$$

5.2 Viscosity

5.2.1 Dynamic Viscosity

The dynamic viscosity is:

$$\mu = (F / A) / (dv/dy) \quad (\text{Eq. 20A})$$

$$(ML^{-1}T^{-1} = (MLT^{-2}/L^2) / (LT^{-1}/L)) \quad (\text{Eq. 20B})$$

In primary units:

$$\mu = \text{dynamic viscosity, (Pa}\cdot\text{s)} - (ML^{-1}T^{-1})$$

$$F = \text{force (N), Newton} - (MLT^{-2})$$

$$A = \text{area (m}^2\text{)} - (L^2)$$

$$dv = \text{variation of velocity, (m/s)} - (LT^{-1})$$

$$dy = \text{normal distance, (m)} - (L)$$

In practical units:

$$\mu = \text{dynamic viscosity, (centipoise)} - (ML^{-1}T^{-1})$$

NOTE: 1 centipoise = 1 mPa s

$$F = \text{force (N), Newton} - (MLT^{-2})$$

$$A = \text{area, (mm}^2\text{)} - (10^{-6} \text{ m}^2, L^2)$$

$$dv = \text{variation of velocity, (m/s)} - (LT^{-1})$$

$$dy = \text{normal distance, (mm)} - (10^{-3} \text{ m}, L)$$

5.2.2 Kinematic Viscosity

It is by definition, the shear force per unit area and per unit mass density due to a unit velocity gradient normal to the flow:

$$\nu = (F / (A \rho)) / (dv/dy) \quad (\text{Eq. 21A})$$

$$(L^2T^{-1} = (MLT^{-2} / (L^2 \times ML^{-3})) / (LT^{-1}/L)) \quad (\text{Eq. 21B})$$

That is:

$$\nu = \mu / \rho \quad (\text{Eq. 22A})$$

$$(L^2T^{-1} = ML^{-1}T^{-1} / ML^{-3}) \quad (\text{Eq. 22B})$$

In basic units:

ν = kinematic viscosity, (m²/s) - (L²T⁻¹)

F = force (N), Newton - (MLT⁻²)

A = area, (m²) - (L²)

ρ = mass density, (kg/m³) - (ML⁻³)

dv = variation of velocity, (m/s) - (LT⁻¹)

dy = normal distance, (m) - (L)

In practical units:

ν = kinematic viscosity, (mm²/s) - (10⁻⁶ m² /s, L²T⁻¹)

F = force, (Newton) - (MLT⁻²)

A = area, (mm²) - (10⁻⁶ m², L²)

ρ = mass density, (kg/liter) - (ML⁻³)

dv = variation of velocity, (m/s) - (LT⁻¹)

dy = normal distance, (mm) - (10⁻³ m, L)

See Table 3 where 1 mm²/s = 1 centistoke.

5.2.3 Fluid Shear Stress

Fluid shear stress (P) is by definition:

$$\tau = \mu v / \delta \quad (\text{Eq. 23A})$$

$$(\text{ML}^{-1}\text{T}^{-2} = \text{ML}^{-1}\text{T}^{-1} \times \text{LT}^{-1}/\text{L}) \quad (\text{Eq. 23B})$$

where:

τ = shear stress, (Pa) - ($\text{ML}^{-1}\text{T}^{-2}$)

μ = absolute viscosity, (mPa s) - ($\text{ML}^{-1}\text{T}^{-1}$)

v = velocity, (m/s) - (LT^{-1})

δ = radial clearance, (m) - (L)

5.2.4 Examples

5.2.4.1 Absolute Viscosity

Calculate the absolute viscosity (μ) of MIL-PRF-83282 hydraulic fluid at -30 °C and 10000 kPa by solving Equation 24:

$$\mu = v \rho \quad (\text{Eq. 24})$$

Viscosity $v = 1400 \text{ mm}^2/\text{s}$ (centistokes) obtained from Figure 2.

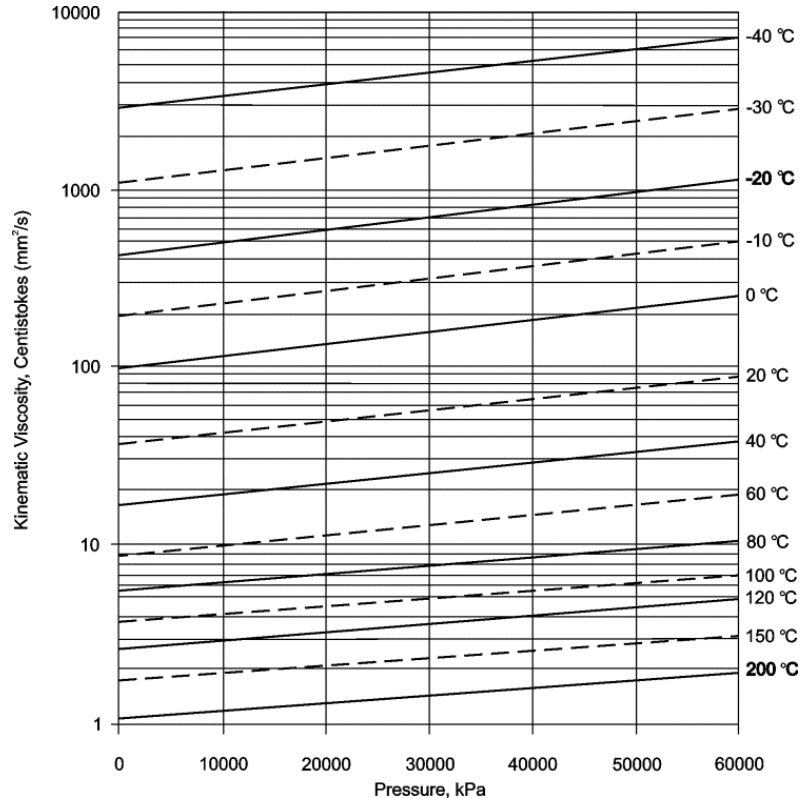


Figure 2 - Kinematic viscosity versus pressure, MIL-PRF-83282

Density $\rho = 0.88 \text{ kg/L}$ obtained from Figure 3.

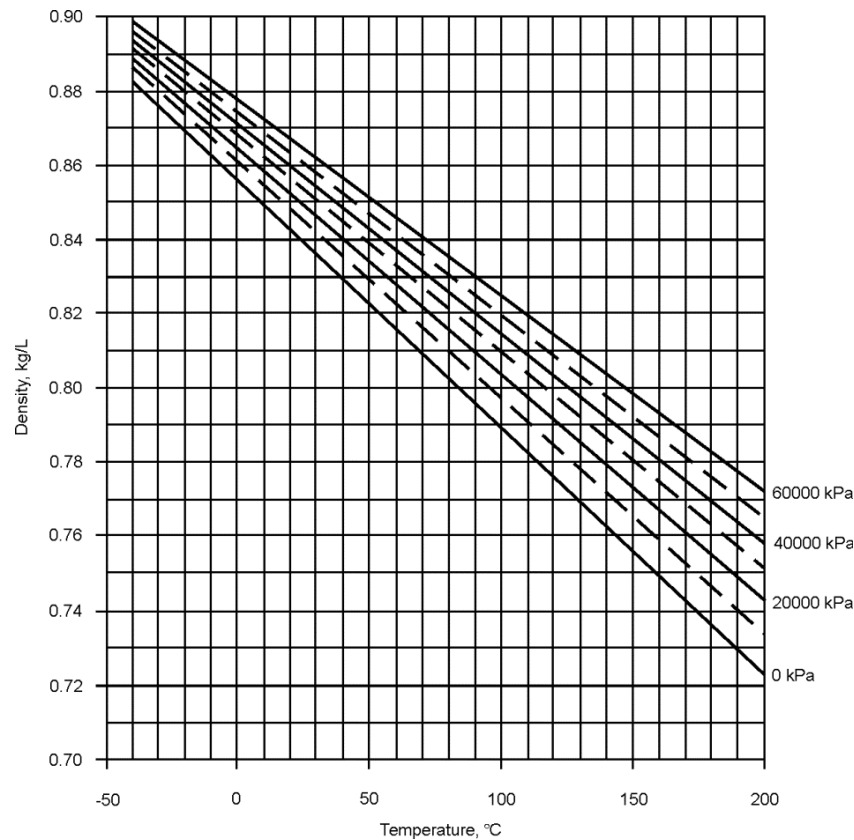


Figure 3 - Mass density of MIL-PRF-83282

In primary units:

$$\mu = 1400 \times 10^{-6} \text{ m}^2/\text{s} \times 880 \text{ kg/m}^3 = 1.23 \text{ Pa s} \quad (\text{Eq. 25})$$

In practical units:

$$\mu = 1400 \text{ mm}^2/\text{s} \times 0.88 \text{ kg/L} = 1230 \text{ mPa s} \quad (\text{Eq. 26})$$

In the usual (CGS) units:

$$\mu = 1400 \text{ cst} \times 0.88 \text{ g/cc} = 1230 \text{ centipoise} \quad (\text{Eq. 27A})$$

$$\mu = 1230 \text{ mPa s} \quad (\text{Eq. 27B})$$

5.2.4.2 Application - Direct Drive Valve

Calculate the fluid shear stress (τ) between a valve slide and its sleeve using MIL-PRF-83282 hydraulic fluid at -30°C and 10000 kPa.

The slide is stroked 0.6 mm in 8 ms at a constant speed.

The diametral clearance is $3.8 \mu\text{m}$ ($150 \mu\text{in}$).

The slide is assumed to be centered.