



AEROSPACE INFORMATION REPORT

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Superseding AIR1678B

Uncertainty of In-Flight Thrust Determination

RATIONALE

AIR1678B has been reaffirmed to comply with the SAE Five-Year Review policy.

FOREWORD

In 1972, the Safety Standardization Advisory Committees of the SAE Aerospace Council, working with the SAE Propulsion Division suggested the need for improved knowledge of aircraft propulsion system in-flight thrust. Later, the U.S. Air Force Aeronautical Systems Division independently made a similar suggestion.

The Propulsion Division and the Aerospace Council concluded that the real need was to establish a forum where this subject could be discussed by knowledgeable experts on a technical basis. SAE undertook to do so. Dr. Robert Abernethy was commissioned by George Townsend, Propulsion Division Chairman, to organize the In-Flight Propulsion Measurement Committee E-33. Mr. Gary Adams, representing the Air Force, was also involved in organizing the E-33 Committee at this early stage. The first meeting took place in December 1978.

The E-33 Committee endeavored to gather industry-wide expertise in in-flight thrust measurement and uncertainty analysis. The committee was organized into Subcommittees A/B which concentrated on thrust-drag bookkeeping and thrust determination methodology and Subcommittee C which addressed the subject of thrust determination uncertainty.

After reviewing the industry state-of-the-art, Committee E-33 determined that it would be appropriate to assemble and publish two companion Aerospace Information Reports. Subcommittee E-33A/B was organized under Chairman John Roberts to produce AIR1703, "In-Flight Thrust Determination." Subcommittee E-33C, under Chairman Gary Adams, produced AIR1678, "Uncertainty of In-Flight Thrust Determination." Together these reports provided a comprehensive survey of in-flight thrust determination, beginning with definitions and concluding with guidelines for planning a total program and estimating the measurement errors.

The original version of AIR1678 was published in 1985 and later reaffirmed in December 1992. In 1996, Subcommittee E-33C under the chairmanship of Mike McGonigle undertook a revision to the document to bring it into compliance with other national and international standards, primarily ANSI/ASME PTC 19.1-1998 and the ISO Guide to the Expression of Uncertainty in Measurement. This work was completed by the committee under the leadership of Glenn Steele and published in 2002. Revision B continued this development and was completed in 2011.

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INTRODUCTION

Estimating the measurement error or uncertainty is an essential element in the complex process of evaluating in-flight thrust. During the pre-flight planning phase, it is necessary to assess the accuracy of the candidate methodologies for determining in-flight thrust and to select those which best satisfy program requirements and resources. Estimation of measurement error defines parameter accuracy, data acquisition, and data reduction requirements for the ground and flight test phases; and post-flight is used to examine the consistency of the test results with pretest predictions and with vehicle performance estimates.

The purpose of this document is to provide information covering methods for estimating the effects of the measurement error or uncertainty of in-flight thrust determination in aircraft employing conventional turbofan/turbojet engines. Methodologies beyond those presented are required in order to evaluate configurations such as vectored thrust or V/STOL. This document is intended to be used as a technical guide. It is not intended to be used as a standard or legal document. A companion document, Reference 2.1.1.1, presents information and guidance on the selection and use of methodologies to predict and assess propulsion system in-flight thrust. Both documents describe comprehensive procedures and tasks for implementing the methodologies. Each program would select those tasks that are appropriate to meet its particular objectives.

The term "in-flight thrust determination" is used synonymously with "in-flight thrust measurement" although in-flight thrust is not directly measured but is determined or calculated using mathematical modeling relationships between in-flight thrust and various direct measurements of physical quantities. The in-flight thrust determination process includes both Ground Testing and Flight Testing, Figure 1. The mathematical modeling relationships between the in-flight thrust and the measurements of the physical quantities are calibrated in Ground Tests. Error effect estimates for each item shown in Figure 1 are required to calculate the uncertainty of the "in-flight thrust measurement."

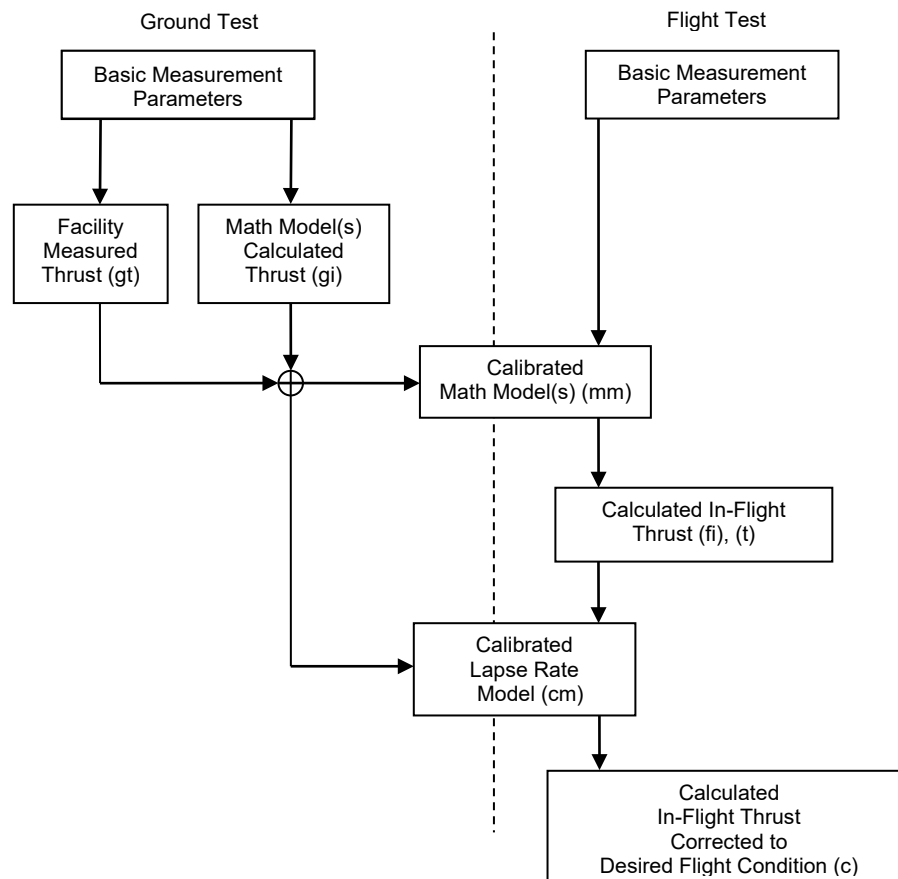


FIGURE 1 - THE IN-FLIGHT THRUST MEASUREMENT PROCESS

The text is organized into the following major topics:

Measurement Uncertainty Methodology (Section 4): This section describes the concepts involved in estimating the uncertainty of in-flight thrust. Its purpose is to provide a common basis for understanding. The methodology includes the use of statistical and engineering concepts to meet the need for an easily interpreted estimate of uncertainty (References 2.1.2.1, 2.1.4.1).

In-Flight Thrust Measurement Processes (Section 5): This section describes the ground test and flight test phases of the in-flight thrust determination process. The five sets of fundamental error components which must be evaluated to determine the uncertainty of in-flight thrust are identified.

Application of Uncertainty Methodology to the In-Flight Thrust Program (Section 6): The uncertainty methodology described in Section 4 is applied to the In-Flight Thrust Measurement Process described in Section 5. Four representative test program phases are defined, i.e., Program Definition Planning, Ground Test, Flight Test, and Results Analysis. The tasks to be accomplished in each phase are described in Reference 2.1.4.5 and shown in Table 8.

Seven Appendices are also included:

- Appendix A - Small Sample Statistical Methods

Special, small sample, statistical methods are required to determine the value for the Student's "t₉₅" distribution when the degrees of freedom of the result is less than 10 (Reference 2.1.4.2). This appendix discusses the t value, degrees of freedom, and the propagation of degrees of freedom. A table of "t₉₅" values is presented for degrees of freedom from one to thirty.

- Appendix B - Impact of Measurement Process on Error Classification and Estimation

This appendix discusses the decision logic for the classification of error based on the actual measurement process. Reclassification is discussed and the concept of fossilized error is introduced.

- Appendix C - Example Influence Coefficient Calculations
- Appendix D - Initial Performance Survey of Thrust Measurement Options

This appendix uses the published results of the Air Launched Cruise Missile Program to illustrate the approach for conducting an initial performance survey of thrust options.

- Appendix E - Calibrated Mathematical Model and the Associated Uncertainty Components for Flight Test

The term "calibrated mathematical model" is discussed and simplified examples of two common approaches are provided.

- Appendix F – Nonsymmetric Uncertainties
- Appendix G – Background of Uncertainty Methodology

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1. SCOPE

This document defines and illustrates the process for determination of uncertainty of turbofan and turbojet engine in-flight thrust and other measured in-flight performance parameters. The reasons for requiring this information, as specified in the E-33 Charter, are:

- determination of high confidence aircraft drag;
- problem rectification if performance is low;
- interpolation of measured thrust and aircraft drag over a range of flight conditions by validation and development of high confidence analytical methods;
- establishment of a baseline for future engine modifications.

This document describes systematic and random measurement uncertainties and methods for propagating the uncertainties to the more complicated parameter, in-flight thrust. Methods for combining the uncertainties to obtain given confidence levels are also addressed. Although the primary focus of the document is in-flight thrust, the statistical methods described are applicable to any measurement process.

The E-33 Committee has endeavoured to gather industry-wide expertise in in-flight measurement and uncertainty analysis to collect and promulgate recommended practices in the subject disciplines. The Committee is organized into subcommittees to address both the analytical and test methodology for determination of in-flight thrust and also the uncertainty of the determination. This document; Uncertainty of In-flight Thrust Determination, AIR1678, addresses the process for determining the uncertainty of in-flight thrust. A companion document, In-Flight Thrust Determination, AIR1703, addresses the basic methodology for determining in-flight thrust.

The Committee, after reviewing recommended changes and clarification in definitions and application of statistical uncertainty items, made small revisions to the original document published in 1985. These changes were incorporated into AIR1678 Rev A.

This Revision B has the same Scope as preceding versions. The nomenclature and methodology used herein are now consistent with evolving world and national standards promulgated primarily by ISO and ASME.

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

2.1.1.1 AIR1703, In-Flight Thrust Determination

2.1.1.2 831439, Application of In-Flight Thrust Determination Uncertainty, SAE Technical Paper, Adams, G. R., Thompson, J. W., Abernathy, R. B., Biesiadny, T., et al., Presented at the Aerospace Congress and Exposition, Long Beach, California, October 1983, doi:120.4271/831439.

2.1.2 ANSI Publications

Available from American National Standards Institute, 25 West 43rd Street, New York, NY 10036-8002, Tel: 212-642-4900, www.ansi.org.

2.1.2.1 ANSI/ASME PTC 19.1-2005 (Revision of ASME PTC 19.1-1998), Test Uncertainty

2.1.2.2 ISO 10012-1 1992, Quality Assurance requirements for measuring equipment - Part 1: Metrological confirmation system for measuring equipment.

2.1.2.3 ISO 1995, Guide to the Expression of Uncertainty in Measurement.

2.1.3 ASME Publications

Available from American Society of Mechanical Engineers, 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900, Tel: 973-882-1170, www.asme.org.

2.1.3.1 80-GT-9, Improvement of Parameter Estimation Through Uncertainty Analysis, Afdams, G. R., ASME Paper, Presented at the Gas Turbine Conference and Products Show, New Orleans, LA, March 1980.

2.1.4 Applicable References

2.1.4.1 H. W. Coleman and W. G. Steele, "Engineering Application of Experimental Uncertainty Analysis," AIAA Journal, Vol. 33, No. 10, Oct. 1995, pp. 1888-1896.

2.1.4.2 W. G. Steele, R. A. Ferguson, R. P. Taylor, and H. W. Coleman, "Comparison of ANSI/ASME and ISO Models for Calculation of Uncertainty," ISA Transactions, Vol. 33, 1994, pp. 339-352.

2.1.4.3 P. J. Campion, J. E. Burns, and A. Williams, Section 5 Recommendations, Her Majesty's National Physical Laboratory Report "A Code of Practice for the Detailed Statement of Accuracy," 1973.

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2.1.4.6 W. G. Steele, P. K. Maciejewski, C. A. James, R. P. Taylor, and H. W. Coleman, "Asymmetric Systematic Uncertainties in the Determination of Experimental Uncertainty," AIAA Journal, Vol. 34, No. 7, July 1996, pp. 1458-1463.

2.1.4.7 K. K. Brown, H. W. Coleman, W. G. Steele, and R. P. Taylor, "Evaluation of Correlated Bias Approximations in Experimental Uncertainty Analysis," AIAA Journal, Vol. 34, No. 5, May 1996, pp. 1013-1018.

2.1.4.8 H. W. Coleman and W. G. Steele, "Implications of Correlated Bias Uncertainties in Single and Comparative Tests," Journal of Fluids Engineering, Vol. 117, Dec. 1995, pp. 552-556.

2.1.4.9 H. W. Coleman and W. G. Steele, "Experimentation, Validation, and Uncertainty Analysis for Engineers", 3rd Edition, John Wiley and Sons, New York, NY, 2009.

2.1.4.10 B. D. Couch, W. O. Boals, and B. M. Bishop "ALCM Preflight-Test Thrust Uncertainty Analysis," AEDC-TR-81-2, July, 1981.

2.1.4.11 R. B. Abernethy and Barbara Ringhiser, United Technologies Corp., "The History and Statistical Development of the New ASME-SAE-AIAA-ISO Measurement Uncertainty Methodology," AIAA paper 85-1403, July 8, 1985.

2.1.4.12 J. W. Thompson, W. F. Kimzey, and W. O. Boals, "An Overview of the Application of Standard Measurement Uncertainty Methodology to Propulsion Testing," AGARD Conference Proceedings No. 429, September 1987.

- 2.1.4.13 R. B. Abernethy, et al, "ICRPG Handbook for Estimating the Uncertainty in Measurements Made with Liquid Propellant Rocket Engine Systems," JANNAP (formerly ICRPG) Performance Standardization Working Group Report CPIA No. 180 (AD 851 127), April 1969.
- 2.1.4.14 "Uncertainty of In-Flight Thrust Determination," SAE AIR1678, 1985.
- 2.1.4.15 R. B. Abernethy, "SAE In-Flight Propulsion Measurement Committee E-33: Its Life and Work," SAE in Aerospace Engineering Volume 1, No. 1, July 1981.
- 2.1.4.16 R. B. Abernethy, et al, "Uncertainty Methodology for In-Flight Thrust Determination," SAE paper 831438, October 1983. See also SAE paper 831439 for application.
- 2.1.4.17 ANSI/ASME MFC-2M, "Measurement Uncertainty for Fluid Flow in Closed Conduits," 1983.
- 2.1.4.18 R. B. Abernethy, R. P. Benedict, and R. P. Dowdell, "ASME Measurement Uncertainty," ASME paper 83WA/FM-3.
- 2.1.4.19 ANSI/ASME PTC 19.1, "Measurement Uncertainty," Performance Test Codes Supplement, 1985.
- 2.1.4.20 AIAA Thrust-Drag Editorial Board, chaired by Dr. E. E. Covert, "Measure and Prediction of Thrust and Drag from Transports to Fighters," 1985.
- 2.1.4.21 D. L. Colbert, B. D. Powell, and R. B. Abernethy, "The ICRPG Measurement Uncertainty Model," AIAA paper No. 69-734.
- 2.1.4.22 R. A. Abernethy and J. W. Thompson, Jr., "Uncertainty in Gas Turbine Measurements," AIAA paper No. 73-1230.
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- 2.1.4.24 R. B. Abernethy and R. P. Benedict, "Measurement Uncertainty: A Standard Methodology," ISA paper 1984, 0-87664-806-5/84.
- 2.1.4.25 ISO TC30 SC9, "Fluid Flow Measurement Uncertainty," Draft of January 1985 (Revision to ISO 5168).
- 2.1.4.26 International Organization for Standardization, "International Vocabulary of Basic and General Terms in Metrology," 2nd Edition, 1993 (Geneva, Switzerland).
- 2.1.4.27 W. G. Steele, R. P. Taylor, R. E. Burrell, and H. W. Coleman, "Use of Previous Experience to Estimate Precision Uncertainty of Small Sample Experiments," AIAA Journal, Vol. 31, No. 10, October 1993, pp. 1891-1896.
- 2.1.4.28 B. N. Taylor and C. E. Kuyatt, "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," NIST Technical Note 1297, 1994 Edition.
- 2.1.4.29 Joint Committee for Guides in Metrology (JCGM), "Evaluation of Measurement Data – Supplement 1 to the "Guide to the Expression of Uncertainty in Measurement" – Propagation of Distributions using a Monte Carlo Method", JCGM 101:2008, France, 2008.
- 2.1.4.30 "Measurement Uncertainty Applied to Cost-Effective Testing", SAE AIR5925, 2002.
- 2.1.4.31 "Advanced Ducted Propulsor In-Flight Thrust Determination", SAE AIR5450 2008
- 2.1.4.32 "Turbofan and Turbojet Gas Turbine Engine Test Cell Correlation", SAE ARP741 Rev B, 2002.

2.2 Symbols

Greek

β	True systematic error: the fixed, systematic or constant component of the total error, δ . It is constant for every measurement and is an unknown theoretical quantity.
$\tilde{\beta}_r$	Observed systematic error between the uncalibrated calculated thrust and the facility measured thrust.
β_r	Observed systematic error between the calibrated calculated thrust and the facility measured thrust.
δ	Total error: the difference between the measurement and the true value : $\delta = \beta + \varepsilon$. It is an unknown theoretical quantity.
ε	True random error, sometimes called repeatability error or sampling error, the random component of δ . Its value is characteristic of each measurement and is an unknown theoretical quantity.
μ	Mean of the parent population (i.e. mean of the set of all population values of a parameter). An unknown theoretical quantity.
ν	Number of degrees of freedom.
σ	Standard deviation of the parent population. It is an unknown theoretical quantity.
σ^2	True variance of a population. An unknown theoretical quantity.
θ	Absolute influence coefficient or absolute sensitivity.
θ'	Relative influence coefficient or relative sensitivity.

Roman

b_X	Systematic standard uncertainty of a single measurement.
$b_{\bar{X}}$	Systematic standard uncertainty of the sample mean. Since the systematic error remains constant for every measurement, the systematic uncertainty of the sample mean equals the systematic uncertainty of a single measurement : $b_X = b_{\bar{X}}$.
B	Expanded systematic uncertainty and is the estimate of the limit of the systematic error β , for a stated confidence level, usually 95%.
B^+, B^-	Expanded systematic uncertainties or estimates of the upper and lower limits of a nonsymmetrical systematic error with a normal distribution. B^+ and B^- are the upper and lower values for the nonsymmetrical interval defined such that $[-B^-, B^+]$ contains the true systematic error with a stated confidence level, usually 95%.
F	Thrust.
$f()$	Functional relationship.
K	Order of curve fit.
M_L, M_U	Range of measurements, Lower and Upper.

N_i	Number of measurements or sample points or observations available for the measurement X_i (sample size).
I	Number of elemental measurements used in the calculation of the performance parameter.
r	Calibration model error, the absolute difference between calculated and measured performance.
r'	Calibration model error, the relative difference between calculated and measured performance.
s_X	Random standard uncertainty of a sample of N_i measurements.
$s_{\bar{X}}$	Random sample standard uncertainty of the mean (of a sample of N_i measurements) where $s_{\bar{X}} = \frac{s_X}{\sqrt{N_i}}$.
s_r	Standard deviation of the observed calibration model error.
$\tilde{s}_r$	Standard deviation of the observed uncalibrated model error.
$\hat{s}_r$	Estimated value of calibration model error standard deviation.
t	Student's t value at a specified confidence level with v degrees of freedom, i.e., $t_{95, v}$.
u	Combined standard uncertainty.
U	Expanded uncertainty, with a defined level of confidence (usually 95 %).
U^+, U^-	Upper and lower limits of a nonsymmetrical uncertainty interval.
X	Value of an individual measurement.
$\bar{X}$	Mean of the sample population (also called sample mean).
Y	A performance parameter.

2.3 Subscripts

c	Flight test corrected to desired test conditions
cm	Lapse rate correction model error
f	Flight test
fi	Flight instrumentation (engine)
g	Ground test
gt	Ground test facility instrumentation
gi	Ground instrumentation (engine)
i	Counter for variables (also defined as elemental measurements or basic measurements)
j	Counter for the individual measurements
k	Counter for sources of elemental errors and uncertainties within a category (p)



p	Counter for error categories
	p = 1 Calibration errors
	p = 2 Test Article and / or Instrumentation Installation Errors
	p = 3 Data acquisition errors
	p = 4 Data reduction errors
	p = 5 Errors of method
m	Counter for multiple results
meas	Measured
mm	Mathematical model error
N	Net
r	Calibration model error
t	As-tested flight condition

2.4 Superscripts

*	Fossilized random error
---	-------------------------

3. GLOSSARY

ACCURACY: This is a qualitative and not a quantitative term for the closeness or agreement between a measured value and the true value.

AVERAGE VALUE ($\bar{X}$): The arithmetic mean of N readings.

CALIBRATION: The process of comparing the response of an instrument to a standard instrument over the measurement range. Corrections may be applied in an attempt to reduce the instrument systematic error.

CALIBRATION HIERARCHY: The chain of calibrations which links or traces a measuring instrument to a primary standard.

CONFIDENCE LEVEL (COVERAGE): The probability that the true value falls within the specified limits.

DEGREES OF FREEDOM (ν): The number of independent observations used to calculate a statistic. One degree of freedom is lost for each previously calculated statistic used to calculate a new statistic.

ELEMENTAL RANDOM ERROR SOURCE: An identifiable source of random error that is a subcomponent of total random error.

ELEMENTAL RANDOM STANDARD UNCERTAINTY ($s_{\bar{X}_{kp}}$): An estimate of the standard deviation of the mean of an elemental random error source.

ELEMENTAL SYSTEMATIC ERROR SOURCE: An identifiable source of systematic error that is a subcomponent of total systematic error.

ELEMENTAL SYSTEMATIC STANDARD UNCERTAINTY ($b_{\bar{X}_{kp}}$): An estimate of the standard deviation of the mean of an elemental systematic error source.

EXPANDED UNCERTAINTY ($U_{\bar{X}}$): An estimate of the plus or minus limits of total error, with a defined level of confidence (usually 95%).

ESTIMATE: A value calculated from a sample of data as a substitute for an unknown population constant. For example, the sample standard deviation (s) is an estimate of the population standard deviation (σ).

FOSSILIZATION: Sometimes the random error from the calibration process becomes part of the fixed systematic error. This conversion from random to systematic error is indicated by an asterisk superscript.

INFLUENCE COEFFICIENT (θ): see SENSITIVITY

LABORATORY STANDARD: An instrument which is calibrated periodically against a primary standard. The laboratory standard may also be called an interlab standard.

MEASUREMENT ERROR: The difference between the measured value and the true value. It includes both systematic and random errors.

PARENT POPULATION: Total theoretical population.

PRIMARY STANDARD: A primary physical standard or one maintained at a National Standards Laboratory.

RANDOM ERROR (ε): The portion of the total error that varies randomly in repeated measurements of the true value throughout a test process.

RANDOM STANDARD UNCERTAINTY (s_X): A value that quantifies the dispersion of any measurement of the data sample

RANDOM STANDARD UNCERTAINTY OF THE SAMPLE MEAN ($s_{\bar{X}}$): A value that quantifies the dispersion of a sample mean.

REPEATABILITY ERROR: See random error.

RESIDUAL STANDARD DEVIATION : The measure of dispersion of the dependent variable about a least squares regression or curve.

RESULT: A value calculated from a number of parameters.

SAMPLE STANDARD DEVIATION : see RANDOM STANDARD UNCERTAINTY

SAMPLE SIZE (N_i): The number of independent measurements of the basic measurement X_i in a sample.

SENSITIVITY: The instantaneous rate of the change in a result due to a change in a parameter.

STANDARD DEVIATION (σ): The true measure of dispersion of a population.

STATISTIC: Any numerical quantity derived from the sample data. $\bar{X}$ and $s_{\bar{X}}$ are statistics

STUDENT'S t: A value used to estimate the uncertainty for a given confidence level.

SYSTEMATIC ERROR (β): The difference between the average of the parent population (μ) and the true value.

SYSTEMATIC STANDARD UNCERTAINTY ($b_{\bar{X}}$): A value that quantifies the dispersion of a systematic error associated with the mean.

TOTAL ERROR (δ) The true, unknown difference between the assigned value of a parameter or test result and the true value.

TRACEABILITY: The documentation of a measuring device's calibration through a chain of calibrations to a primary standard. Traceability does not guarantee accuracy but helps to assure it by the discipline of a documented hierarchy.

TRANSFER STANDARD: A laboratory instrument which is used to calibrate working standards and which is periodically calibrated against the laboratory standard.

TRUE VALUE: The unknown actual value of the parameter being measured.

TYPE A UNCERTAINTY: Uncertainties are classified as Type A when data is used to calculate a standard deviation for use in estimating the uncertainty (used in Reference 2.1.2.3).

TYPE B UNCERTAINTY: Uncertainties are classified as Type B when data is not used to calculate a standard deviation, requiring the uncertainty to be estimated by other methods (used in Reference 2.1.2.3).

UNCERTAINTY INTERVAL: The interval expected to contain the true value with a defined probability: $\pm U$ for symmetrical systematic uncertainty intervals and $+U^+$ and $-U^-$ for nonsymmetrical systematic uncertainty intervals.

STANDARD UNCERTAINTY: The estimate of the expected error limit defined as $u = \sqrt{b^2 + s^2}$.

WORKING STANDARD: An instrument which is calibrated in a laboratory against an interlab or transfer standard and is used as a standard in calibrating measuring instruments.

4. MEASUREMENT UNCERTAINTY METHODOLOGY

The methodology for determining the uncertainty of in-flight thrust must be structured to combine statistical and engineering concepts in a manner which can be applied to each step in the assessment procedure. This section addresses each of these steps and formulates the appropriate methodology. The result is an easily interpreted, single value estimate of thrust uncertainty.

The essential steps in the procedure for determining thrust uncertainty are:

- a. Identify all elemental error sources for each measurement in the ground test facility and in-flight related to determining thrust.
- b. Preliminarily classify elemental error sources as systematic or random.
- c. Estimate the magnitude of each elemental uncertainty.
- d. For each flight test measurement process, make the final systematic or random classification.
- e. Combine elemental uncertainties to define measurement systematic and random uncertainty components.
- f. Propagate measurement uncertainty components of calculated thrust uncertainty.
- g. Calculate thrust uncertainty from uncertainty components.
- h. Report results.

The following section defines measurement error and its components, identifies the sources of the errors, indicates how to classify the errors and provides guidance in estimating the error components. Combining the uncertainty for the elemental error sources to define measurement uncertainty components (systematic and random) and how to propagate these uncertainty components to net thrust uncertainty is also discussed. Section 4.2 introduces the concept of measurement uncertainty obtained by combining the components of measurement uncertainties. The uncertainty interval and its interpretation are explained for a basic measurement. Guidelines are provided for validating the estimated measurement uncertainties. The final section provides guidelines for reporting the uncertainty components of the basic measurements and the uncertainty intervals of both the basic measurements and calculated thrust. Definitions are presented in Section 3.

4.1 Measurement Error

All quantities, ranging from basic measurements to calculated performance, have error. This error is the difference between the measured or calculated observable value and the true, unknown value, as illustrated in Figure 2.

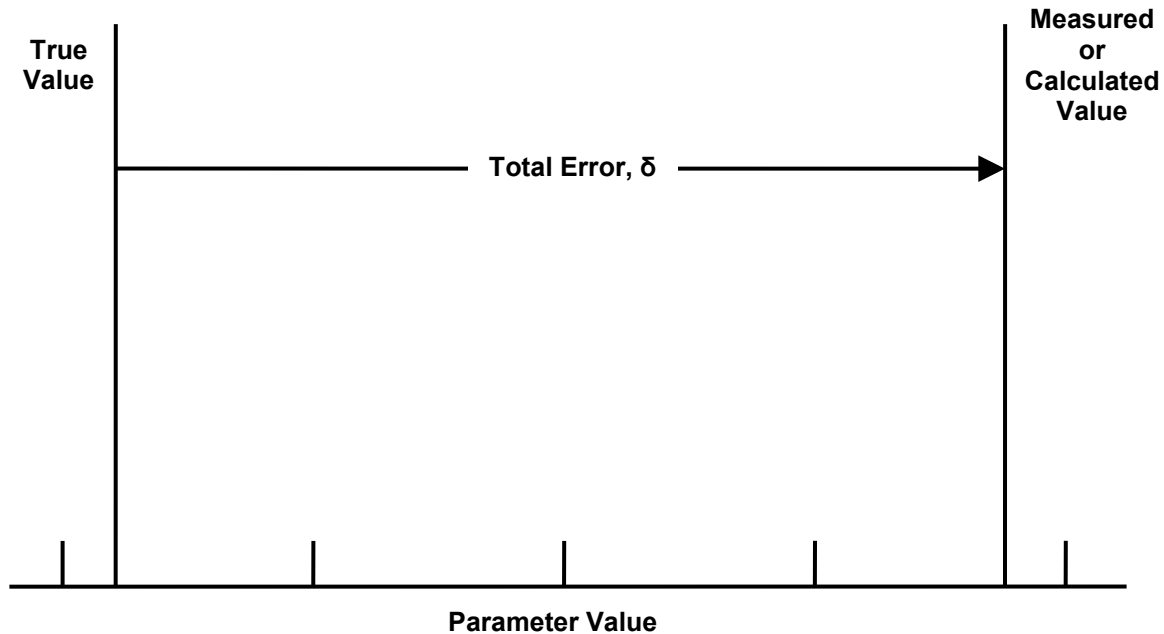


FIGURE 2 - MEASUREMENT ERROR, δ

4.1.1 Error Classification

Measurement errors can be classified as one of two components: a random error or a systematic error. Total error is the sum of a systematic error (β) and a random error (ϵ_j) (see Figure).

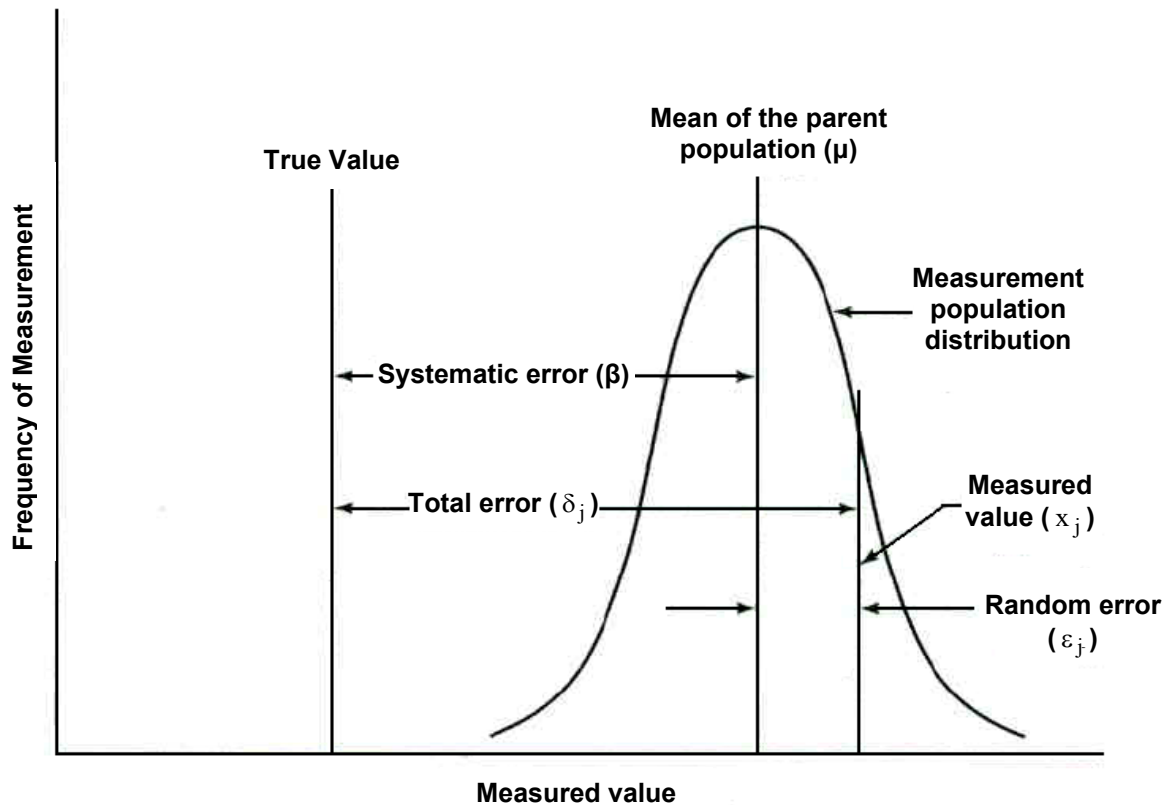


FIGURE 3 - ERROR CLASSIFICATION

A random error is one which can be estimated by statistical analysis of repeated measurements, while systematic error estimation is at least partially dependent on non-statistical methods (Reference 2.1.4.3).

An accurate measurement here is one that has small error in both random and systematic components, as is shown in Figure 2a.

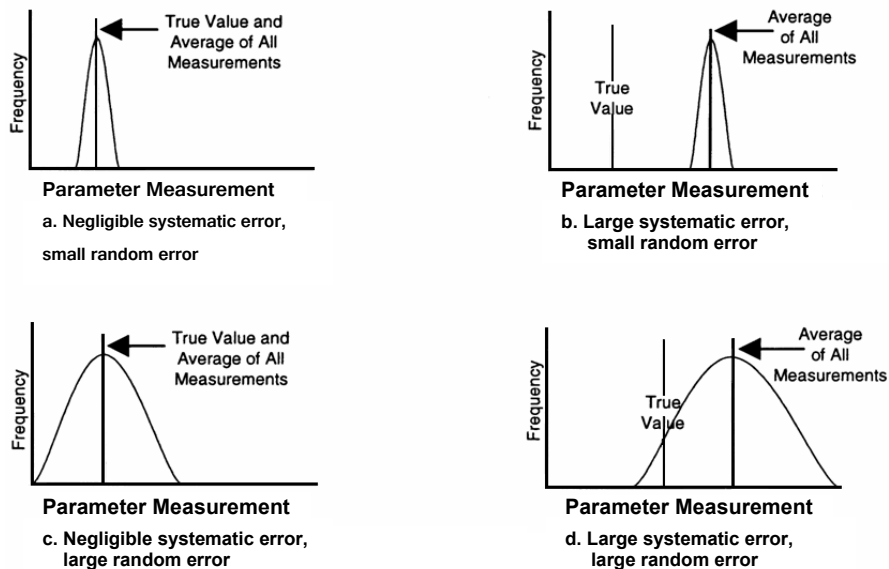


FIGURE 4 - MEASUREMENT ERROR COMPONENTS (SYSTEMATIC AND RANDOM)

RANDOM ERROR

Random errors are encountered in repeated measurements and are the differences between the observed values (each value x_j has its own random error ε_j) and the mean value of the parent population (μ). Repeated measurements at steady-state (constant) conditions are not expected to produce precisely the same data. There are numerous, small effects which may cause measurement variations. These variations tend to spread about an average value in the fashion of a normal distribution curve, Figure 5. The curve is characterized by the population standard deviation, σ .

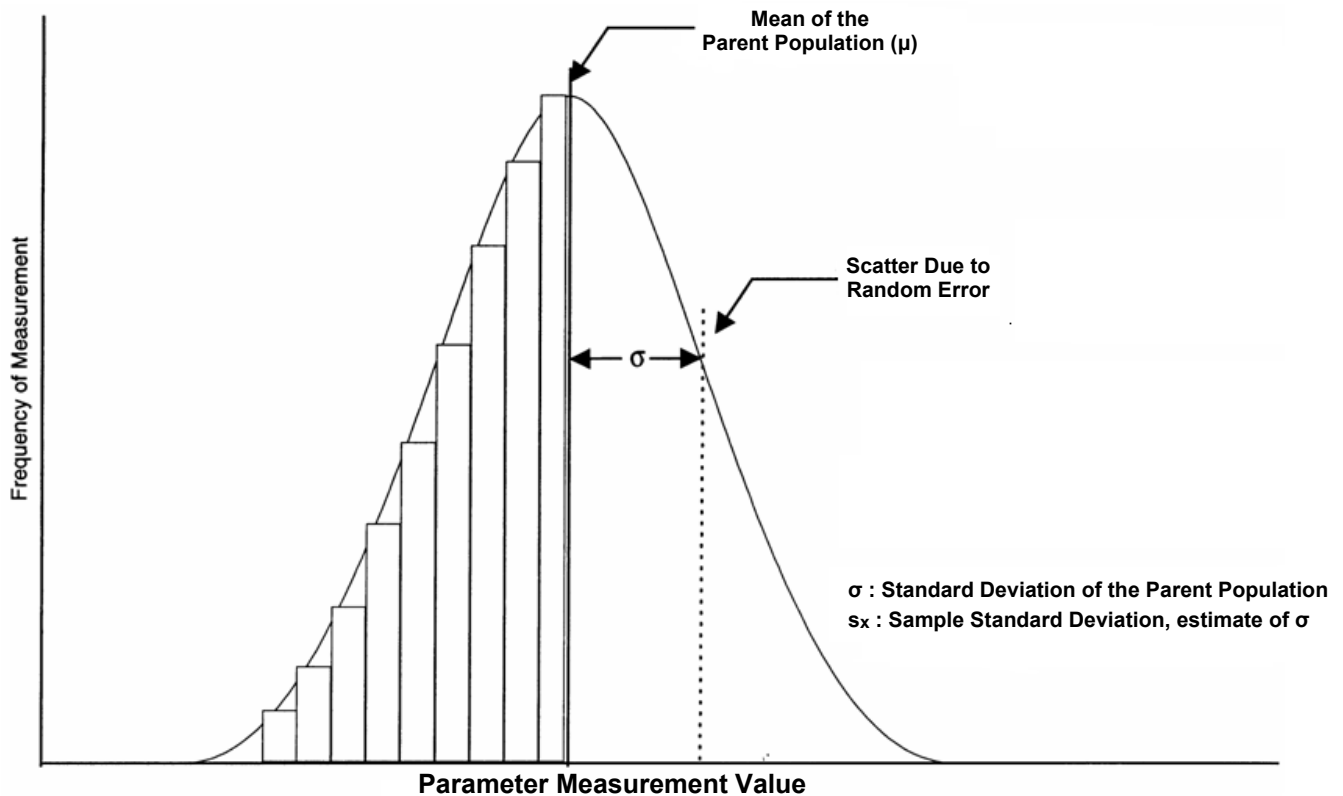


FIGURE 5 - NORMAL DISTRIBUTION CURVE RESULTING FROM TYPICAL STEADY-STATE DATA MEASUREMENT

The sample standard deviation for the variable X_i , s_{x_i} , is the computed estimate of the population standard deviation, σ , and is calculated as follows:

$$\text{Sample Standard Deviation} = s_{x_i} = \sqrt{\frac{\sum_{j=1}^{N_i} (x_{ij} - \bar{X}_i)^2}{N_i - 1}} \quad (\text{Eq. 1})$$

where:

N_i = Number of measurements for the variable X_i

x_{ij} = Individual measurements of the variable X_i

$$\bar{X}_i = \text{Sample mean, average value of } X_{ij}, \text{ calculated as } \bar{X}_i = \frac{\sum_{j=1}^{N_i} X_{ij}}{N_i} \text{ (Estimate of the mean of the parent population } \mu \text{)}$$

The goodness of the sample standard deviation as an estimate of the true population standard deviation is dependent on the number of measurements (observations) used in the initial estimation process. A large sample, thirty or more, is usually available for this estimation.

SYSTEMATIC ERROR

An uncertainty analysis assumes a carefully controlled measurement process within which every known calibration correction has been made. Because the calibration corrections are not ideal, some small systematic errors will remain.

The (unknown) systematic error β remains unchanged for the duration of a test. In repeated measurements at steady-state (constant) test conditions each measurement has the same systematic error magnitude, therefore the average measurement has the same systematic error as any single measurement. The relationship between systematic and random errors and the true value of the measured quantity is depicted in Figure 6. The systematic error β is the difference between the parent population mean μ , which can be approximated by the sample average $\bar{X}$ if the sample is large, and the true value.

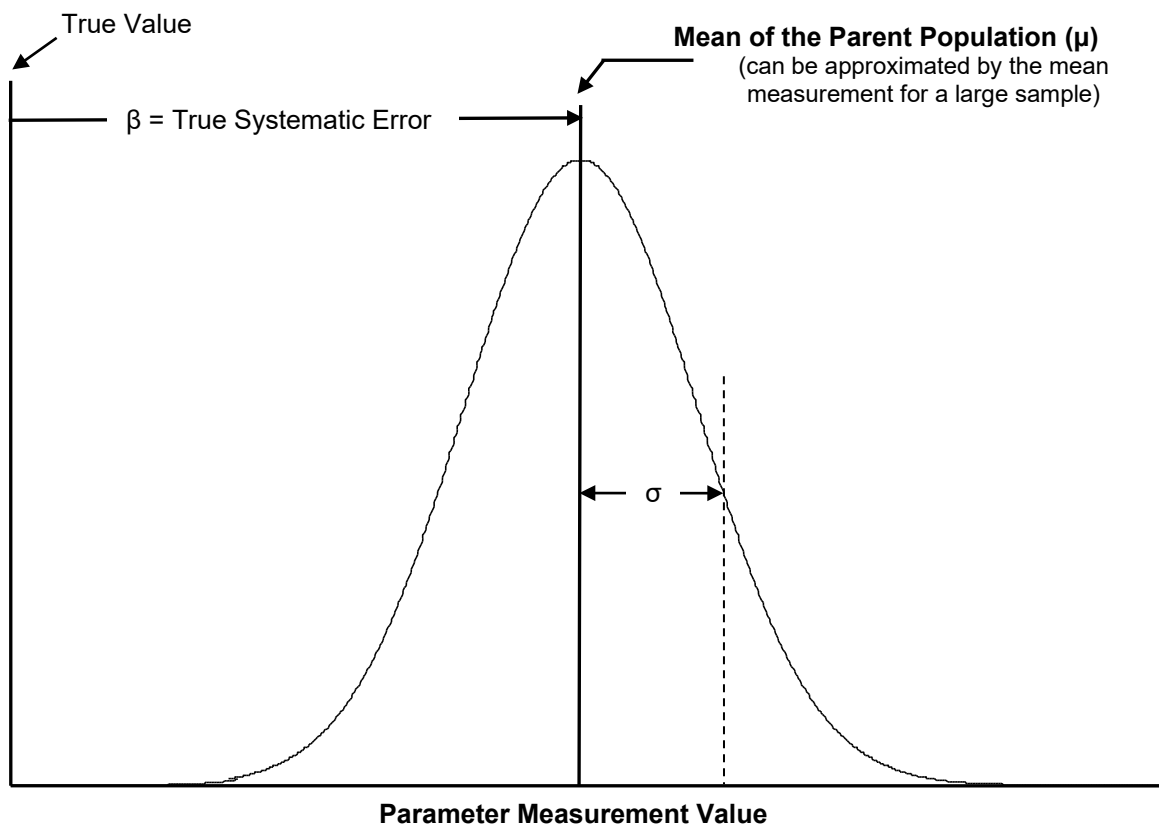


FIGURE 6 - SYSTEMATIC ERROR

4.1.2 Sources of Measurement Errors

A measurement system has many potential sources of error. The errors for each source are referred to as elemental errors. It is preferable to estimate the uncertainty for each elemental error source; however, it is permissible to estimate the uncertainty for a combination of elemental error sources. For convenience, these elemental error sources for any measurement can be divided into five categories indicated by the subscript p :

p Category

1. Calibration Errors
2. Test Article and / or Instrumentation Installation Errors
3. Data Acquisition Errors
4. Data Reduction Errors
5. Errors of Method

Elemental error sources associated with each of the above five categories are discussed in the following paragraphs.

CALIBRATION ERRORS (Category $p=1$)

The measurement uncertainty analysis assumes a well controlled measurement process in which there are no gross mistakes or errors. It also assumes that reasonable calibration corrections have been applied. Calibrations are performed to improve the test accuracy and to provide test measurement traceability to a national standards laboratory. By applying the calibration corrections, some systematic errors are reduced, but in the process some other errors may be introduced. The systematic error of the uncalibrated test instrument is exchanged for the systematic error of the standard or master meter. In addition, the random error of the calibration process will contribute to the calibration error. In some cases, the random error of the calibration process will contribute to the random error of the test result, and in others, it will be converted (fossilized) into systematic error. For a fuller discussion, see Appendix B.

Traceability is established and maintained through a calibration hierarchy. Each comparison in the hierarchy constitutes an elemental error source, k . Figure 7 is an example instrument calibration hierarchy.

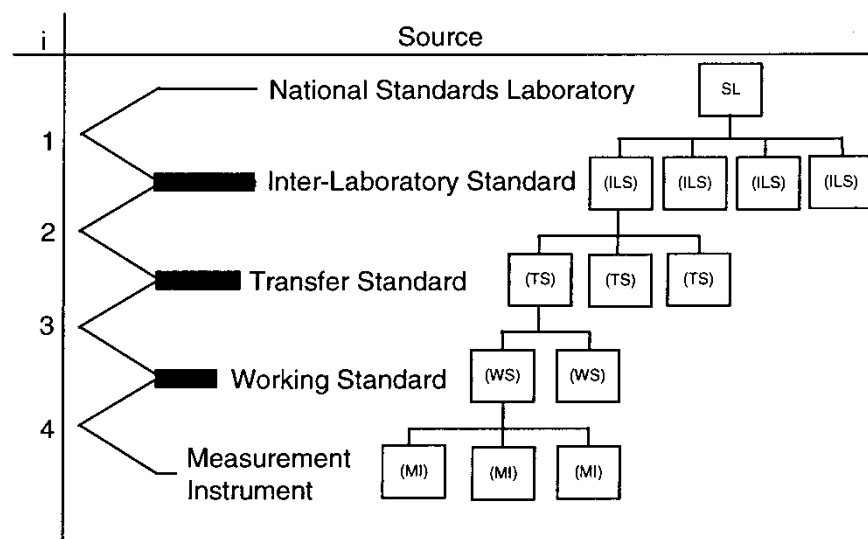


FIGURE 7 - MEASUREMENT CALIBRATION HIERARCHY

Each comparison in the calibration hierarchy has elemental errors associated with it. Estimates of these elemental errors provide standard deviations for the random and systematic uncertainties in each level of the hierarchy, as listed in Table 1.

TABLE 1 - CALIBRATION HIERARCHY ERROR SOURCES

Calibration (p=1)			
k	Error source	Systematic Uncertainty	Random Uncertainty
1	NSL - ILS	b_{11}	s_{11}
2	ILS - TS	b_{21}	s_{21}
3	TS - WS	b_{31}	s_{31}
4	WS - MI	b_{41}	s_{41}

TEST ARTICLE AND / OR INSTRUMENTATION INSTALLATION (Category p=2)

Interactions between (a) test instrumentation (for instance a probe) and test article (for instance the air inlet of an engine), or between (b) the test article and the test facility can be encountered.

The case (a) can be illustrated by the measurement of the airflow in the air intake of an engine. The presence of a Pitot tube may cause intrusive disturbance effects, may affect the measurement and the calculated airflow. Also, heat transfers if any (conduction, convection and radiation) or swirl in the vicinity of the sensor may affect the measurement.

The case (b) can be illustrated with the testing of an engine in an altitude facility. If the rate of flow of secondary cooling air is changed during a test, to cool the test cell, there may be a change in thrust measurement. Such a change in measured thrust represents an error in this category.

Facility limitations for testing may require extrapolations to other conditions, which introduces a category 2 error.

Typical test article and / or instrumentation error sources are listed in Table 2.

TABLE 2 - TEST ARTICLE AND / OR INSTRUMENTATION INSTALLATION ERROR SOURCES

Interactions (p=2)			
k	Error source	Systematic Uncertainty	Random Uncertainty
1	Heat transfer	b_{12}	s_{12}
2	Installation effect	b_{22}	s_{22}

DATA ACQUISITION ERRORS (Category p=3)

Table 3 illustrates some of the error sources, k , associated with the data acquisition system illustrated in Figure. Data for this example are acquired by measuring the electrical output resulting from pressure applied to a strain-gage-type pressure measurement instrument. Other error sources, such as electrical simulation, probe errors, and environmental effects are also present.

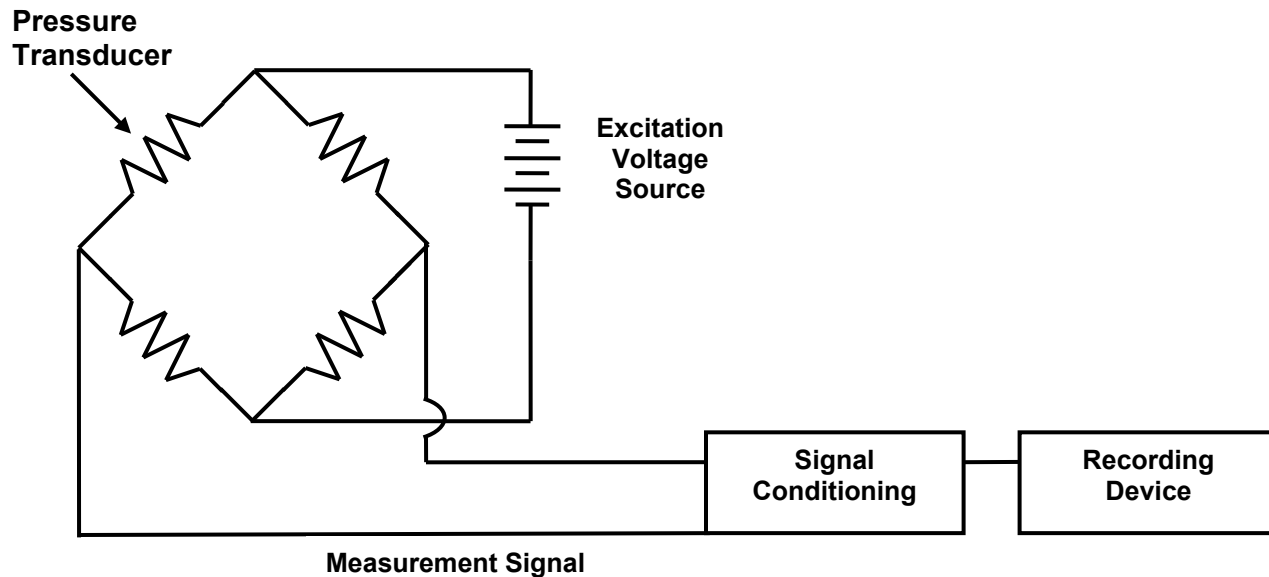


FIGURE 8 - EXAMPLE DATA ACQUISITION SYSTEM

TABLE 3 - TYPICAL DATA ACQUISITION ERROR SOURCES

Data Acquisition (p=3)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	Excitation Voltage	b_{13}	s_{13}
2	Electrical Simulation	b_{23}	s_{23}
3	Signal Conditioning	b_{33}	s_{33}
4	Recording Device	b_{43}	s_{43}
5	Pressure Transducer	b_{53}	s_{53}
6	Probe Errors	b_{63}	s_{63}
7	Environmental Effects	b_{73}	s_{73}

DATA REDUCTION ERRORS (Category p=4)

Computers operate on raw data to produce output in engineering units. Typical data reduction error sources are listed in Table 4.

TABLE 4 - TYPICAL DATA REDUCTION ERROR SOURCES

Data Reduction (p=4)			
k	Error source	Systematic Uncertainty	Random Uncertainty
1	Curve Fit	b_{14}	s_{14}
2	Computer Resolution	b_{24}	s_{24}
3	Assumptions or constants contained in calculation routines	b_{34}	s_{34}
4	Using approximating engineering relationships or violating their assumptions	b_{44}	s_{44}
5	Using an empirically derived correlation (fluid properties)	b_{54}	s_{54}

ERRORS OF METHOD (Category p=5)

There are other sources of errors in the measurement process not captured in categories 1 to 4. Such errors may be categorized as errors of method, or real effects. Typical errors of method are listed in Table 5.

TABLE 5 - TYPICAL ERRORS OF METHOD SOURCES

Non-Instrumentation (p=5)			
k	Error source	Systematic Uncertainty	Random Uncertainty
1	Spatial Nonuniformity	b_{15}	s_{15}
2	Temporal Nonuniformity	b_{25}	s_{25}
3	Test Procedures (e.g. stabilization time)	b_{35}	s_{35}

4.1.3 Estimating Measurement Uncertainty

The following paragraphs outline approaches and considerations for estimating and classifying the random and systematic error components.

Nonetheless the best method to minimize the total uncertainty due to the effects of these different sources is to carry out overall system calibrations (end-to-end method for example) when possible.

RANDOM STANDARD UNCERTAINTY ESTIMATION

When sufficient data are available, the random standard uncertainty is taken to be the sample standard deviation s_x . For the data set to be sufficient, a large sample (over 30) should be taken over a period of time during which all random error sources are active.

In the absence of adequate data, engineering judgment can be used to estimate the random standard uncertainty.

The standard deviation, s_{X_i} is calculated from test data. When large samples are used in estimating the standard deviation, the interval defined by plus and minus the sample standard deviation around any measurement ($X_{ij} \pm s_{X_i}$) will contain the population mean 68% of the time if the random error population is normally distributed. In other words if they are normally distributed the random errors ε_{ij} will be contained 68% of the time in the interval $[-s_{X_i}, +s_{X_i}]$.

For repeated or simultaneous observations, the standard deviation can often be reduced by treating the mean of the multiple independent observations as a single measurement. The distribution of the means will have a smaller standard deviation than the sample standard deviation, as indicated in Figure 9 and defined by Equation 2. Indeed if several independent samples of size N_i are taken from a large, normally distributed population, and the mean of every sample is calculated, then the standard deviation of the distribution of the means will be the standard deviation of the individual measurements divided by the square root of the size of the samples N_i .

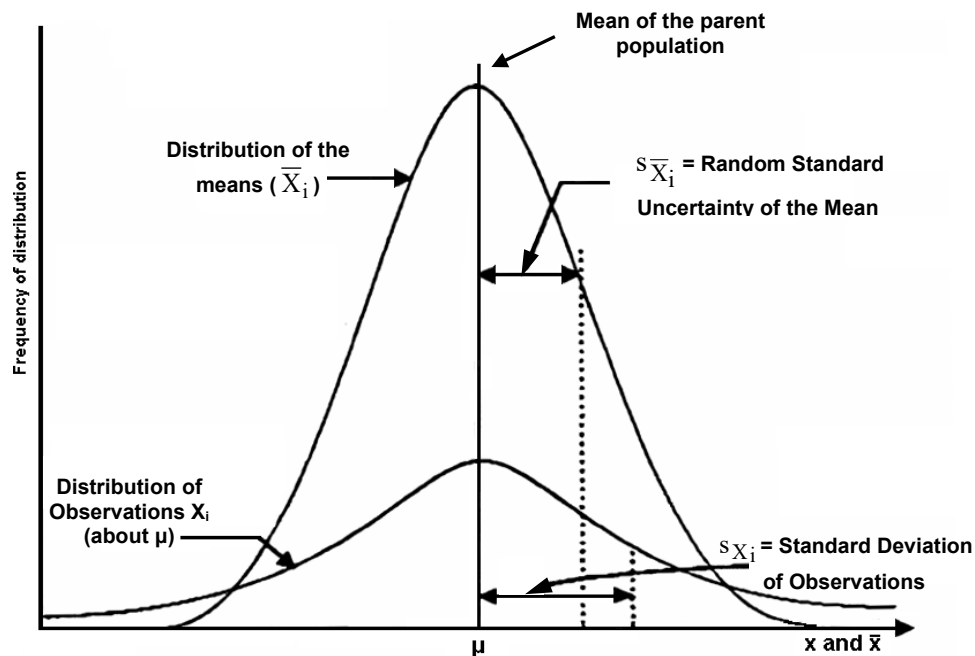


FIGURE 9 - DISTRIBUTION OF X_i AND $\bar{X}_i$

$$\text{Random standard uncertainty of the sample mean} = \text{Standard deviation of the sample mean} = s_{\bar{X}_i} = \frac{s_{X_i}}{\sqrt{N_i}} \quad (\text{Eq. 2})$$

The random standard uncertainty of the sample mean $s_{\bar{X}_i}$ quantifies the error when using the sample mean $\bar{X}_i$ to estimate the population mean μ .

When large samples (more than thirty measurements) are used, the interval defined by plus and minus the standard deviation of the sample means around the mean of any sample ($\bar{X}_i \pm s_{\bar{X}_i}$) will contain the population mean 68% of the time if the random error population is normally distributed. If the systematic error is equal to zero then the population mean is equal to the true value, therefore this interval will contain the true value 68% of the time.

Since the standard deviation of the sample mean is lower than the standard deviation of the sample, the sample mean is a better estimate of the mean of the parent population. Therefore it is preferable to use a measurement mean in order to reduce the random uncertainty and the overall uncertainty of the measurement.

SYSTEMATIC STANDARD UNCERTAINTY ESTIMATION

Information from special tests and engineering judgment is the principal source for estimating the systematic standard uncertainty b_{X_i} . The systematic uncertainty must account for small known fixed errors, which are too difficult to correct, as well as for small unknown fixed errors. These systematic errors will include fossilized random error from the calibration process, as discussed in Appendix B. In a well controlled measurement process, the assumption is that there are no large unknown systematic errors. To ensure that a controlled measurement process exists, all measurements should be monitored with statistical quality control charts (Reference 2.1.4.4).

The magnitude of the systematic error cannot be determined unless the measurements can be compared with the true value, which is not feasible. Therefore the systematic error is assumed to be a single value drawn from a population of possible error values. The standard deviation of this population of possible errors β , which quantifies the dispersion of the values, is the systematic standard uncertainty b_{X_i} . The systematic error remains constant for every measurement, and therefore the systematic standard uncertainty of the data sample is the same as the systematic standard uncertainty of the mean: $b_{\bar{X}_i} = b_{X_i}$.

Since the systematic error β remains constant and unknown during successive measurements, the evaluation of the standard deviation b can hardly be inferred from experiments. Engineering judgment and applicable test information can therefore be used to estimate a symmetrical interval $[-B, +B]$ which is expected to contain the systematic error β with 95% confidence. The population of possible β values is assumed to be normally distributed (unless information indicates the contrary), and so the systematic standard uncertainty can be estimated as: $b_{\bar{X}_i} = \frac{B}{2}$, as shown in Figure 10. If the error distribution is not normal the factor used to convert the 95 % confidence level interval estimate B into the systematic standard uncertainty b may be different. For example for a rectangular distribution on the interval $[-a, +a]$ the relation is

$$b_{\bar{X}_i} = \frac{a}{\sqrt{3}} = \frac{B}{0.95 \times \sqrt{3}}.$$

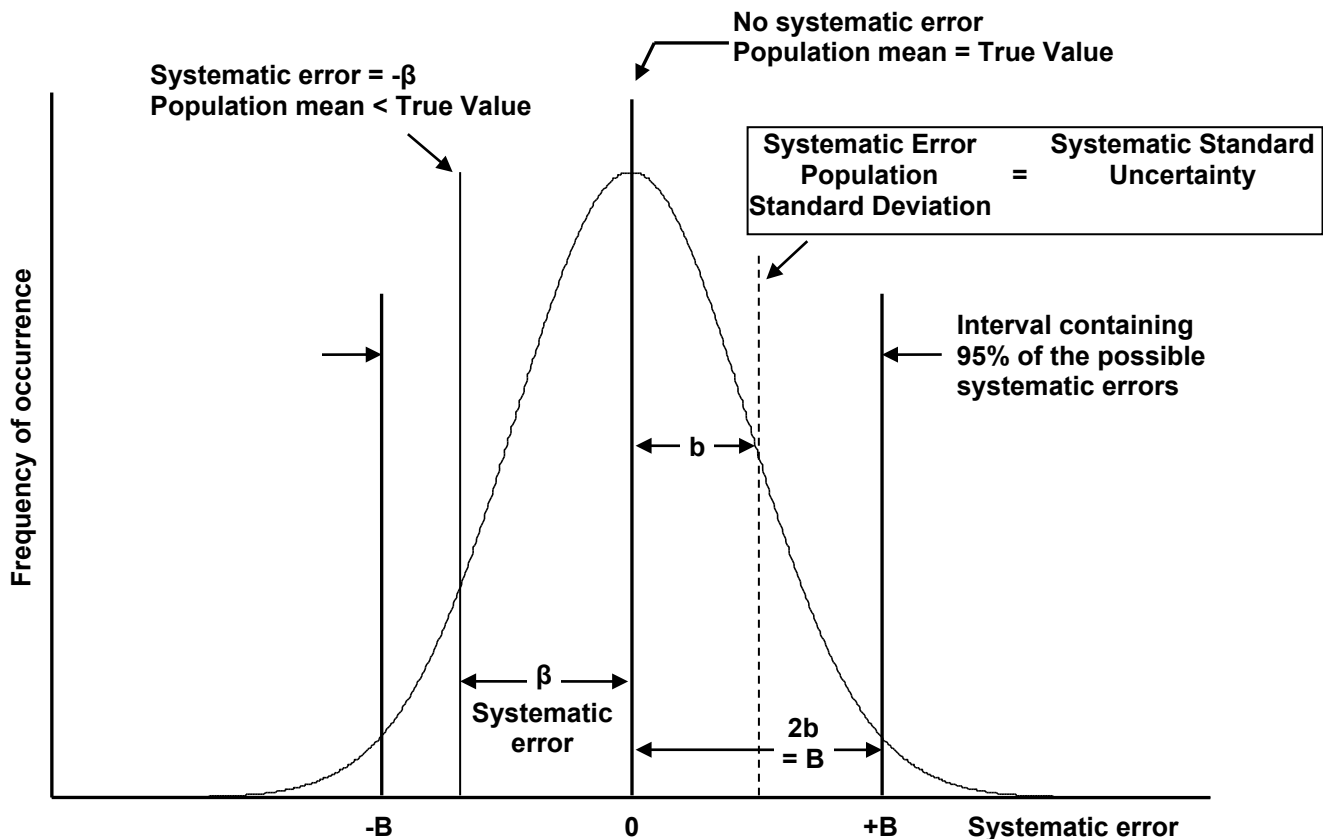


FIGURE 10 - SYSTEMATIC STANDARD UNCERTAINTY ESTIMATION (WITH A NORMAL DISTRIBUTION)

The unknown systematic uncertainty intervals ($\pm B$) are assumed to be symmetrically distributed about the true value. However, sometimes the physics of the problem indicates that the systematic uncertainty intervals are nonsymmetric : in these cases there are a lower and upper value B^- and B^+ for the nonsymmetrical interval (see appendix F). The intent of the systematic uncertainty interval estimate is to identify the limits of the actual systematic error of a given measurement process.

4.1.4 Combining Measurement Uncertainties

Combination of the elemental uncertainties is essential for modeling the basic measurement uncertainty, for propagating measurement uncertainties to performance parameters, and for uncertainty reporting and validation. Uncertainties associated with all five categories of error sources are combined to define the basic measurement uncertainty.

COMBINING RANDOM UNCERTAINTIES

The random standard uncertainty (s), can be either calculated directly using Equations 1 and 2 or inferred from the root-sum-square of the elemental random uncertainties from all sources.

$$s = \sqrt{\sum_p \sum_k s_{kp}^2} \quad (\text{Eq. 3})$$

where p defines error categories: (1) calibration, (2) test article / instrumentation installation, (3) data acquisition, (4) data reduction, and (5) errors of method and k defines elemental error sources within a category.

For example, the random uncertainty for the calibration category ($p=1$) is the root-sum-square of the elemental random uncertainties (see Table).

$$s_1 = s_{\text{cal}} = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2} \quad (\text{Eq. 4})$$

In like manner, the random uncertainties for the other categories are defined by the root-sum-square of their individual elemental random uncertainties.

The basic measurement random uncertainty is then the root-sum-square of the random uncertainties of the five categories.

$$s = \sqrt{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2}$$

$$s = \sqrt{\sum_k s_{k1}^2 + \sum_k s_{k2}^2 + \sum_k s_{k3}^2 + \sum_k s_{k4}^2 + \sum_k s_{k5}^2} = \sqrt{\sum_p \sum_k s_{kp}^2} \quad (\text{Eq. 5})$$

COMBINING SYSTEMATIC UNCERTAINTIES

Most measurement processes will contain a large number of systematic error sources. The systematic standard uncertainty used herein is the root-sum-square of the elemental systematic uncertainties of all categories (Reference 2.1.4.5).

$$b = \sqrt{\sum_p \sum_k b_{kp}^2} \quad (\text{Eq. 6})$$

The individual category error limits; calibration (b_1), test article / instrumentation errors (b_2), data acquisition (b_3), data reduction (b_4), and errors of method (b_5), are defined by the root-sum-square of their individual elemental systematic uncertainties.

The systematic uncertainty for the basic measurement process is then